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ON THE LIGHT-VARIATIONS OF ASTEROIDS AND SATELLITES

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§ 1. It has often been suggested that the light-changes of those variable stars which are regularly periodic may be due to the existence of spots on their surfaces, which are hidden or brought into view as the star rotates about its axis. The same explanation has been given more plausibly for the variability shown by certain satellites and asteroids, for in this case the only rival hypothesis is that which ascribes the light-changes to the departure of the body from a spherical form.

It may therefore be worth while to discuss some results of these hypotheses, and consider (1) what is the character of the light-curve produced by the rotation of an arbitrarily spotted body, and (2) how far it is possible to reason backward from such a light-curve to the spots which produce it.

§ 2. The simplest case of a spotted body is a self-luminous sphere without absorbing atmosphere. In this case the apparent brightness of any part of the surface is independent of its inclination to the line of sight, so that a sphere of constant intrinsic brightness appears as a uniformly illuminated disk.

The case of a body shining by reflected light is greatly complicated by the effects of phase; but if we confine ourselves to planets close to opposition, the appearance of the full Moon shows that

the hypothesis of a uniformly luminous disk is not very far from representing the facts, at least for a body devoid of atmosphere. Bodies with a dense atmosphere, like the Sun and *Jupiter*, are likely to appear darker near the limb, and our theory must be modified to include them.

If B denotes the intrinsic brightness of an element $d\sigma$ of the surface of the sphere, γ the angle between the (outward) normal to the sphere at this point and the line of sight, and R the distance of the element from the observer, the apparent area of the element will be $\frac{d\sigma \cos\gamma}{R^2}$, and the light which the observer receives from it will be

$$dL = \frac{B \cos\gamma\, d\sigma}{R^2}, \tag{1}$$

provided that there is no absorbing atmosphere.

If there is such an atmosphere, its effective thickness will increase as we approach the limb, and the percentage of transmitted light will decrease. This percentage will be a function of γ alone. If we denote it by A, we shall have for the light received from the element $d\sigma$

$$dL = \frac{AB \cos\gamma\, d\sigma}{R^2}. \tag{1a}$$

This expression may also be made to include the effects of any brightening or darkening of the limb due to peculiarities of the law of diffuse reflection (which is known to vary from planet to planet).

To find the total light L which the observer receives from the sphere, we must integrate the expressions (1) or (1a) over the visible portion S of the sphere (that is, the region in which $\cos\gamma$ is positive), and thus we obtain the equations

$$L = \int_S \frac{B \cos\gamma\, d\sigma}{R^2}, \tag{2}$$

which holds good when there is no absorption; and

$$L = \int_S \frac{AB \cos\gamma\, d\sigma}{R^2}, \tag{2a}$$

which includes the effect of absorption.

These expressions are perfectly general, and hold good for any point outside the sphere.

We will now confine ourselves to the consideration of very distant points of observation for which the radius of the sphere is negligible in comparison with its distance. We may then without sensible error disregard the variations in the distance and direction of different parts of the sphere from the observer, so that in (2) we may treat R as constant and equal to the distance of the sphere's center, while γ becomes the angle at the center between lines drawn to the observer and to any given element. Our equations then become

$$L=\frac{1}{R^2}\int_S B\cos\gamma\, d\sigma\ , \tag{3}$$

or, if we include the effects of an atmosphere,

$$L=\frac{1}{R^2}\int_S A\,B\cos\gamma\, d\sigma\ , \tag{3a}$$

where the integrals extend over the visible hemisphere, for which $\cos\gamma$ is positive.

§ 3. Let us now choose any system of polar co-ordinates ρ, θ, ϕ, fixed with reference to the sphere, and let r denote the radius of the sphere. The brightness B at any point of the sphere's surface will be a function of its co-ordinates θ, ϕ. We may show this explicitly by writing it in the form $B\ (\theta,\ \phi)$.

If we denote the co-ordinates of the observer (referred to this system) by R, θ_0, ϕ_0, the light which he receives from the sphere will depend only on these three quantities. From (3) we see that it may be expressed in the form $L=\frac{1}{R^2}\,j\ (\theta_0,\ \phi_0)$.

Since we have

$$d\sigma=r^2\sin\theta\, d\theta\, d\phi\ ,$$

and since from the definition of γ it follows that

$$\cos\gamma=\cos\theta\cos\theta_0+\sin\theta\sin\theta_0\cos\left(\phi-\phi_0\right)\ ,$$

it is clear that the equations (3) or (3a) enable us to determine the value of $j\,(\theta_0,\ \phi_0)$ for any particular values of θ_0 and ϕ_0, when the function $B\ (\theta,\ \phi)$ is known. We wish, however, to obtain general relations between the functions $j\,(\theta_0,\ \phi_0)$ and $B\ (\theta,\ \phi)$. Since both these functions are given by their values over the surface of a sphere, it is natural to seek to express them in terms of surface spherical harmonics.

From physical considerations $B(\theta, \phi)$ must always be finite and positive (or zero). It will be expansible in a spherical harmonic series if it satisfies Dirichlet's conditions; that is, if it has only a finite number of maxima, minima, and points or lines of discontinuity. These conditions impose no limitation of practical importance upon our function, and we shall hereafter suppose that they are satisfied, so that there exists an expansion of the form

$$B(\theta, \phi) = Y_0 + Y_1(\theta, \phi) + Y_2(\theta, \phi) \ldots . + Y_n(\theta, \phi) , + \ldots . \quad (4)$$

where Y_n is a surface spherical harmonic of the nth order.

§ 4. It will be convenient at this point to recall certain properties of such expansions. The spherical harmonic Y_n is the sum of $2n+1$ terms involving Legendre's functions of the form

$$Y_n(\theta, \phi) = A_{n,0} P_n(\cos\theta) + \sum_{m=1}^{m=n} (A_{n,m} \cos m\phi + B_{n,m} \sin m\phi) P_n^m (\cos\theta) . \quad (5)$$

The numerical coefficients are given by the definite integrals

$$\left.\begin{aligned} A_{n,0} &= \frac{2n+1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} B(\theta, \phi) P_n(\cos\theta) \sin\theta \, d\theta \\ \begin{matrix} A_{n,m} \\ B_{n,m} \end{matrix} &= \frac{2n+1}{2\pi} \int_0^{2\pi} d\phi \int_0^{\pi} B(\theta, \phi) P_n^m(\cos\theta) \begin{matrix} \cos \\ \sin \end{matrix} m\phi \sin\theta \, d\theta \end{aligned}\right\} \quad (6)$$

Y_0 is a constant, independent of θ and ϕ.

The series (4) is thus uniquely determined. When B (θ,ϕ) satisfies Dirichlet's conditions, it is convergent. Its sum, however, will in general present discontinuities, like that of a Fourier series. But if we multiply each term of such a series by a quantity which is numerically less than $\frac{A}{n}$, where A is a constant, the series so obtained will have a sum which is continuous for all values of θ and ϕ.

If the multiplier is always numerically less than $\frac{A}{n^2}$, the sum of the new series will be continuous, and will possess continuous derivatives of the first order for all values of θ and ϕ.

If we take any two antipodal points, at the extremities of a diameter of the sphere, the values of Y_0, ... Y_2 and all the even harmonics will be equal at the two points, while those of the odd harmonics Y_1, Y_3 . . . . will be numerically equal, but of opposite sign.

Finally, we may express $Y_n(\theta, \phi)$ as a function of the angle γ, already defined, by introducing another angle λ, which may be defined geometrically as the position angle of any point on the apparent disk of the sphere, relative to its center, as seen by the observer, r, γ, and λ will then form a system of polar co-ordinates, and there will exist an expansion of the form,

$$Y_n(\theta, \phi) = a_0 P_n(\cos\gamma) + \sum_1^n (a_m \cos m\lambda + b_m \sin m\lambda) P_n^m(\cos\gamma) \; . \quad (7)$$

If in this expression we set $\gamma = 0$, then θ and ϕ will have the values θ_0 and ϕ_0, while $P_n(\cos\gamma) = 1$, and $P_n^m(\cos\gamma) = 0$ for all values of m, and so we must have

$$a_0 = Y_n(\theta_0, \phi_0) \; . \quad (8)$$

§ 5. We are now in a position to evaluate the function $L(\theta_0, \phi_0)$. Substituting in (3) $d\sigma = r^2 \sin\gamma \, d\gamma \, d\lambda$, and considering only the term $Y_n(\theta, \phi)$ in B, we find for the corresponding term in L,

$$\frac{r^2}{R^2} \int_0^{2\pi} d\lambda \int_0^{\frac{\pi}{2}} \left\{ Y_n(\theta_0, \phi_0) P_n(\cos\gamma) + \sum_1^n (a_m \cos m\lambda + b_m \sin m\lambda) P_n^m \cos\gamma \right\} \cos\gamma \sin\gamma \, d\gamma \, ,$$

where the limits correspond to the condition that the integral shall be taken over the visible hemisphere.

Integrating with respect to λ, all the terms except the first vanish and we obtain

$$\frac{2\pi r^2}{R^2} Y_n(\theta_0, \phi_0) \int_0^{\frac{\pi}{2}} P_n(\cos\gamma) \cos\gamma \sin\gamma \, d\gamma \; . \quad (9)$$

Setting $\cos\gamma = x$, the integral becomes

$$\int_0^1 P_n(x) x \, dx \; .$$

This is a known integral. Its value is $\frac{1}{2}$ when $n=0$; $\frac{1}{3}$ when $n=1$; $\frac{1}{8}$ when $n=2$; zero when n is odd and greater than 1; and $\frac{1, 3, 5 \ldots\ldots (n-3)}{2, 4, 6 \ldots\ldots (n+2)} (-1)^{\frac{n}{2}+1}$ when n is even and greater than 2. Hence, collecting the terms of L, we obtain

$$L(\theta_0, \phi_0)=\frac{\pi r^2}{R^2}\left(Y_0+\frac{2}{3}Y_1(\theta_0, \phi_0)+\frac{1}{4}Y_2(\theta_0, \phi_0)-\frac{1}{24}Y_4(\theta_0, \phi_0)\right.$$
$$\left.+\frac{1}{64}Y_6(\theta_0, \phi_0)-\frac{1}{128}Y_8(\theta_0, \phi_0)\ldots\ldots\right) \quad (10)$$

Since the spherical harmonics Y are completely determined when the surface-brightness $B(\theta, \phi)$ is known, the equation (9) contains the complete solution of our problem. We may deduce from it several important properties of this solution:

(1) We may write the coefficient of Y_n in the form

$$\pm\frac{\pi r^2}{R^2}\cdot\frac{3, 5, 7\ldots\ldots(n-3)}{4, 6, 8\ldots\ldots(n-2)}\cdot\frac{1}{n(n+2)},$$

which shows that it is always numerically less than $\frac{\pi r^2}{R^2 n^2}$. It follows that the series (9) is absolutely and uniformly convergent, and that the function L which it represents is continuous, as well as its first derivatives, for all values of θ_0 and ϕ_0. Hence the light-curve due to the rotation of the sphere about any axis must be continuous, and free from abrupt changes of direction, even though the distribution of brightness on its surface is discontinuous.

(2) Since the odd harmonics Y_3, Y_5, etc., do not appear in L, it is clear that a great variety of distributions of brightness on the sphere may give rise to the same light-curve.

(3) Let us now take the axis of rotation as the axis of our polar co-ordinates θ, ϕ. Then, as the sphere rotates (while the positions of its center and the observer remain the same), θ_0 will be constant, and equal to the inclination of the axis of rotation to the line of sight, while ϕ_0 will increase uniformly with the time. If ω is the angular velocity of rotation, we will have $\phi_0=\omega(t-t_0)$, t_0 being the time when the meridian $\phi=0$ crosses the center of the disk.

Introducing these into (9), we obtain the expression for the light received by the observer as a function of the time—that is, the equation of the light-curve.[1] Each of the spherical harmonics Y_n becomes a finite Fourier series in ϕ_0, whose coefficients are functions of θ_0, terminating with the terms in $\begin{matrix}\cos\\ \sin\end{matrix} n\phi_0$. Since the series (9)

[1] By "light-curve" throughout the present discussion is meant the curve which gives the light itself as a function of the time and not that which gives the corresponding stellar magnitude.

is absolutely convergent, we may rearrange it, and collect the terms involving like multiples of ϕ_o. We thus obtain the equation of the light-curve, in the form of the Fourier series

$$L=\frac{2\pi r^2}{R^2}\begin{pmatrix} C_o+C_1\cos\omega(t-t_o)+C_2\cos 2\omega(t-t_o)\ldots. \\ +D_1\sin\omega(t-t_o)+D_2\sin 2\omega(t-t_o)\ldots. \end{pmatrix}, \tag{11}$$

where the coefficients C and D are functions of θ_o of the form

$$C_o(\theta_o)=\frac{A_{1,o}}{3}\cos\theta_o+\sum_{n=o}^{n=\infty}a_{n,o}P_n(\cos\theta_o)$$

$$C_1(\theta_o)=\frac{A_{1,1}}{3}\sin\theta_o+\sum_{n=2}^{n=\infty}a_{n,1}P_n^1(\cos\theta_o)$$

$$C_2(\theta_o)=\sum_{n=2}^{n=\infty}a_{n,2}P_n^2(\cos\theta_o)\ , \tag{12}$$

and in general

$$C_m(\theta_o)=\sum_{n=m}^{n=\infty}a_{n,m}P_n^m(\cos\theta_o)\ ,$$

where the summations extend only over *even* values of n. The expressions for D_1, D_2, etc., are obtained by introducing the numerical constants $b_{n,m}$ in place of $a_{n,m}$.

Now, $P_n^m(\cos\theta_o)$ is an even or odd function of $\cos\theta_o$, according as $n-m$ is even or odd. It follows that $C_m(\theta_o)$ is an odd or even function of θ_o according as m is odd or even. Hence we have, except for C_o and C_1,

$$\begin{aligned} C_m(\theta_o)&=(-1)^m C_m(\pi-\theta_o) \\ D_m(\theta_o)&=(-1)^m D_m(\pi-\theta_o)\ . \end{aligned} \tag{13}$$

The corresponding relations for C_o and C_1 are

$$\left.\begin{aligned} C_o(\theta_o)-C_o(\pi-\theta_o)&=\frac{2}{3}A_{1,o}\cos\theta_o \\ C_1(\theta_o)+C_1(\pi-\theta_o)&=\frac{2}{3}A_{1,1}\sin\theta_o\ . \\ D_1(\theta_o)+D_1(\pi-\theta_o)&=\frac{2}{3}B_{1,1}\sin\theta_o \end{aligned}\right\} \tag{14}$$

When $\theta_o=\frac{\pi}{2}$ (that is, when the observer is in the equatorial plane), all the odd coefficients except C_1 and D_1 must vanish, and all the even ones except C_o must be at a maximum or a minimum.

When $\theta_0 = 0$, $P_n(\cos\theta_0) = 1$ and $P_n^m(\cos\theta_0) = 0$, so that all the coefficients vanish except C_0, and the light is constant. This is geometrically obvious, as in this case the observer always sees the same hemisphere.

§ 6. The case of a spotted sphere with an absorbing atmosphere may be discussed in a similar fashion, starting from equation (3*a*) instead of (3), the only difference being the presence of the transmission-factor A.

When we come to integrate the equation (9), we may expand $A\cos\gamma$ (which is a function of γ alone) in a series of Legendre functions.

$$A\cos\gamma = a_0 + a_1 P_1(\cos\gamma) \ldots + a_n P_n(\cos\gamma) \ldots \quad .$$

We shall then be led to integrals of the form $\int_0^1 P_n(x)P_m(x)dx$. The value of such an integral is finite and different from zero if $n-m$ is odd, but is zero if $n-m$ is even (except for $n=m$, when it is $\frac{1}{2n+1}$).

We thus obtain the coefficient of $Y_n(\theta_0, \phi_0)$ in the expansion of L, in terms of the constants $a_0 \ldots a_n \ldots$ so that, when the law of absorption is known, the expansion of L is completely determined. In this case, however, the expansion will in general contain the odd harmonics Y_3, Y_5, etc., and consequently the relations (13) and (14) between the coefficients of different light-curves will no longer hold good.

If therefore we know the light-curves of a rotating sphere for a sufficient range of values of θ_0, we can determine whether or not it has an absorbing atmosphere (or some other peculiarity of surface which has a similar effect on the appearance of its disk).

It can be proved that, if the transmission-factor A is a continuous function of γ, the coefficient of Y_n in the expansion of L will be of the order of $\frac{1}{n^2}$, so that it will still be true that the light-curve can have no sharp angles.

§ 7. We may extend our results to include the case of any convex solid, arbitrarily spotted.

By a convex solid we mean one such that any tangent plane

meets its surface in only one real point. (This definition includes solids bounded by portions of several intersecting convex surfaces, but excludes polyhedra with plane or conical faces.) On such a surface there will be one, and only one, point where the (outward) normal has a given direction.

Just as in § 2, we shall have, for the light received at a distant point,

$$L = \frac{1}{R^2} \int_S B \cos \gamma \, d\sigma \,, \tag{3}$$

where $d\sigma$ is an element of the surface, and γ is the angle between the outward normal to this element and the line of sight, and the integral extends over all elements for which $\cos \gamma$ is positive.

Now, let θ, ϕ be the polar co-ordinates corresponding to the direction of this normal. Since the surface is convex, their values define uniquely the position of the given element, and hence the value of B, which may therefore be regarded as a function of θ, ϕ. The Gaussian curvature C of the surface at this point will also be a function of θ, ϕ. But by the definition of this kind of curvature we have

$$C d\sigma = \sin \theta \, d\theta \, d\phi \,. \tag{15}$$

Hence we may write (3) in the form

$$L = \frac{1}{R^2} \int_S \frac{B}{C} \cos \gamma \sin \theta \, d\theta \, d\phi \,.$$

This differs from the integral discussed in § 3 only by the presence of the denominator C, and we may proceed just as we did then, except that $\frac{B}{C}$ is the function which is expanded in the spherical harmonic series (4).

The resulting values of L will be identical with those given by a sphere of radius r and surface brightness $\frac{r^2 B}{C}$ (corresponding points on the sphere and surface being those for which θ, ϕ are equal).

It thus appears that there is no gain in generality in passing from a spotted sphere to a spotted convex surface—that is, in assuming that the curvature is variable, as well as the surface brightness.

On the other hand, if we assume that the surface brightness is constant, and only the curvature variable, we lose somewhat in generality.

The boundary of our solid is necessarily a closed surface. Hence its orthogonal projection on any plane must be zero. This condition gives us $\int_S d\sigma \cos\gamma = 0$, where γ is the angle between the normals to the element $d\sigma$ and the plane, and the integral extends over the whole surface. Introducing the variables θ and ϕ, and expanding $\frac{1}{C}$ in a spherical harmonic series, we easily find that this equation will be satisfied when, and only when, the expansion of $\frac{1}{C}$ contains no harmonic of the first order. Since B is constant, it follows that the expansion of L contains no harmonic of the first order. By § 5 all the other odd harmonics are absent, so that the expansion contains only even harmonics and the relations (13) hold good for all values of n, including 0 and 1.

This may also be proved geometrically. Since the surface is of uniform brightness, the light received at any distant point will be proportional to the area of the celestial sphere which the body appears to cover. For two points at equal (great) distances in opposite directions, the apparent areas of the body will be the same, and the values of L identical. It follows at once that the expansion of L contains no odd harmonics.

§ 8. This proof has the advantage that it is available even if the body is not convex. It may even consist of several separate parts (all of the same brightness), provided the system rotates like a rigid body.

In this case, unlike the previous ones, the light-curve may have abrupt changes of direction. For example, a cube, rotating about an axis parallel to one of its edges and seen from a point in the equatorial plane, gives a light-curve consisting of segments of different sine-curves, which cut at sharp angles. There may be such angles in the light-curve even when the surface of the body itself is smooth—for example, when it consists of two equal spheres, which eclipse one another centrally.

A spotted body which is not convex may give us still greater variety. It is clear, however, that the light-curve itself must always be continuous, for otherwise a rotation through an infinitesimal

angle must produce a finite change in the light received, which is impossible if the area and brightness of the surface are finite.

Hence in all cases L may be expanded in a spherical harmonic series in θ, ϕ, but in this series the terms of both even and odd orders will in general be present. It will still be possible to obtain the equation of the light-curves in the form (11), where the coefficients are functions of θ_0, but no general relations will exist between the values of the coefficients for different values of θ_0, except that, in all cases, the light will be constant if the body is viewed from a point on its axis of rotation, so that all the coefficients except C_0 must vanish when $\theta_0=0$ or $\theta_0=\pi$.

§ 9. We may now consider the inverse problem: Given the observed light-curves of a body, to determine the position of its axis of rotation and the character of its surface.

In the case of a star this problem is indeterminate. All that we can know about the inclination θ_0 of the axis of rotation to the line of sight is that it is not zero, if the star is variable (provided, of course, that the variability is to be explained by the star's rotation). Even if we knew the value of θ_0, we could not hope to find out much about the form or spottedness of the star's surface, for our knowledge of the values of the function $L(\theta_0, \phi_0)$, for the given value of θ_0, does not inform us how it behaves for other values of this variable.

It is only when we may start with a much less generalized hypothesis (for example, the eclipse theory for *Algol* variables) that we can hope to gain much information about the surface conditions of variable stars.

§ 10. The case of an asteroid is much more promising. We may observe it from all directions, approximately in the plane of its orbit, and the values of θ_0 will have a range equal to twice the inclination of its equator to this plane. In what follows we shall suppose that we know the light-curves of the planet (corrected for the influence of its varying distance from the Earth and Sun) for a series of oppositions, well distributed around the orbit.

We have first to find the position of its equator. We shall take the planet's equator and orbit as fundamental planes, and, for brevity, shall use the terms "longitude," "right ascension," and

the like, as they would be used by an observer living on its surface. If we suppose him to be situated on the initial meridian (from which longitudes on the surface are measured), then θ_0 and ϕ_0 will be the north polar distance and hour-angle of the Earth, as seen by this observer.

Let Ω denote the longitude of his vernal equinox, measured from some fixed point in the orbit plane (e. g., the perihelion), and i the inclination of the equator to this plane. Let l be the Earth's longitude, measured from the same point, and b its latitude, and let ψ be the Earth's right ascension (referred to this equator and equinox). l and b are given by our observations; b will usually be small, and for the present we shall neglect it. We shall then have

$$\begin{aligned} \cos\psi \sin\theta_0 &= \cos(l-\Omega) , \\ \sin\psi \sin\theta_0 &= \sin(l-\Omega)\cos i ; \\ \cos\theta_0 &= \sin(l-\Omega)\sin i . \end{aligned} \tag{17}$$

If, then, we can determine the values of ψ and θ_0, we can find Ω and i.

Now, the form of the light-curve depends only upon θ_0, while its phase varies with ϕ_0. From the last of the equations (17) we see that θ_0 reaches any given value for two different values of l. Calling them l_1 and l_2, we have, for all values of θ_0

$$l_1 + l_2 = 2\Omega + \pi . \tag{18}$$

That is, the points whose longitudes are l_1 and l_2 are equidistant from the point whose longitude is $\Omega + \frac{\pi}{2}$ (the solstitial point). The light-curves of the planet for oppositions in these two longitudes will be identical in form.

If, then, we arrange the observed light-curves in order according to the opposition-longitudes l, it will be an easy matter to determine the solstitial point, and hence the value of Ω.

For any two oppositions for which the values of l differ by 180°, the values of $\cos\theta_0$ will be equal and of opposite sign. If the planet's surface is of uniform brightness (whatever its form), we see from (13) that the light-curves at these two oppositions must be identical. If it is convex, and has no absorbing atmosphere, it follows from (14) that the two light-curves can differ only in the terms involving C_0, C_1, and D_1.

We therefore have the following three rules for determining the character of the planet's surface:

I. If it is not possible to find a point such that the light-curves at oppositions equidistant from it in longitude are identical, then the planet's variability cannot be accounted for by the rotation of any body which permanently maintains its form, markings, and axis of rotation.

II. If the light-curves at oppositions in diametrically opposite longitudes are not identical, the planet must be spotted, or have an absorbing atmosphere.

III. If the difference between the light-curves at two such oppositions cannot be reduced to a simple sine-curve, by a proper choice of initial epochs for the two curves, then the planet cannot be a convex body, unless it has an absorbing atmosphere, or some equivalent surface peculiarity.

We have now to determine the inclination i. Consider two oppositions, in longitudes l_1 and l_2, for which the light-curves are the same. Similar phases of the two curves will correspond to the times of transit of the Earth across the meridian of our imaginary observer on the planet. If ψ_1 and ψ_2 are the (planeto-centric) right ascensions of the Earth at the two oppositions, the angle through which the planet has turned between the times of corresponding phases of the light-curve at the two oppositions will be some integral number of revolutions *plus* $\psi_2-\psi_1$. If we know the rotation period accurately, we can therefore determine $\psi_2-\psi_1$. But from (17) we have $\tan\psi_1=\cos i\tan(l_1-\Omega)$, $\tan\psi_2=\cos i\tan(l_2-\Omega)=-\tan\psi_1$, whence

$$\cos i=\frac{\tan\frac{1}{2}(l_2-l_1)}{\tan\frac{1}{2}(\psi_2-\psi_1)}\,. \tag{19}$$

Every such pair of oppositions will give us a determination of $\cos i$. It is easy to show that the most favorable pairs are those for which l_2-l_1 is near 90° or 270°.

To determine the accurate value of the rotation period we may use either two oppositions in the same longitude for which the values of θ_0 and ψ are equal, or two near the two nodes of the equator on the orbit, for which $\theta_0=0$ and $\psi_2-\psi_1=180°$, whatever the value of i.

Since i is given by its cosine, it is clear that small inclinations cannot be accurately determined from the observations. Even if $i=30°$, the maximum difference between $\psi_2-\psi_1$ and l_2-l_1 is only about $9°$, corresponding to a displacement of similar phases of the light-curve by $\frac{1}{40}$ of the period. In any case, the sign of i remains indeterminate, as it is clear geometrically that it should do.

When the Earth's latitude b is not negligible, the equations (17) become much more complex. But we may reduce this case to the one already studied, as soon as we have found two oppositions with identical light-curves, by choosing as our fundamental plane one which passes through the places of the Earth at these two oppositions, and measuring longitude anew from some point in this plane. The equations (18) and (19) then give the position of the planet's equator, relative to this plane.

§ 11. We have now to find out what we can about the character of the planet's surface. We will first consider the case when Rule III of the preceding section shows that the light-changes can be accounted for by the rotation of a spotted sphere. Referring to § 5, we see that, in order to find an expression for the surface brightness B, we must expand $L(\theta_0, \phi_0)$ in a spherical harmonic series.

Knowing the position of the planet's equator, and its exact rotation-period, we may find for each opposition an epoch t_0, when the same initial meridian crosses the center of the apparent disk, and so obtain the equation of the light-curve in the form (11), where we may set $\omega(t-t_0)=\phi_0$. The value of θ_0 is known for each curve, so that we may express the coefficients C and D as functions of this variable. The conditions (13) will be satisfied, since they are equivalent to Rule III. The equations (14), with the aid of (17), will then give us a new determination of the node and inclination of the planet's equator, which must agree with the previous one. They serve also to determine completely the harmonic Y_1, in the expansion (10) of L.

Now, θ_0 may have any value included between $\frac{\pi}{2}+i$ and $\frac{\pi}{2}-i$, but it cannot exceed these limits. We therefore know the function $L(\theta_0, \phi_0)$, not over the whole sphere, but over an equatorial zone, whose limiting latitudes are $\pm i$. But we must have its values over

the whole sphere in order to determine the spherical harmonic series (10) uniquely. All that we know about its behavior in the rest of the sphere is (1) that it itself, and its first derivatives, must be everywhere finite and continuous; and (2) that the difference between its values at diametrically opposite points must be equal to the difference of the values of Y_1 at these points (since the higher odd harmonics are absent). We may therefore assign to it any arbitrary system of values, satisfying the first condition, over one of the two polar regions, when the second condition will determine it over the opposite polar region.

To every such assigned system of values there will correspond a different harmonic expansion of the form (10), and all of these expansions will perfectly represent the observed light-curves, while each of them will lead us to a different distribution of brightness on the planet's surface. The form of the expansion will be completely determined by the observations only when $i=\frac{\pi}{2}$; that is, when the planet's axis of rotation lies in the plane of its orbit.

Knowing the expansion of L, we may at once obtain that of B, by multiplying each term of the former by a suitable constant factor. Comparing (10) and (4), we see, however, that the expansion of B is not completely determined, for the odd harmonics Y_3, Y_5 .... which appear in it do not affect L at all, and so we have no means of finding their values.

If, then, we write the result obtained from our observations in the form

$$L(\theta_0, \phi_0)=S_0+S_1(\theta_0, \phi_0)+S_2(\theta_0, \phi_0)+S_4(\theta_0, \phi_0)+S_6(\theta_0, \phi_0)+ \dots \quad (20)$$

We shall have

$$B(\theta,\phi)=\frac{R^2}{\pi r^2}\left\{\begin{aligned}S_0+\tfrac{3}{2}S_1(\theta,\phi)+4S_2(\theta,\phi)-24S_4(\theta,\phi)+64S_6(\theta,\phi)+\dots\\ +Y_3(\theta,\phi)+Y_5(\theta,\phi)+Y_7(\theta,\phi)+\dots\end{aligned}\right\} \quad (21)$$

where Y_3, Y_5, etc., denote spherical harmonics of the orders 3, 5 .... , which may be chosen at random.

This is the complete solution of our problem. It shows that, even in the most favorable case, there is an infinite variety of distributions of brightness on an asteroid's surface, which will account for its observed variations in light.

§ 12. The solution as it stands is, however, only formal, for we have taken no account of the physically necessary condition that the surface-brightness B can never be negative. It may be that, although the series (20) gives always positive (and so physically possible) values for L, the series (21) gives negative values for B.

We may distinguish two cases, if we set $B=B_1+B_2$, where B_1 denotes the sum of the odd, and B_2 that of the even harmonics. B_2 is completely determined by the observations, while in B_1 only the term ${}^{3}_{2}S_1$ is known. Now, B_2 may be defined as half the sum of the values of B at any point and the diametrically opposite point, and B_1 as half their difference. If then B_2 is negative at any point, B must be negative either at this point or the opposite point, whatever the value of B_1 may be.

In this case it is physically impossible to account for the observed variability by the rotation of a spotted sphere.

If B_2 is everywhere positive, or zero, B will be so too, provided the numerical value of B_1 is never greater than that of B_2.

If ${}^{3}_{2}S_1$ is never greater than B_2, this condition may be satisfied by setting $Y_3=Y_5 \ldots . =0$. When this is not the case, it may still be possible to assign such values to Y_3, Y_5, etc., that B_1 is not greater than B_2.

Let us denote the expression for B thus found by B_0. It will be a particular solution of our problem. The general solution may be expressed in the form $B=B_0+B_3$, where B_3 denotes any sum of odd harmonics, of order greater than 1, such that $B_0+B_3 \geqq 0$ for all points on the sphere.

The problem is in general still indeterminate. In certain cases it may not be so. Suppose, for example, that $L=0$ for some position of the observer. Then the whole hemisphere which is visible from his direction must be dark. That is, any physically possible solution must have $B=0$ over this hemisphere. The difference of two such solutions vanishes over the hemisphere. But since it consists wholly of odd harmonics, it must vanish over the whole sphere; that is, the two solutions are identical.

If the rules of § 10 show that the planet has an absorbing atmos-

phere, we may assume a law of absorption, and then, by reversing the reasoning of § 6, determine the markings on its surface. In this case all the harmonics, odd and even, will usually be determinate. But as we do not know what the actual law of absorption is, we are no better off than before.

§ 13. Instead of assuming that our planet is spherical, we may assign to it any other convex form. The expansion (21) will then give the values of $\frac{B}{C}$ at every point of the surface, which serve to define the markings.

Since the surface is convex, C is always positive. For a surface without sharp edges, C will everywhere be finite and the conditions for a physically possible solution will be those discussed in the last section. For a body with sharp edges there will be certain ranges of value of θ, ϕ for which C is infinite, and a physically possible solution must give $\frac{B}{C}=0$ for all such values of the variables, in addition to satisfying the previous conditions.

When the first harmonic is absent in the expansion of $\frac{B}{C}$, we might assume that B is constant, and seek to account for the light-changes by the form of the surface alone. But this leads to difficult problems in the theory of surfaces,[1] and will not be attempted here.

§ 14. We have so far confined ourselves to the study of the light-curves of the planet at opposition. At other phases the light received from a given element $d\sigma$ will depend not only on γ, but also on the angle of incidence i of the Sun's rays, and the expression for the light received by the observer from this element will be $dL=\frac{B}{R^2}f(i, \gamma)d\sigma$, where $f(i, \gamma)$ vanishes when either $\cos i$ or $\cos \gamma$ is zero. To obtain the total light L, this must be integrated over that part of the surface for which $\cos i$ and $\cos \gamma$ are both positive.

It will still be true that any two convex surfaces for which the values of $\frac{B}{C}$ at corresponding points are the same will give identical values of L. But in this case the odd harmonics Y_3, Y_5, etc., in

[1] For example, the equation $C=$constant leads not only to the sphere, but to all the surfaces applicable to a sphere.

the expansion of $\frac{B}{C}$ will in general influence the value of L, and from this their values may sometimes be determined.

§ 15. We may summarize the results of this discussion as follows:

If a variable asteroid has been observed at a series of oppositions in all parts of its orbit:

(1) We can determine by inspection of its light-curves whether or not they can be accounted for by its rotation alone, and, if so, whether the asteroid (*a*) has an absorbing atmosphere, (*b*) is not of a convex form, (*c*) has a spotted surface, or whether these hypotheses are unnecessary.

(2) It is always possible (theoretically) to determine the position of the asteroid's equator, (except that the sign of the inclination remains unknown).

(3) It is quite impossible to determine the shape of the asteroid. If any continuous convex form is possible, all such forms are possible.

(4) In this case we may assume any such form, and then determine a distribution of brightness on its surface which will account for the observed light-curves. This can usually be done in an infinite variety of ways.

(5) The consideration of the light-curve of a planet at phases remote from opposition may aid in determining the markings on its surface, but cannot help us to find its shape.

PRINCETON UNIVERSITY OBSERVATORY,
April 28, 1906.

POLARIZATION AND SELECTIVE REFLECTION IN THE INFRA-RED SPECTRUM

By A. H. PFUND

INTRODUCTION

Object and general survey of the work.—The intimate relationship existing between the refractive index, extinction coefficient, and reflecting power of an absorbing medium was first pointed out by Cauchy in his well-known formulæ for metallic reflection. That these formulæ do actually give a true account of existing conditions has been proved in recent years by Minor,[1] Pflueger,[2] Edmunds,[3] and others, for conductors as well as for insulators. In deducing his formulæ, Cauchy made the very natural assumption that the intensity of the light, as it penetrated into the medium, fell off according to some exponential law. Now, although this assumption leads to formulæ which are verified by experiment, no insight is obtained into the actual mechanism of reflection, and, in fact, up to the present time a satisfactory theory bearing upon this subject is wanting. It was with the object of collecting data which, it was hoped, might be of some help in solving the problem, that the present work was taken up. For the sake of greater clearness, I wish to give in the following a brief survey of the several investigations carried out.

As indicated by the title, the work naturally falls under two headings:

1. Polarization in the infra-red.
2. Selective reflection in the infra-red.

Heretofore investigators working with polarized radiations in the infra-red have simply assumed that these radiations were polarized, without actually proving them to be so. Therefore the first point to be taken up was to show definitely that these radiations were susceptible of polarization. In the course of the work a new polarizer and analyzer were discovered which are believed to be a decided

[1] *Ann. d. Phys.*, **10**, 581, 1903.

[2] *Ibid.*, **65**, 214, 1898.

[3] *Phys. Rev.*, **18**, 193, 385, 1904.

improvement upon the forms now in use. With these new instruments it was readily proved, as far as the experiment could be carried (i. e., up to 13 μ), that infra-red radiations can be polarized by reflection.

One of the distinguishing properties of a metal is its ability to convert plane into elliptically polarized light by reflection. It was thought of interest to investigate whether an insulator reflecting metallically in the infra-red would also have this property. The substance chosen was Iceland spar, and it was shown conclusively that the conversion of plane into elliptically polarized light does take place—as might have been expected.

Originally it was intended to determine the refraction and extinction curves for Iceland spar within the region of metallic reflection by a katoptric method which involved a determination of the constants of elliptically polarized light. The method employed was the well-known Brewster's double mirror method as used by Quincke,[1] Pflueger (*loc. cit.*), and others. It was shown by the use of silver mirrors that such measurements can easily be carried out into the infra-red. Unfortunately, however, in the case of Iceland spar the amount of energy reaching the radiometer was too small to make accurate measurements possible.

The object of the work on selective reflection was the following:

According to modern conceptions, selective reflection from insulators is occasioned by particles with definite free periods which are capable of vibrating in resonance with certain incident radiations. As the phenomenon is looked upon as taking place entirely within the molecule, it was thought of interest to determine, first, whether the selective reflection of a substance was dependent on its physical state, and, secondly, whether the mechanism giving rise to this selective reflection was localized within some definite portion of the molecule. As to the first point, the selective reflection of a salt was studied, both when the salt was solid and when molten; and as to the second point, an investigation was carried out on the selective reflection of a number of salts having a certain radical in common.

Having found that a molten salt is capable of reflecting selectively, a number of other molten salts and liquids were investigated for

[1] *Pogg. Ann.*, Jubelband, 336, 1874.

similar effects. In the case of sulphuric acid some very remarkable changes in the reflection-curve were found as the acid was diluted. Each of these subjects in turn will be discussed in detail.

DESCRIPTION OF APPARATUS

Radiometer-spectrometer.—The general arrangement of the apparatus is shown in the following diagram (Fig. 1):

Radiations coming from a Nernst glower (N) placed in front of the slit (S_1) were rendered parallel by the concave mirror (M_1),

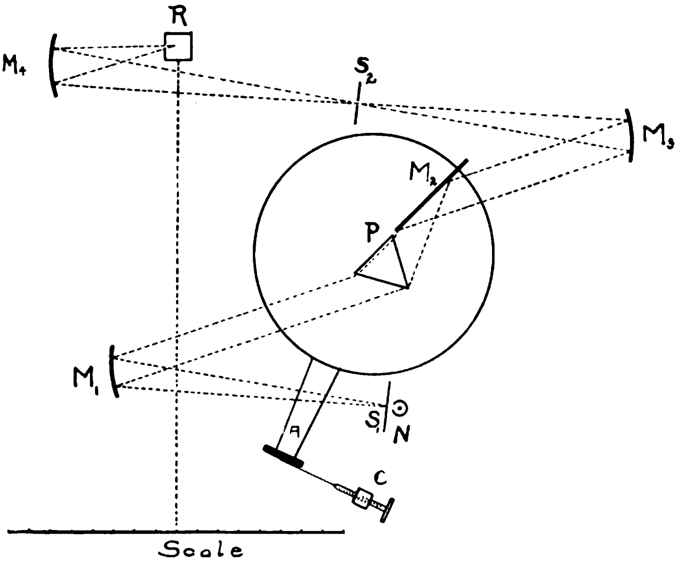


FIG. 1

and then passed through the Wadsworth mirror and prism arrangement (PM_2). The mirror (M_3) brought the spectrum of the source to a focus on the second slit (S_2). Thus, by a rotation of the prism and mirror system (PM_2) mounted on the spectrometer, it was possible to cause the entire spectrum to pass over the slit S_2, and in this manner any desired portion could be brought to a focus on one of the blackened vanes of the radiometer R. The slits S_1 and S_2 were usually opened to a width of 0.5 mm.

As mentioned, the source of radiation was a Nernst glower inclosed in a brass cylinder which had but one narrow opening along its

side. In this manner the disturbing effects of air currents were removed, as shown by actual tests of the constancy of the radiation of the glower. The concave mirrors (M_1) and (M_3) had a radius of curvature of 52 cm, while the mirror M_4, which concentrated the radiations on the radiometer vane, had a radius of curvature of only 26 cm. The rock-salt prism (P) was about 5 cm on an edge and had a refracting angle of 60° 10′ 6″. The calibration was carried out in the usual manner of calculating from the known dispersion-curve of rock salt, the deviations corresponding to definite wave-lengths. To check the results thus obtained, the position of the emission bands of CO_2 from a Bunsen burner (2.70 μ and 4.40 μ), and also the position of the bands of metallic reflection from quartz (8.49 μ and 9.03 μ)[1] and from Iceland spar (6.69 μ and 11.41 μ), were observed, and were found to lie exactly on the curve.

Concerning the spectrometer the only point worth mentioning is that the rotation of the prism and mirror system, which was mounted on the spectrometer table, was measured by means of a micrometer (c) connected with the projecting arm (A) through a piece of steel tape. The length of the arm was so chosen that one division on the divided barrel of the micrometer corresponded to 6 seconds of arc.

The radiometer was of the usual Nichols type and was supplied with a rock-salt window. The sensibility of the instrument could easily be varied by changing the length of the quartz fiber which was partly wound up on a miniature reel arrangement. In the polarization experiments the sensibility was such that a meter-candle gave rise to a deflection of about 1000 mm, while in the later reflection experiments the sensibility was reduced to six-tenths of this value. With the exception of the rock-salt window and the minute concave mirrors about to be described, the radiometer was identical with the one employed by J. T. Porter[2] and described by him.

Throughout the course of the entire investigation this radiometer-spectrometer remained unmodified, all of the special apparatus being brought out in front of the slit S_1.

Minute concave mirrors.—Undoubtedly the easiest way of deter-

[1] These values, obtained by Rosenthal (*Ann. der. Phys.*, **68**, 783, 1899), are probably the most accurate known.

[2] *Astrophysical Journal*, **22**, 229, 1905.

mining the deflection of a radiometer is to observe the motion of the image of an incandescent lamp filament projected on a ground glass scale by means of a small concave mirror. Heretofore this method has not been used, for the reason that small concave mirrors weighing a milligram or two have not been obtainable. It occurred to me that they might very easily be made in the following manner.

A small concave mirror or good spectacle lens of about 1 m radius of curvature is heavily silvered and polished on its concave side, and is then cut up into strips about 4 mm wide. If, now, one of these strips be struck a smart tap on its edge by means of a flat file, small scales of glass bearing silver on one side can be caused to fly off. These scales are, of course, the concave mirrors in question. With a little practice it is easy to produce mirrors of this kind varying from 7 or 8 mm in diameter down to infinitesimal dimensions. These mirrors are found to retain their figure well, and to give images of surprising sharpness and brilliancy.

In making such small mirrors care must be taken to select a concave mirror of good figure, and an easy way to carry out a test is to diaphragm off all of the mirror but an area about 2 mm square, and then to examine the image produced. If it be necessary to use these mirrors in the open air, where silver will tarnish, it would be advisable to platinize the reflecting surface by cathode discharge.

POLARIZATION IN THE INFRA-RED

Reflection from glass and selenium.—Ultra-violet, visible, and short infra-red rays may readily be polarized by means of some such device as a Nicol or Rochon prism. However, beyond 2.5 μ in the infra-red spectrum this method fails on account of the opacity of the doubly refracting substance, Iceland spar, used in the construction of the above-named polarizers. Probably the most convenient method of producing polarized radiations in the regions of great wave-lengths is that involving reflection at the polarizing angle from some transparent substance. The conditions which such a substance must fulfil are:

1. It must have a fairly high reflecting power.

2. Its polarizing angle must be practically constant for different wave-lengths.

3. It must polarize very completely.

The obvious manner of attacking the problem is to make a study of the reflection-curves of various substances which appear to give promise of fulfilling the above-mentioned conditions. The apparatus used in carrying out such measurements is shown in the following diagram (Fig. 2).

Radiations from the Nernst glower at N were reflected at an angle of incidence of 10° from the two mirrors (M_2) and (M_3), and were finally brought to a focus on the slit S_1 of the spectroscope already described. The mirror whose reflecting power was to be determined

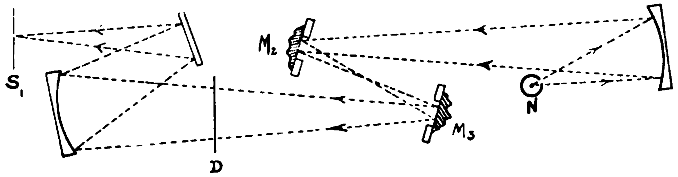


FIG. 2

was held in position at (M_3) by being pressed against a brass plate into which a hole had been cut. Any desired mirror could be placed behind the opening of a similar brass plate at (M_2). In order to keep the deflections on the scale, a diaphragm was placed at D which could be made to cut down the vertical aperture of the beam. Measurements of reflecting power were obtained in the following manner. The mirror to be tested was placed in position at M_3, and the radiometer deflection was noted; then a silver mirror was made to replace the former, and again the deflection was noted. The ratio of the latter deflection to the former gave the reflecting power of the substance for the wave-length corresponding to a given spectrometer setting.

Considerable time was lost in trying to adjust the two mirrors (i. e., the one of silver and the other of the material under investigation) upon a rotating table which could be made to move between stops. Due to the fact that the replacement of one mirror by another was not absolutely exact, very discordant results were obtained. Consequently this method was abandoned, and the one already described was used—thereby making the replacement of one mirror by another exact beyond a question. As might be expected, very concordant results were obtained.

In looking over some unpublished results on the reflecting power of amorphous selenium, it appeared to me that the reflecting power of this substance gave promise of becoming constant for the longer wave-lengths, and it was decided to make some exact measurements. Furthermore, since glass has been repeatedly used in the infra-red as a polarizer, it was thought worthy of interest to determine in what regions of the spectrum such a procedure would be permissible. The results as plotted in the following curves (Fig. 3) speak for themselves.

That glass would be unfit to act as a polarizer throughout the entire spectrum might have been predicted from a knowledge of

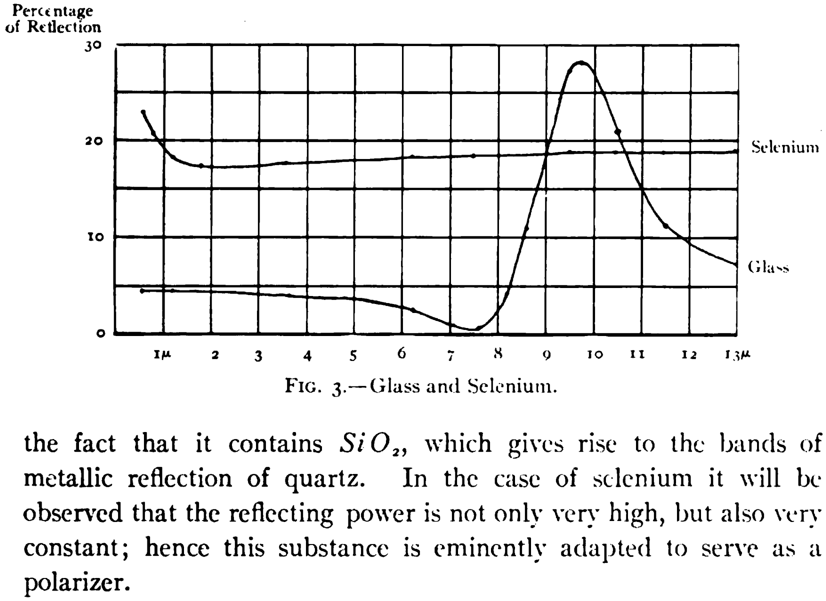


FIG. 3.—Glass and Selenium.

the fact that it contains SiO_2, which gives rise to the bands of metallic reflection of quartz. In the case of selenium it will be observed that the reflecting power is not only very high, but also very constant; hence this substance is eminently adapted to serve as a polarizer.

Since in experiments with polarized light two reflections are necessary, one from the polarizer and one from the analyzer, the maximum attainable energy after two reflections is to the incident energy as $r^2 : 1$, where r is the reflecting power of the polarizing medium. This statement would be rigorously true if the incident energy were already polarized in the plane of incidence. Taking into account the fact that this is not the case—i. e., considering the fact that the incident radiations are unpolarized—calculations have been made upon the basis of Fresnel's formulæ to show how great

the gain in energy is when selenium is used as polarizer and analyzer in place of other substances.

Refractive Index of Substance	Fraction Refl. from Polarizer	Fraction Refl. from Analyzer	Ratio of Emergent to Incident Energy	Gain in Using Selenium
2.565 (selenium)	0.27	0.54	0.146	
1.60	.095	.19	.018	8.1 times
1.50 (rock-salt)	.0745	.149	.011	13.3 "
1.40 (fluorite)	.0515	.103	.0053	27.6 "
1.30 (fluorite)	.0375	.075	.0028	52.0 "

Since selenium is comparatively transparent in the infra-red, it will be permissible to apply Fresnel's reflection formulæ and calculate the refractive index (n):

$$n=\frac{1+\sqrt{r}}{1-\sqrt{r}}\ .$$

Near 13 μ, where the reflection-curve becomes flat, $r=19$ per cent., and the refractive index as calculated is $n=2.565$. Now, according to Maxwell's equations the relation $n^2=\epsilon$ (where ϵ is the dielectric constant) is fulfilled if the measurements be made sufficiently far removed from the region of absorption bands. Recently Schmidt[1] has found that for amorphous selenium $\epsilon=6.60$, while from the foregoing measurements we have $\epsilon=n^2=6.58$. This shows that the Maxwellian relation is already fulfilled, and it seems only reasonable to suppose that the reflecting power, and hence the polarizing angle, will remain constant throughout the remainder of the infra-red spectrum.

The selenium mirrors used in the experiments about to be described were prepared in the following manner. Some pure selenium was melted down in a porcelain crucible and was poured on a plate of hot glass; a second plate of hot glass was then quickly pressed down upon the pool of molten selenium, and the whole was laid on an iron plate to cool. The layer of selenium, after being pressed out, was usually about 1 mm in thickness. Heretofore considerable difficulty was experienced in getting the second plate to come off. I have found, however, that this can be accomplished very easily

[1] *Ann. d. Phys.*, **2**, 114, 1903.

if the second plate be made considerably longer than the first, thereby making it possible to bend it slightly and thus cause it to tear itself away from the selenium. Very beautiful mirrors may be obtained in this manner. Those actually used were of oval shape, about 10 cm long and 5 cm wide, and had a figure as perfect as that of the glass plate which had covered them.

By means of such mirrors as these a polarizer of the following form was constructed (Fig. 4):

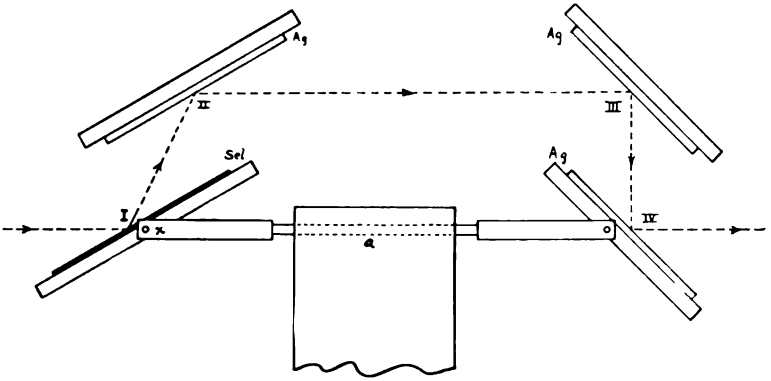


FIG. 4

Selenium polarizer and analyzer.—The selenium mirror (1) and the silver mirror (2) were adjusted parallel, and then rigidly connected with one another, so that a rotation about the axis (x), changing the angle of incidence, would not change the direction of the emergent beam. The two other silver mirrors (3) and (4) were also adjusted parallel to one another, and the entire system could be rotated about the axis a (the axis a being perpendicular to the axis x). It is easy to see that in an apparatus of this kind the incident and emergent beams lie along the same straight line, and a rotation about the axis a will not affect the direction of the emergent beam. Therefore, if this beam be concentrated upon the slit of a spectrometer, it will remain there in spite of the rotation of the mirror system.

In the actual experiments the instrument just described was used as the analyzer, while the polarizer was made to consist of the mirrors (1) and (2) alone. In order to measure the rotations of the system of mirrors, small protractors graduated to 30′ were fas-

tened to the axis *a*. By means of a telescope tenths could be easily estimated, thus making it possible to read to 3′ of arc.

Complete polarization of infra-red radiations.—As was mentioned in the beginning, the first question taken up was whether this apparatus would actually polarize infra-red radiations. The manner in which the experiment was carried out is shown in the accompanying diagram (Fig. 5). The rays from the Nernst glower (*N*), after being rendered parallel by the concave mirror *M*, were polarized at *P*, and then analyzed at *A*. The concave mirror (M_2) focused

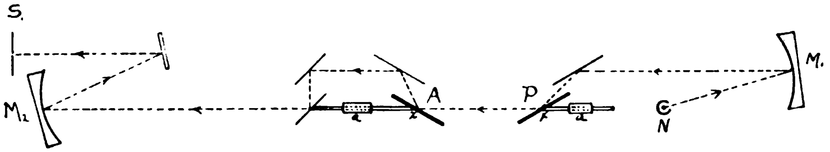


FIG. 5

the image of the Nernst glower on the slit S_1 of the spectrometer already described. Having roughly adjusted the selenium mirrors of the polarizer and analyzer in the position of maximum polarization, they were "crossed," and the position of least reflection was determined by successive rotations of the systems about the axes *x* and *a*. This position could be found very easily, and the fact was established that variations in the angle of incidence of a degree or two did not markedly influence the results. Deflections of the radiometer were observed up to 13 μ corresponding to the condition of parallel and crossed polarizer and analyzer. Results of the following character were obtained throughout this entire range of wave-lengths:

Relative Position of Polarizer and Analyzer	Radiometer Deflection
Parallel	> 1000 mm
Crossed	< 1 mm

This shows, then, that the radiations are polarized to a very high high degree. Of course, no claim is made to complete polarization, as it is found that, if the incident energy be increased very greatly, a small deflection of a few divisions is obtained. All things taken into consideration, however, selenium is found to fulfil the conditions imposed upon it as a polarizer very well indeed.

Elliptical polarization from Iceland spar in the region of metallic

reflection.—In order to determine whether a non-metallic substance in its region of metallic reflection behaves as a metal toward polarized light, a parallel beam of radiations (Fig. 6) from a Nernst glower (N) polarized in an azimuth of 45° at P was caused to be reflected from a surface of Iceland spar (I) (polished on one of its

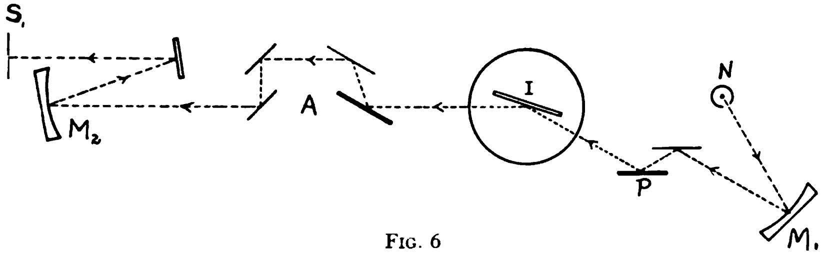


FIG. 6

cleavage planes) at an angle of incidence of 65°. After passing through the analyzer (A), these radiations eventually reached the slit (S_1) of the spectrometer. For a given setting of the spectrometer, radiometer reflections were observed corresponding to the two positions of the analyzer giving maximum and minimum energy. The readings here recorded are for $\lambda = 4\ \mu$, where Iceland spar reflects vitreously, and for $\lambda = 6.70\ \mu$, where it reflects metallically.

	Maximum Deflection	Minimum Deflection	Ellipticity (Ratio of Axes)
4.0 μ........................	501 div.	0 div.	10:0
6.7 μ........................	90 "	55 "	10:6

These results show conclusively that Iceland spar transforms plane into elliptically polarized light within the region of metallic reflection.

On account of the difficulty of preparing plates of Iceland spar or quartz sufficiently thin to make transmission and absorption measurements possible within the regions of metallic reflection, it was thought feasible to make these measurements by an indirect, katoptric method. As stated earlier in the paper, the amount of energy reaching the radiometer was found insufficient to make such determinations possible. When it is considered that the radiations suffered fifteen reflections before reaching the radiometer, it is not surprising that the available amount of energy was found too small.

However, the work of Pflueger (*loc. cit.*) and Nichols[1] on insulators, and of Hagen and Rubens[2] and Minor on metals, has unquestionably established the fact that strong absorption and metallic reflection go together. In view of the fact that any explanation of metallic reflection involving the conception of resonance is not applicable to metals, one is led to suspect that metallic reflection, wherever found, is, in all probability, to be attributed to strong absorption alone.

SELECTIVE REFLECTION IN THE INFRA-RED

Reflection from solid and molten salt.—In order to determine the selective reflection of various substances, such investigators as Rubens and Nichols,[3] Aschkinass,[4] and Porter (*loc. cit.*) made use of the method of multiple reflections, commonly known as the method of "Reststrahlen." On account of the weak reflecting power of most of the substances investigated in the present work, it was decided not to use the above method, but rather to make direct determinations of the reflecting power. In the case of solids the same method was used as that described in the chapter dealing with the reflection from selenium, while in the case of liquids the method used involved the reflection of energy from the free liquid surface.

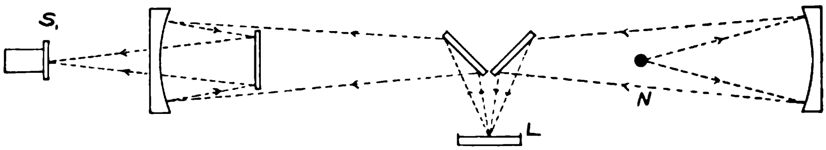


FIG. 7

The general arrangement of apparatus is shown in elevation in the diagram (Fig. 7):

Radiations from the Nernst glower N were first brought to a focus on the surface of the liquid at L, and finally on the slit S_1 of the spectrometer. Measurements were obtained in the following manner: With the liquid surface in a fixed position a complete series of readings was taken throughout the entire spectrum; then the liquid surface was replaced by one of silver, and a similar series

[1] *Sitzungsberichte der Kgl. Preuss. Akad. der Wissenschaften*, **44**, 1, 1896.

[2] *Ann. d. Phys.*, **8**, 1, 1902.

[3] *Ibid.*, **60**, 418, 430, 1897.

[4] *Ibid.*, **65**, 241, 1898.

was taken. By proceeding in this manner it was possible to make corrections for a rising or falling energy-curve of the source. The principal reason for adopting this method of procedure in preference to the one used before was that certain liquids, especially the molten salts, had a most annoying tendency to form "surface skins," and it was only by stirring the molten mass vigorously, and then taking the readings in the shortest time possible, that concordant readings could be obtained. It might be added that the radiations sent out by the Nernst glower were of an amply sufficient constancy to make

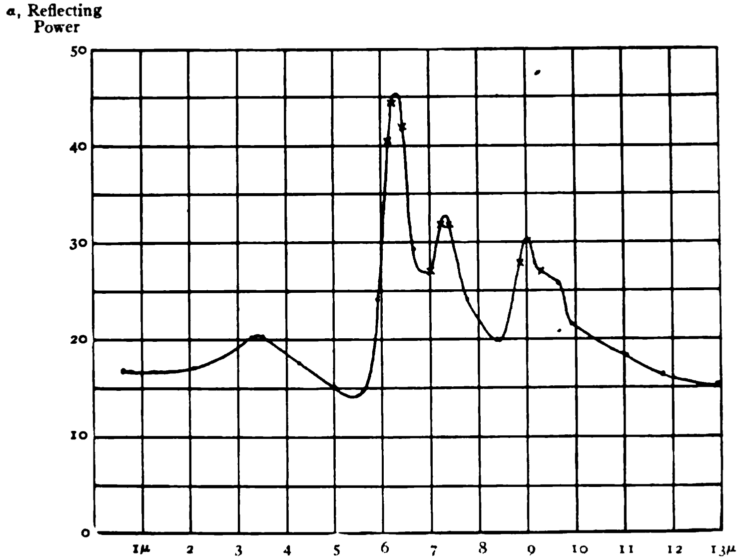


FIG. 8.—Sodium-Potassium Tartrate.

such a procedure allowable. On each of the curves plotted it is indicated whether the ordinates are proportional or equal to the reflecting power.

The intensity of reflection from most of the liquids examined, even at the maxima, lies under 15 per cent. The only exception is sulphuric acid, which at its highest maximum reflects 20.8 per cent.

The first point to be taken up was to show that, so long as the molecule as a whole remains unaltered and no changes take place in the nature of the surrounding medium, the position of the bands of metallic reflection will remain the same. The substance chosen

to show this was sodium potassium tartrate. A reflection-curve was obtained from a polished crystal, and then an attempt was made to obtain a similar curve for the molten salt. Unfortunately,

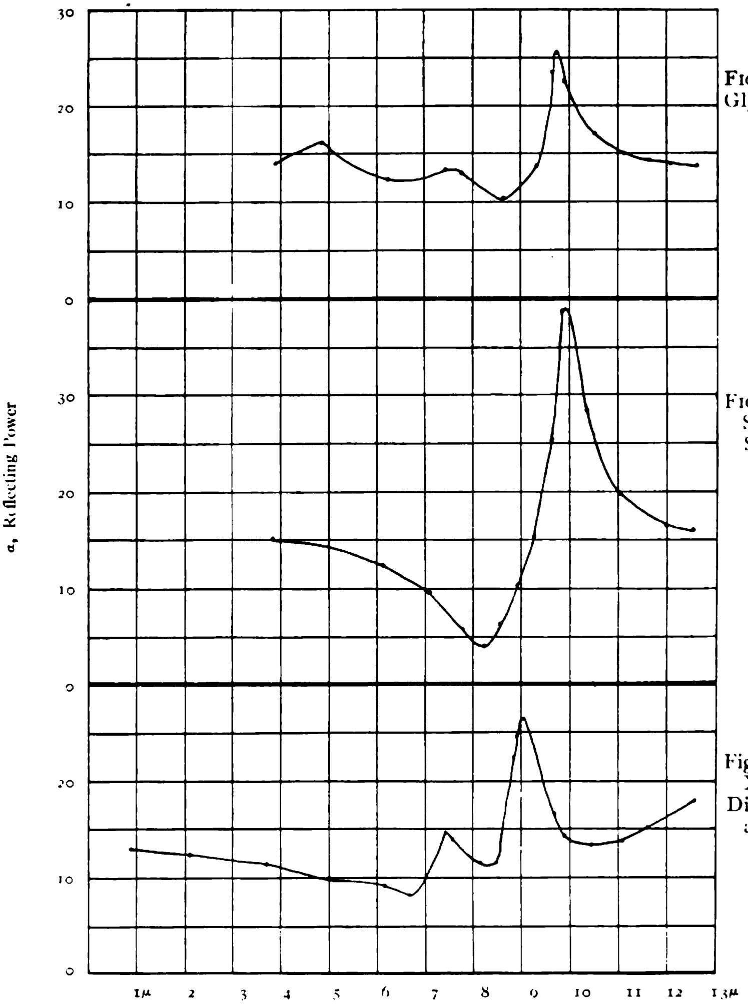


FIG. 9.—Glycerine.

FIG. 10.—Sodium Silicate.

Fig. 11.—Nitroso-Dimethyl-aniline.

however, it was not possible to obtain a complete curve of this kind, on account of the rapid formation of a surface skin, and in consequence I had to content myself with a determination of the positions of the maxima, which were very marked. This was done by setting

the spectrometer approximately in the position of a band, then removing the surface skin and making an accurate setting before a new skin had time to form. The readings thus obtained are indicated by crosses on Fig. 8, and show that the position of the bands remain unaltered.

Reflection from liquids.—In view of the fact that molten sodium potassium tartrate was found to possess bands of metallic reflection,

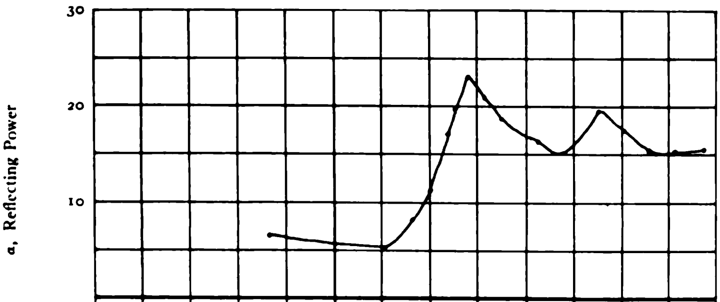


FIG. 12.—Nitric Acid.

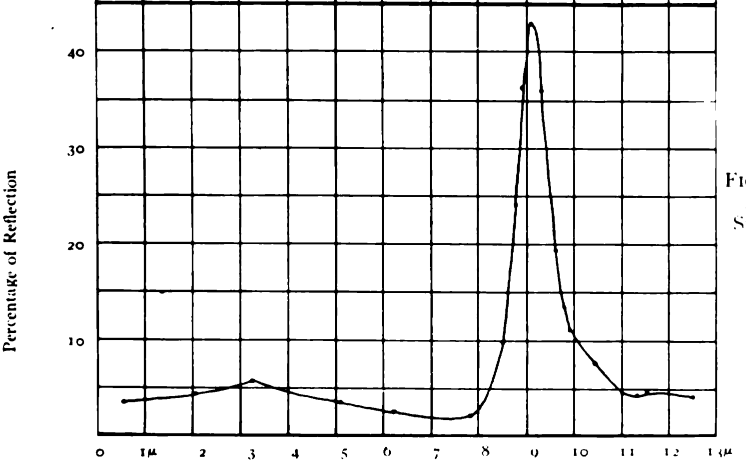


FIG. 13.—Nickel Sulphate.

it was deemed of interest to investigate other molten salts and liquids for similar effects. The accompanying curves (Figs. 9–12, 14, and 15) for glycerin, liquid sodium silicate, molten nitroso-dimethyl-aniline, nitrates of calcium, cobalt, magnesium and ammonium, nitric acid, and sulphuric acid show in a most striking manner that selective reflection is by no means confined to solids.[1] It might be added

[1] No marked bands of selective reflection were found for wave-lengths shorter than 3 μ.

that no such marked maxima were obtained when water, alcohol, molten caustic potash, phosphoric acid, acetic acid, and hydrochloric acid were tried.

Localization of mechanism of selective reflection within molecule.— Upon looking over the results obtained, it became very evident that

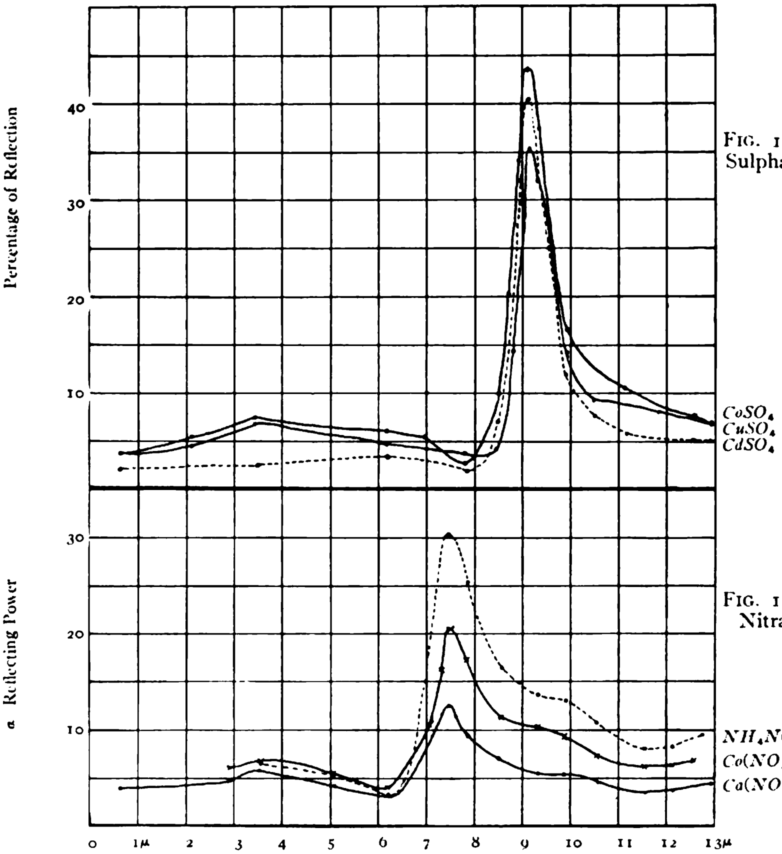


FIG. 14.— Sulphates.

FIG. 15.— Nitrates.

salts of the same acid had curves whose principal maxima were situated in the same region of the spectrum, and it was decided to investigate this point more fully. Consequently, the selective reflection of the following substances was investigated: sulphates of copper, nickel, cadmium, iron, sodium, potassium, and cobalt; also the nitrates of silver and potassium in addition to those of calcium, ammonium, magnesium, and cobalt already investigated. Whenever

crystals were obtained whose surfaces were of sufficient size and perfection, the absolute values of the reflecting powers were determined. Where this was not the case, it was possible to obtain only

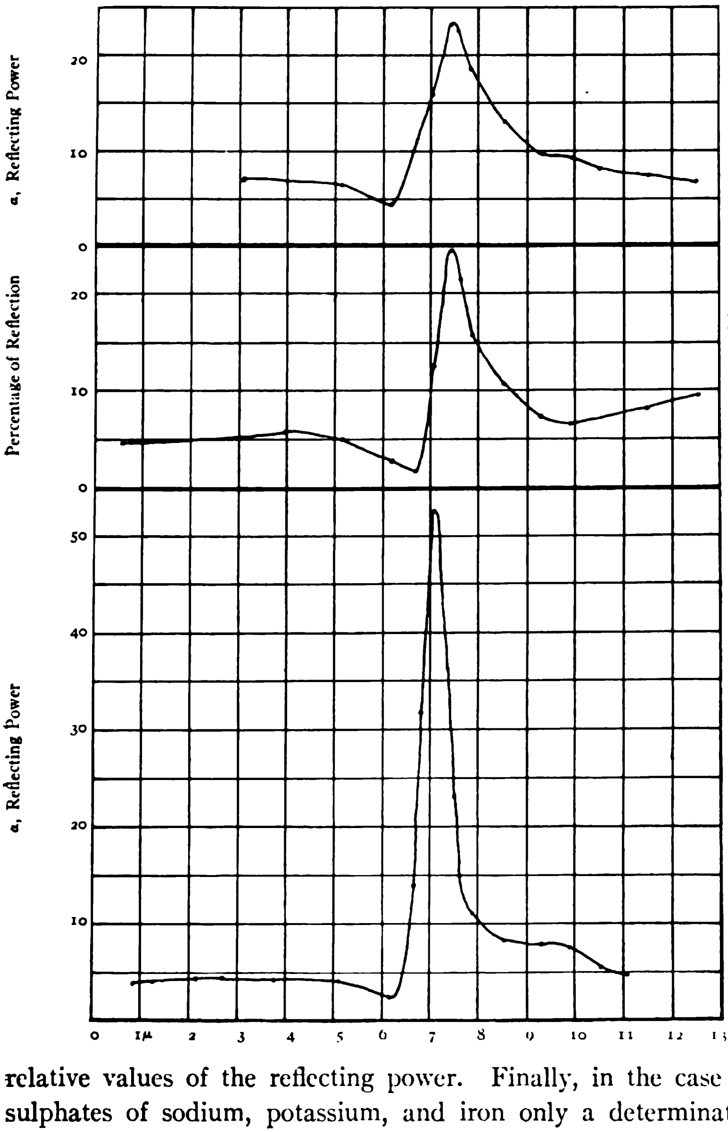


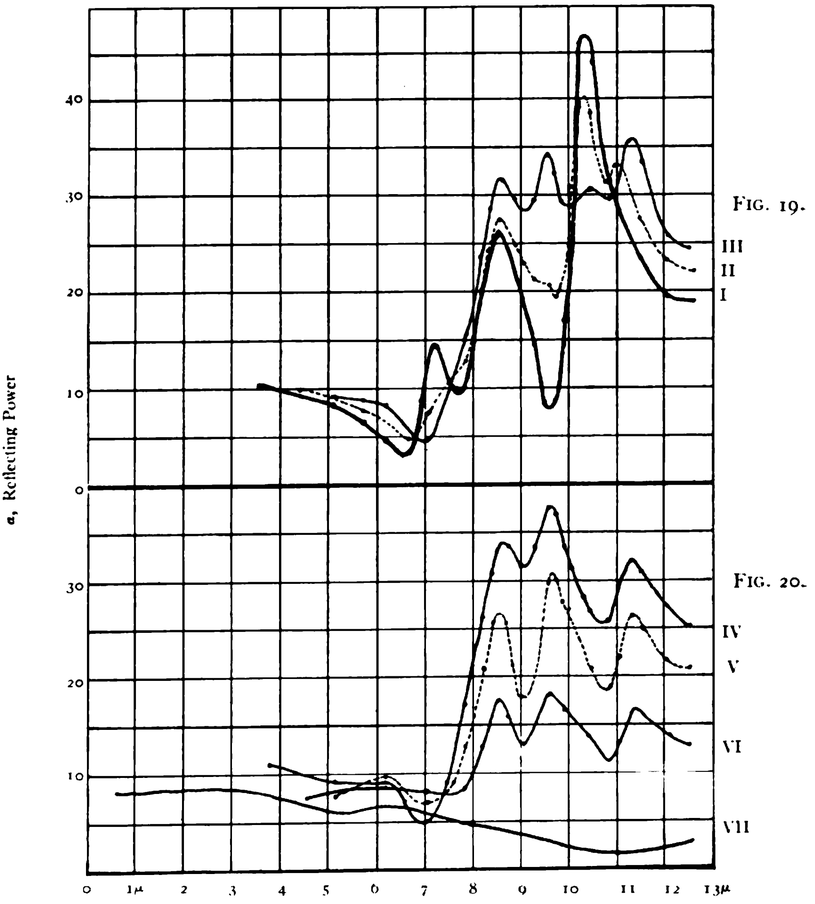
FIG. 16.—Magnesium Nitrate.

FIG. 17.—Silver Nitrate.

FIG. 18.—Potassium Nitrate.

relative values of the reflecting power. Finally, in the case of the sulphates of sodium, potassium, and iron only a determination of the positions of the maxima was made.

An inspection of the following curves, as for example those of the sulphates (curves 13 and 14), cannot fail to impress one with the fact that the maximum near 9.05 μ is present under all circumstances, no matter what the nature of the metal in the molecule may



FIGS. 19 and 20.—Fuming Sulphuric Acid.

be. Again in the case of the nitrates, it is equally evident that the strong maximum near 7.40 μ appears in all cases. Attention must, however, be called to the fact that the position of the maxima for the salts of a given acid are not identically the same as will be observed if the curves for silver nitrate and potassium nitrate (Figs. 17 and

18) are compared; nor do the maxima of sulphuric and nitric acid coincide with those of the sulphates and nitrates respectively. These points will be discussed later on. However all this may be, one cannot help but acknowledge that the similarity of the curves, especially of those shown in Figs. 14 and 15, is too striking to be without significance. Since every salt of a given acid thus far studied shows a maximum in approximately the same region of the spectrum, in spite of marked changes in the nature of the metal, one is strongly tempted to localize the mechanism giving rise to this maximum in the acid radical of the molecule, i. e., that portion of the molecule which in solution becomes the negative ion.

Reflection from fuming sulphuric acid at various dilutions.—In order to reassure myself of the correctness of the results which had thus far been obtained, a second series of measurements was carried out for all of the substances studied. All of the results were verified, with the exception of those for sulphuric acid, which were totally different from the first. As was later found out, this was due to the fact that the wrong sulphuric acid bottle (containing weak acid) had been used. On account of the striking difference in the two curves, it was decided to make a systematic investigation of the changes in the forms of the curves corresponding to different degrees of dilution of the acid. The curves Figs. 19 and 20 represent the results obtained.

Curve I is for undiluted fuming sulphuric acid of density 1.87
" II " " 12 parts of this acid and 1 part of water
" III " " 12 " " " " " 2.4 " " "
" IV " " 3 " " " " " 1 " " "
" V " " 1 part " " " " 1 " " "
" VI " " 1 " " " " " 3 parts " "
" VII " " pure water (distilled).

These proportions of acid to water are given in terms of volumes.

In curve I three distinct maxima are discernible—at 7.20 μ, 8.65 μ, and 10.30 μ; and also three minima—at 6.62 μ, 7.73 μ, and 9.64 μ. Curves II and III, which represent intermediate stages, show that the maximum at 10.30 μ is rapidly disappearing; the minimum at 9.46 μ is being filled in; the maximum at 8.65 μ remains, while the maximum at 7.20 μ also disappears. In addition to these

changes, other maxima are coming in, whose positions from curves IV and VI are seen to be at 9.60 μ and 7.40 μ. As will be observed, these maxima are distinctly new and are not the old ones displaced to another position. Furthermore, it is evident from curve VII that water superimposes no peculiarities of its own on the curves for sulphuric acid.

I have tried to interpret these results in the following manner.

It is generally conceded that in sulphuric acid of the greatest strength used in these experiments there is a large excess of undissociated H_2SO_4 molecules. By an addition of water, these molecules are first broken down into $\overset{+}{H}\ \overset{-}{HSO_4}$ ions, and finally into $\overset{+}{H}\ \overset{+}{H}\ \overset{-}{SO_4}$ ions—the complete change to the latter form taking place only at infinite dilution. Considering the fact that these changes in the molecules and ions are accompanied by marked changes in the reflection-curves, it seems only reasonable to suppose that those maxima, which are at first present and then disappear with increasing dilution, are due to molecules or ions (or possibly both) which also disappear with increasing dilution. Similarly, the new maxima which appear are supposed to be due to new ions which are formed in consequence of increasing dilution. It would, indeed, be premature to attempt to attribute with definiteness certain reflection maxima to certain molecules or ions; for the state of our knowledge as to the condition of affairs existing in solutions is as yet too incomplete. In the present case, for example, we have fuming sulphuric acid (which is a solution of SO_3 in H_2SO_4) and water. Not only is it possible to have a large number of different kinds of ions, but in addition we may have a large number of complexes or hydrates which the molecules of the acid might form with those of the water. From this it will be seen that much work remains to be done to clear up the subject.

But let us return once more to the work on the sulphates and nitrates. As already pointed out, this work has made it seem probable that the mechanism giving rise to certain strong maxima of reflection is localized within the acid radical. Now, this conclusion is quite in accord with the results of some calculations which Drude[1]

[1] *Annalen der Physik*, 14, 677, 1904.

has recently carried out. In these it is shown that the particle which, in consequence of its sympathetic vibrations, gives rise to the ultra-violet absorption band has a charge and a mass identical with that of the corpuscle, while the particle giving rise to the infra-red absorption band has a mass of the order of magnitude of the molecule itself. Since it is a large portion of the molecule which is looked upon as being thrown into vibration, it does not seem difficult to account for the fact that the positions of the reflexion maxima for the salts of the same acid are not the same. There is but little doubt in my mind that the positions would be the same if the acid radicals could execute their vibrations independently of other particles surrounding them—a condition approached at infinite dilution. In the cases actually studied the freedom of motion of the acid radical depended not only on the closeness of the bond between it and the metal ion, but also upon molecules of water clustering around it. It does not seem unreasonable, therefore, to suppose that these influences might affect the period of vibration of the acid radical, and hence the position of the reflection maximum.

The following table gives a list of the substances studied, together with the positions of the principal maxima of reflection:

Substance	Formula	Position of Reflection Maxima			Remarks
Na-K tartrate	$C_8H_4KNaO_6 . 4H_2O$	6.35	7.35	9.05 μ	Solid crystal
Magnesium nitrate..	$Mg(NO_3)_2 \quad .6H_2O$	7.45			Molten
Cobalt nitrate	$Co(NO_3)_2 \quad .6H_2O$	7.45			"
Ammonium nitrate..	NH_4NO_3	7.45			"
Calcium nitrate.....	$Ca(NO_3)_2 \quad .4H_2O$	7.45			"
Silver nitrate	$AgNO_3$	7.45			Solid crystal
Potassium nitrate ..	KNO_3	7.05			" "
Nickel sulphate....	$NiSO_4 \quad .7H_2O$	9.05			" "
Cobalt sulphate	$CoSO_4 \quad .7H_2O$	9.05			" "
Copper sulphate	$CuSO_4 \quad .5H_2O$	9.15			" "
Cadmium sulphate.	$CdSO_4 \quad .4H_2O$	9.10			" "
Ferric sulphate	$Fe_2(SO_4)_3 \quad .9H_2O$	9.05			" "
Sodium sulphate....	$Na_2SO_4 \quad .10H_2O$	9.02			" "
Potassium sulphate..	K_2SO_4	8.85			" "
{ Fuming sulphuric .	H_2SO_4 and SO_3	7.20	8.60	10.35 μ	Naturally fluid
{ Acid and water....	in H_2O	8.60	9.60	11.35	" "
Nitric acid	HNO_3	7.85	10.55		" "
Glycerine..........	$C_3H_8O_3$	4.80	9.70		" "
Na-Silicate	Na_2SiO	9.05			" "
Nitroso-dimethyl aniline.............	$(CH_3)_2NC_6H_4NO$	7.40	9.00		Molten

SUMMARY

The results of this investigation may be summed up briefly as follows:

1. The reflecting power of amorphous selenium has been studied out to 13 μ. In consequence of the high and constant reflecting power of this substance, it was used in the construction of a polarizer and analyzer, adapted for work throughout the entire infra-red spectrum.

2. It was shown that infra-red radiations are capable of being polarized out to a wave-length of 13 μ, which was as far as the experiments could be carried.

3. It was shown that the non-metallic substance, Iceland spar, in its region of metallic reflection transforms plane into elliptically polarized light by reflection. This shows that, so far as its behavior toward plane polarized light is concerned, there is nothing to distinguish a non-metal from a metal.

4. Upon finding that the positions of the bands of selective reflection from a solid salt (*Na-K* tartrate) remain unchanged when the salt is molten, it is concluded that the mechanism giving rise to these bands is not affected by the freedom of motion of the molecule as a whole, and is therefore in all probability localized within the molecule itself.

5. By examining numerous liquids, it was found that these, as well as the solids, possess bands of selective reflection in the infra-red.

6. In the case of fuming sulphuric acid it was found that marked changes in the reflection-curves appeared when the acid was diluted. It was concluded that these changes were due to the breaking-down of certain compounds in solution and the consequent formation of new ones.

7. From the marked similarity in appearance and position of reflection maxima of the salts of a given acid (nitrates and sulphates) it was concluded that the mechanism giving rise to these maxima was localized within the acid radical.

Most of the substances used in this work were obtained through the kindness of Professor Harry C. Jones, of the Chemical Labora-

tory, and it is with pleasure that I acknowledge my indebtedness to him.

The present investigation has been carried out during the past year under the direction of Professor Ames, whom I wish to thank most heartily for his many valuable suggestions and his never-failing interest in the progress of the work.

Johns Hopkins University,
April, 1906.

ON A NEW FORM OF SPECTROHELIOGRAPH

By G. MILLOCHAU and M. STEFÁNIK

In presenting a new form of spectroheliograph, we consider it advantageous to recall the principal points in the history of this instrument.

In 1869 M. Janssen[1] described an apparatus for the observation of monochromatic images of luminous objects; this apparatus still exists at the Meudon Observatory, and we reproduce a photograph of it herewith (Fig. 1). It consists of a direct-vision spectroscope, in which the collimating lens is movable between two screws, permitting the spectrum to be displaced slightly. At the focus of the telescope lens is a second slit, the two jaws of which can be independently adjusted and used to isolate the desired radiation. This slit is observed with a positive eyepiece. The spectroscope thus described is contained within a tube, which can be moved rapidly about its axis by means of a system of gears. This instrument thus constitutes a spectrohelioscope, and was intended for the visual study of the prominences; but by substituting a sensitive plate for the eyepiece it might be immediately transformed into a spectroheliograph. M. Janssen's apparatus embodies the principal characteristics of the spectroheliograph, although his idea did not receive practical application during a score of years.

FIG. 1

[1] *C. R.*, **68**, 94, 713, 1869. *Ibid.*, *Inst.*, XXXVII, p. 307, 1869. *Mondes*, (2) XXI, p. 420.

During this period Braun and Lohse devoted themselves to the question. Braun, director of the Kalocsa Observatory, described a spectroheliograph in the *Astronomische Nachrichten* in 1873. His idea was to cause a two-slit spectroscope to move about a pivot passing through the point of intersection of the axes of the collimator and telescope. The image of the second slit was to be photographed by means of a camera. This apparatus was to be attached to an equatorial and moved by a mechanical device bearing upon a fixed point. Unfortunately, this instrument was not constructed, because of a lack of funds, and the idea remained unknown. Subsequently Lohse, of Potsdam, made unsuccessful experiments with a spectroheliograph of his invention.

Hale, who was not then acquainted with the investigations of his predecessors, devised two types of spectroheliographs and described them in 1889. In one of these devices the solar image was allowed to move across the first slit, while the photographic plate, placed immediately behind the second slit, was moved simultaneously; in the other plan the two slits were movable and the other parts of the apparatus stationary. In 1891 he obtained, with an instrument of the second type, the first photographs of the chromosphere and prominences, and in 1892 he extended his method to the study of the entire disk of the Sun.

A little later Mr. Evershed mounted his spectroheliograph (with direct-vision prism), which may be regarded as a special case of Braun's.

At the end of 1893 M. Deslandres published in the *Comptes Rendus* of the Paris Academy of Sciences the first results obtained with his spectroheliograph. His apparatus consists of a one-prism spectroscope, of small dispersion, placed on a carriage, by means of which it can be moved in a horizontal direction, perpendicular to the axis of the collimator. The photographic plate is moved behind the second slit by a system of levers attached to a fixed point.

A general defect of spectroheliographs is the tendency to record on the photographs all the vibrations produced by the various rolling or sliding parts comprised in their construction. This defect is due to the very principle of the instrument, according to which the solar image is built up by the integration of a line. Hence, in order

to obtain the best possible results, it is necessary to construct these instruments with special care, which renders them very costly and also difficult to operate.

In the arrangement which we have the honor to propose, we believe we have reduced the rolling and sliding friction to a minimum, and consequently have provided a means of diminishing, in large measure, the difficulty just mentioned. A two-slit spectrograph, of any form, is mounted so as to move around a horizontal axis perpendicular to the plane containing the optical axes of the spectrograph (Fig. 2). This arrangement is realized by the use of an axis turning between two centers. The motion is produced by a Brashear clepsydra, mounted vertically. It is connected with the spectro-

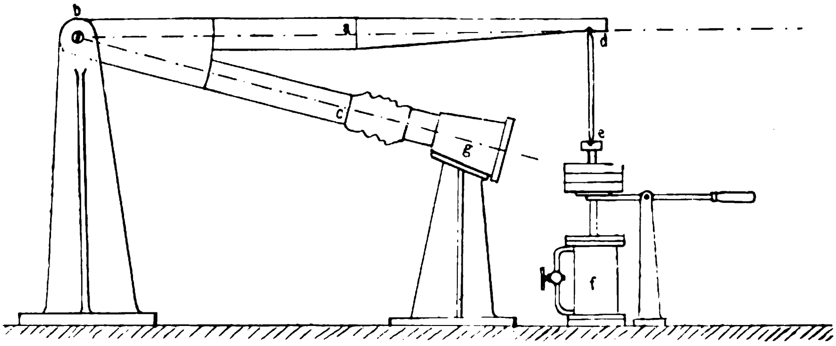


FIG. 2.—Diagram of Spectroheliograph.

a, *b*, *c*, spectrograph with grating.
b, horizontal axis of rotation.
a, first slit; *c*, second slit.
d, *e*, stem of junction with Brashear clepsydra, *f*.
g, fixed photographic apparatus for securing image given by movement of second slit.

graph by a bar with pointed extremities, which enter two conical holes, one of which is on the spectrograph in the prolongation of the optical axis of the collimator, while the other is at the end of the piston-rod of the clepsydra. The axis of rotation of the spectroheliograph must pass through the point of intersection of the axis of the collimator and that of the telescope of the spectrograph. The distances between this axis and the two slits should be in the ratio of the focal lengths of the collimator and telescope objectives.[1]

[1] These two principles were pointed out by Braun in 1873 (*Astronomische Nachrichten*, 80, 33–41, 1873).

In case a grating is employed,[1] the second slit may be stationary, and placed in the axis of the telescope; the setting on the spectral line can be accomplished by a slow rotation of the grating.

At its two extremities the slit is widened for the purpose of taking a photograph of a portion of the spectrum of the diffuse light of the sky, thus providing a simple means of determining the exact radiation with which the monochromatic photograph was obtained. A photographic plate may be placed immediately behind the second slit, supported by the stationary part of the instrument (Hale's arrangement), or the image may be photographed with a separate camera (Braun's arrangement).

This spectroheliograph may receive light from a siderostat or a cœlostat, or it may be attached directly to an equatorial. In the last case its mounting should be capable of rotation about the optical axis of the objective of the equatorial, in order to render the plane of the spectrograph nearly vertical, and the Brashear clepsydra should be arranged so as to be placed nearly vertical for any position of the Sun.

For solar investigations the invention of the spectroheliograph has the same importance as the discovery of the telescope in astronomy. It is a true monochromatic telescope, without which certain details of the solar constitution would perhaps have remained unknown. The thanks of the scientific world are due to M. Janssen, who devised and realized the first spectrohelioscope, and to Mr. Hale, who, though unacquainted with the earlier work, succeeded in constructing the first practical spectroheliograph and in obtaining the first monochromatic photographs of the Sun.

OBSERVATOIRE DE MEUDON,
April, 1906.

[1] The recent discovery by Hale of the dark hydrogen flocculi has shown the advantage of employing a grating for this class of investigation.

Minor Contributions and Notes

PLANETARY INVERSION

In view of the discussion between Professor Moulton and Professor W. H. Pickering in the number of the *Astrophysical Journal* for December last, perhaps a brief abstract of the results obtained by applying Sir George Darwin's theory of tidal friction to the question of planetary inversion may be of interest.[1] It seems certain to be the case that if a planet unattended by any satellites has an initial retrograde rotation, its axis of rotation will, under the influence of tidal friction, tilt over until the planet reaches a position of stable equilibrium in which its rotation will be direct. The stable value for the obliquity will lie somewhere between 0° and 90°, its exact value depending on the planet's viscosity, rate of rotation around its axis and of revolution in its orbit.

If satellites are introduced, the question becomes more complicated, and the stable value of the obliquity will vary with the different conditions as to the number of the satellites, their masses, and mean distances. There will be three possible equilibrium values for the obliquity according to different circumstances—values very near 0°, 90°, and 180°, respectively. *Jupiter* is certainly approaching the first of these values; assuming for it initial retrograde rotation, its satellite system must have been evolved after its obliquity had under the influence of tidal friction alone decreased to some value less than 90°. *Saturn*, on the other hand, evolved *Phœbe*, and possibly also *Japetus* and *Hyperion*, while its obliquity was still greater than 90°. As its obliquity approached this value, *Phœbe's* orbit moved down to the ecliptic (thus remaining retrograde), while *Japetus* and *Hyperion* followed *Saturn's* equator over. Later on, the inner satellites were evolved, and under their influence and that of the ring *Saturn* is moving into a stable position of small obliquity. In the cases of *Uranus* and *Neptune*, lack of sufficient data makes it impossible to say with any accuracy what is happening but it seems most likely that *Neptune* is being driven by its one satellite into a stable position with an obliquity of 180°. The obliquity of *Uranus*, too, is possibly being increased at present, but in this case the result is very doubtful.

[1] The paper containing the details of the investigation, which was suggested to me by Professor H. H. Turner, has been published in the *Monthly Notices* of the Royal Astronomical Society, April, 1906.

As I have stated in my paper, the way in which Professor W. H. Pickering stated his theory led Professor Moulton to a not unnatural misconception as to the couple which would be responsible for the inversion of the planet. The misconception has in large part invalidated Professor Moulton's brief criticism of the theory and it underlies certain assumptions necessary for the validity of this criticism, but not, as I view the question, necessary for the validity of the theory. A careful examination of the objections raised by Professor Moulton has shown that they do not apply to the form of the theory which I have discussed.

It remains to add that the theory is beset by many difficulties, such as the great extent of time involved and the doubtful factor introduced by the heterogeneity of the planets. It does not, so far as I can see at present, explain the high obliquities of *Jupiter's* recently discovered satellites; but it is an hypothesis which does offer an explanation of the retrograde motion of *Phœbe* in its orbit, and of the retrograde rotations of *Uranus* and *Neptune*.

F. J. M. STRATTON.

CAIUS COLLEGE, CAMBRIDGE.

A MECHANICAL ILLUSTRATION OF THE PLANE GRATING

The experiment described below was devised for a semi-popular lecture to illustrate by mechanical waves the production of the various spectra by a plane grating. The scheme was to produce, on the surface of mercury, ripples emanating from a series of equidistant points distributed along a straight line. These points of course correspond to the spaces between the rulings of the grating, and the ripples to the secondary waves coming from these spaces.

In order to produce the ripples, a thin sheet of iron was cut into the form of a comb of sixteen teeth spaced 5 mm apart, and attached to the lower prong of an electrically driven tuning-fork, arranged to vibrate in a vertical plane. The frequency of the fork was not known, but was probably under 100. The apparatus was then arranged so that the teeth of the comb dipped into the surface of the mercury, which was contained in a large shallow, circular tray; the comb, however, was not set in the middle of the tray, but rather near one edge.

The waves set up when the fork was put in motion moved much too fast to be distinguishable by unaided vision, only the streaked appearance of the disturbed surface being visible in this way. However, when the surface was viewed through a stroboscope rotating at the proper speed, it was seen that close to the comb there was a chaotic mass of waves,

which farther out resolved themselves into several series of regular rectilinear wave-trains, advancing in different directions, and symmetrically distributed around a line normal to the comb at its middle point. These different series of wave-trains correspond respectively to the spectrum of order zero (moving out in a direction normal to the comb), and spectra of orders one and two on each side.

In order to project the waves on the wall for lecture purposes, use was made of a large cheap lens, of 20 cm diameter, and of 90 cm focal length. By means of this a considerable area of the mercury surface (a circle about

15 cm in diameter) was illuminated by *convergent* light from an arc, striking the surface with an angle of incidence of about 20°. Where the reflected light came to a focus (forming a clear image of the arc when the mercury was undisturbed), the projecting lens was placed to focus the image of the waves upon the wall, a good mirror being inserted in the path to deflect the rays in the proper direction. The scheme of focusing the light from the arc upon the projecting lens is of course essentially the principle of lantern-slide projection, and its great advantages, when applied to the projection of mercury waves, is sufficiently obvious. The

edges of the projected area are sharp and clear, and the illumination brilliant and practically uniform.

The figure is a reproduction of a photograph taken by placing a camera so that its lens replaced the projecting lens at the focus of the arc. The exposure was made by rapidly drawing across the path of the light an opaque screen containing a narrow slit; for the fastest speed on an ordinary commercial camera shutter, rated as $\frac{1}{100}$ second, was found to be too slow for the purpose.

Some trouble was caused by reflection from the walls of the tray, causing the formation of standing waves of small amplitude all over the surface. This was alleviated by lining the walls with corrugated strips of sheet iron, which to a certain extent dispersed the waves, instead of reflecting them regularly.

HERBERT M. REESE.

UNIVERSITY OF MISSOURI,
June 1906.

AN OCCULTING SHUTTER FOR CONCAVE GRATING SPECTROSCOPES

In almost any spectroscopic work with photography, if the plates are to be accurately measured, a comparison spectrum is a prime requisite; and in order to get this comparison spectrum on the photographic plate without any accidental shift with reference to the principal spectrum, some mechanical contrivance is usually necessary. For use with the concave grating, probably the best known and most efficient of these contrivances is the simple device due to Rowland. This consists of a flat bar, about one inch wide, one-eighth inch thick, and as long as the plate, having down its middle a slit of the same width as the thickness of the bar. It is mounted immediately in front of the plate, and arranged so that it can be turned through ninety degrees about its longest axis. When the plane of the flat face is vertical, only a strip down the middle of the plate is exposed to the grating; when it is horizontal, this central strip is covered, and the regions above and below are exposed.

This simple and convenient device has two difficulties. In order to secure room to turn, it cannot be placed very close to the plate, so that the edges of the exposed regions show a penumbral effect which sometimes becomes an annoyance. Moreover, the bar is mounted on the camera-box, and there is always the possibility that in turning it some slight jar will cause an appreciable movement of the plate lengthwise and so introduce between the lines on the two spectra a lateral shift which might be falsely ascribed to a difference in wave-length.

To obviate the first of these difficulties Hale replaced Rowland's rotating "shutter" by a pair of metal plates with suitable openings in them which are slid into grooves in the plate-holder close up to the surface of the photographic plate. First one of these plates is put in place and the central strip of the photographic plate is exposed. Then it is withdrawn, the other slid into its place, and the second exposure made, this time the central strip being covered and the edges exposed. Although this device successfully eliminates the penumbra effect, the chances for accidental displacements to occur are vastly greater than is the case with Rowland's simple shutter, so that it can hardly be recommended unless elaborate precautions be taken to detect such displacements.

In a plane-grating spectroscope or a prism spectroscope the natural place to introduce an occulting-bar is at the slit, for in both cases the slit is in focus on the plate both for horizontal and for vertical lines. There is of course some chance that a displacement of the slit will occur while the occulting-bar is being changed, but with moderate care such an accident is very unlikely.

In the case of a concave grating, owing to the great astigmatism, a horizontal line at the slit is very much out of focus on the plate, and therefore an occulting-bar at the slit would not occult. However, there is in front of the slit (i.e., on the side remote from the grating) a point for which a line at right-angles to the slit will be in focus on the plate. Consequently, it would seem that an occulting-bar placed at this point would secure the desired sharpness of focus, and at the same time be absolutely free from any possibility of causing accidental displacements in either plate or slit. The writer would be glad if this were seriously tried. At present he has not access to a concave grating, but some time ago he made a few preliminary trials with partial success. A bar about one-eighth inch wide was set up horizontally in front of the slit and a position found for which it was in approximate focus on the plate. The edges of the shadow did not appear very sharp; but whether this was due to lack of careful adjustment, to spherical aberration at the grating, or to diffused light from the grating (which was not a very good one) is not known. Of course a careful adjustment must be made to get the edges of the bar accurately straight and at right-angles to the slit, for the astigmatism of the grating affects the sharpness of this shadow, just as it affects the sharpness of the spectral lines. The bar may conveniently be made of the same shape as Rowland's shutter, but need be only two or three inches long—no longer, in fact, than the width of the beam of light at that point. Of course, the position of the bar would have to be changed whenever

the position of the grating is altered with reference to the slit; but this would cause less trouble than one might suppose, for in most cases of actual use the position of the grating is not frequently changed.

HERBERT M. REESE.

UNIVERSITY OF MISSOURI,
February 15, 1906.

A SHORT METHOD OF COMPUTING AN APPROXIMATE VALUE OF THE REDUCTION TO SUN IN RADIAL VELOCITY DETERMINATIONS

The chief function of the following method is to check roughly and quickly the rigorous computation, but it may also be used in reducing approximate measures of plates when an accuracy of 1 km is sufficient. In order to make it as simple as possible, the Earth is supposed to move in a circle with a constant velocity of 29.8 km per second. With this assumption the formula is reduced to

$$29.8 \cos\beta \sin(\odot - \lambda) \quad ,$$

where λ and β are the longitude and latitude of the star, and $\odot$ the longitude of the Sun. The factor $29.8 \cos\beta$ is constant for any star and may be determined once for all.

To avoid the same constants used in the rigorous computation, we may employ the time which has elapsed since conjunction with the Sun in longitude, instead of $(\odot - \lambda)$. If we let

T = date of conjunction with Sun.
t = " " observation.

we have

$$(\odot - \lambda) = f(t - T) \ .$$

If T and t be expressed in days of the year, we shall have

$$(\odot - \lambda)^\circ = \frac{360}{365}(t - T)^{\mathrm{d}} .$$

If we now have a table giving $\sin\left\{\frac{360}{365}(t-T)\right\}$ for $(t - T)$ as argument, the whole operation becomes simplified to the following:

Subtract the date of conjunction (one of the constants of the star) from the date of observation, take out the sine with this as argument, and multiply this sine by $29.8 \cos\beta$ (the second constant).

A small error is introduced in using $(t - T)$, because of the eccentricity of the Earth's orbit. As most of the observations are made near opposition, it would be more accurate to use the date of opposition as

T, but this would introduce an opportunity for an error in the sign. As a compromise I have found a fictitious date of conjunction by first finding the date of opposition, and then subtracting $182^{d}6$. Table II has been computed in this way. In computing, the dates were taken to tenths of a day, and then cut down to the nearest full day, as that is accurate enough and the constants will not change appreciably from year to year.

To test the accuracy of this method, the reduction to Sun was computed for several stars near the ecliptic at different times of the year and compared with the rigorous computations already made. The largest difference found was 0.6 km. As it was thought that this was not the largest difference possible, a test case was tried of a star on the ecliptic observed in October (when i has a maximum value) and only 30 days from conjunction. This gave a difference of 1.3 km, but for a star 150 days from conjunction, observed in October, the difference was reduced to 0.6 km, which was to be expected, as the tables are so computed as to make these differences a minimum in the vicinity of opposition. The average difference found for all actual observations was about 0.3 km.

For cases where T is greater than t—that is, when the preceding conjunction comes in the preceding year—we must add 365 to t, and the form becomes $(t+365-T)$. To avoid this extra addition a third column is given in Table II with the constant $365-T$, and when this is used it is to be *added* to the date of observation. For convenience we have been in the habit of writing the two on the card for the star, thus: 200−165, where the first is to be used if the observation and preceding conjunction occur in the same year, and the second when they occur in different years.

Table I facilitates finding the day of the year; Table II gives the dates of (fictitious) conjunction with the Sun for different longitudes; Table III gives the values of $29.8 \cos\beta$ for different latitudes; and Table IV gives the values of $\sin\left\{\frac{360}{365}(t-T)\right\}$. In this table the values for every tenth day are tabulated in the column headed "0," and those for the other days in the other columns.

As an example of the method of computing the reduction to Sun we will take the observation made on June 14, 1905, of *λ Andromedae:*

$$
\begin{array}{rlrl}
\lambda = & 16^\circ\ 54' & \beta = & +43^\circ\ 48' \\
t = & 165 & 29.8\cos\beta = & 21.4 \\
T = & 101 & & \\
t-T = & 64 \text{ days} & \sin j(t-T) = & 0.89
\end{array}
$$

V_a $= +19.0$ km

V_a rigorously computed $= +19.4$

TABLE I

Day of Year

	Ordinary	Leap Year
January 0	0	0
February 0	31	31
March 0	59	60
April 0	90	91
May 0	120	121
June 0	151	152
July 0	181	182
August 0	212	213
September 0	243	244
October 0	273	274
November 0	304	305
December 0	334	335

TABLE II

λ	Date of Conjunction	λ	Date of Conjunction	λ	Date of Conjunction
0°	84^d—281	120°	203^d—162	240°	324^d— 41
10	94 —271	130	213 —152	250	334 — 31
20	104 —261	140	222^5—142^5	260	345 — 20
30	114 —251	150	232 —133	270	355 — 10
40	124 —241	160	242 —123	280	1 —364
50	134 —231	170	252 —113	290	11 —354
60	144 —221	180	262 —103	300	22 —343
70	154 —211	190	272^5— 92^5	310	32 —333
80	164 —201	200	283 — 82	320	43 —322
90	173 —192	210	293 — 72	330	53 —312
100	183 —182	220	303 — 62	340	63 —302
110	193 —172	230	313^5— 51^5	350	74 —291

TABLE III

β		V cos β
0°	. . . .	29.8 km
10	. . . .	29.4
20	. . . .	28.0
30	. . . .	25.8
40	. . . .	22.8
50	. . . .	19.2
60	. . . .	14.9
70	. . . .	10.2
80	. . . .	5.2
90	. . . .	0.0

TABLE IV

$$\mathrm{Sin}\left\{\frac{360}{365}(t-T)\right\}^{\circ}$$

$t-T$	0	1	2	3	4	5	6	7	8	9
0°	0.00	+0.02	+0.04	+0.05	+0.07	+0.09	+0.11	+0.12	+0.14	+0.16
10	+ .17	+ .19	+ .20	+ .22	+ .24	+ .26	+ .27	+ .29	+ .30	+ .32
20	+ .34	+ .36	+ .37	+ .39	+ .40	+ .24	+ .44	+ .46	+ .47	+ .49
30	+ .50	+ .52	+ .54	+ .55	+ .57	+ .58	+ .59	+ .60	+ .61	+ .63
40	+ .64	+ .65	+ .66	+ .67	+ .68	+ .69	+ .70	+ .72	+ .73	+ .74
50	+ .75	+ .76	+ .77	+ .78	+ .79	+ .80	+ .81	+ .82	+ .83	+ .84
60	+ .85	+ .86	+ .87	+ .88	+ .89	+ .90	+ .91	+ .92	+ .92	+ .93
70	+ .94	+ .95	+ .95	+ .69	+ .96	+ .97	+ .97	+ .98	+ .98	+ .99
80	+ .99	+ .99	+ .99	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00
90	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00	+ .99
100	+ .99	+ .99	+ .99	+ .99	+ .98	+ .98	+ .98	+ .97	+ .97	+ .96
110	+ .96	+ .95	+ .94	+ .93	+ .92	+ .91	+ .90	+ .90	+ .89	+ .88
120	+ .87	+ .86	+ .85	+ .85	+ .84	+ .83	+ .82	+ .81	+ .80	+ .79
130	+ .78	+ .77	+ .76	+ .75	+ .74	+ .73	+ .71	+ .70	+ .69	+ .68
140	+ .67	+ .65	+ .64	+ .63	+ .61	+ .60	+ .59	+ .58	+ .57	+ .55
150	+ .54	+ .52	+ .50	+ .49	+ .48	+ .47	+ .45	+ .43	+ .41	+ .40
160	+ .38	+ .37	+ .35	+ .33	+ .31	+ .30	+ .28	+ .26	+ .24	+ .23
170	+ .21	+ .19	+ .18	+ .16	+ .14	+ .12	+ .11	+ .09	+ .08	+ .06
180	+ .05	+ .03	+ .01	− .01	− .03	− .05	− .07	− .09	− .11	− .12
190	− .14	− .15	− .17	− .19	− .21	− .22	− .23	− .25	− .27	− .28
200	− .30	− .31	− .32	− .34	− .36	− .37	− .39	− .40	− .42	− .43
210	− .44	− .46	− .48	− .50	− .51	− .53	− .54	− .55	− .57	− .59
220	− .60	− .61	− .62	− .63	− .64	− .66	− .67	− .69	− .70	− .71
230	− .73	− .74	− .76	− .77	− .78	− .79	− .80	− .81	− .82	− .83
240	− .84	− .85	− .86	− .87	− .88	− .89	− .90	− .91	− .91	− .92
250	− .92	− .93	− .94	− .94	− .95	− .95	− .96	− .96	− .97	− .97
260	− .98	− .98	− .99	− .99	− .99	− .99	−1.00	−1.00	−1.00	−1.00
270	−1.00	−1.00	−1.00	− .00	−1.00	−1.00	−1.00	−1.00	−1.00	− .99
280	− .99	− .99	− .99	− .99	− .98	− .98	− .98	− .97	− .96	− .96
290	− .95	− .95	− .94	− .93	− .93	− .92	− .92	− .91	− .91	− .90
300	− .89	− .89	− .88	− .88	− .87	− .86	− .85	− .84	− .83	− .82
310	− .81	− .80	− .79	− .78	− .77	− .75	− .74	− .73	− .72	− .71
320	− .70	− .68	− .66	− .65	− .64	− .63	− .61	− .60	− .58	− .56
330	− .55	− .53	− .51	− .50	− .48	.47	− .46	− .44	− .42	− .40
340	− .39	− .37	− .36	− .34	− .33	− .31	− .30	− .28	− .27	− .25
350	− .24	− .23	− .21	− .19	− .17	− .16	− .15	− .13	− .11	− .10
360	− .08	− .07	− .06	− .04	− .02	− .00				

H. K. PALMER.

MOUNT HAMILTON,
January 29, 1906.

LETTER FROM PROFESSOR J. LARMOR

BURLINGTON HOUSE, LONDON, W., June 28, 1906.

The Editor of the Astrophysical Journal.

DEAR SIR: Inclosed is a brief sketch of the result of the deliberations of the Royal Society regarding improved methods of publication of their journals. I think it will be of advantage to your readers to draw their attention to it as a piece of news, as we find that advertisements do not arrest attention. I need hardly say that what the Society desires in the first place is effective circulation, not least in America.

Very faithfully yours,

J. LARMOR,
Secretary, R. S.

Of the *Proceedings* of the Royal Society of London, as divided about a year ago into two series, Vols. 76–77 of Series "A," containing papers of a mathematical and physical character, and Vols. 76–77 of series "B," containing papers of a biological character, have now appeared, each running to about 600 pages royal octavo, with illustrations. A main object of this new arrangement was to render the *Proceedings* more accessible to workers by placing the two groups of subjects on sale separately, at a stated price attached to each separate part of a volume when it first appears. Moreover, with a view to promoting the circulation of the complete series it has been directed that a subscription paid in advance to the publishers, at the reduced price of 15*s.* per volume for either series, shall entitle subscribers to receive the parts as soon as published, or else the volumes when completed, in boards or in paper covers, as they may prefer.

With a view to further increasing the accessibility of the various publications of the Royal Society, each number of *Proceedings* now contains an announcement on the cover, of the more recent memoirs of the *Philosophical Transactions* as published separately in wrappers and the prices at which they can be obtained.

It is hoped that by this arrangement the difficulties which have been found to impede the prompt circulation of the journals of the Society, which are of necessity published in a somewhat different manner from a regular periodical, may be finally removed.

Reviews

Analysis of the Spectrum Lines of Mercury, Cadmium, Sodium, Zinc, Thallium, and Hydrogen; By L. Janicki.

This interesting research was presented as a dissertation at Halle in 1905, and appeared in the *Annalen der Physik*, **19**, 36–79, 1906.

The study of the composition of the lines in the spectrum of many elements has been the subject for many researches. The lack of good agreement between the different results led the author, as it has led others, to make further experiments.

An echelon grating made by A. Hilger was employed. It consisted of 32 plates 1 cm thick, with a step-width of 1 mm. The instrument seemed to be a very accurate one, giving very clear lines without ghosts, as is shown in the reproductions. The estimated limit of error in the measurement of the components was from 0.01 to 0.02 Å. U. for the very weak lines, 0.005 for the hazy lines, and 0.003 for the sharp lines.

The observations were made with many kinds of sources. The flame and the spark were not satisfactory, although by placing from six to ten large Leiden jars with sufficient self-induction in the secondary the sharpness of the radiations from the spark was much improved. He did not investigate the arc in air. For the mercury spectrum he employed the following sources: Arons' lamp of the form suggested by Lummer, large and small quartz lamps of Heraeus' make, Geissler tubes, vacuum tubes of the form used by Eder and Valenta, and tubes with external electrodes. The results as to the structure of the lines from all these sources were the same. In some cases the weaker components were invisible, due to the small intensity of the source. While the components often showed reversal and changes in their intensity, new satellites never appeared.

Mercury.—The results of Janicki's observations upon the mercury lines are as follows: The difference in wave-length of the components from the principal line is given in Å. U. Their relative intensity follows in brackets.

$\lambda = 5790.$

+0.230 ($\frac{1}{7}$); +0.168 ($\frac{1}{10}$); +0.132 ($\frac{1}{7}$); +0.084 ($\frac{1}{5}$);
Principal line (1); −0.119 ($\frac{1}{5}$); −0.187 ($\frac{1}{10}$); −0.251 ($\frac{1}{10}$).

$\lambda = 5769.$

+0.120 ($\frac{1}{10}$); +0.087 ($\frac{1}{10}$); +0.046 ($\frac{1}{3}$);
Principal line (1); −0.050 ($\frac{1}{4}$); −0.113 ($\frac{1}{4}$).

$\lambda = 5461$.

+0.133 ($\frac{1}{8}$); +0.088 ($\frac{1}{3}$);
Principal line (1); −0.066 ($\frac{1}{7}$); −0.099 ($\frac{1}{10}$); −0.232 ($\frac{1}{5}$).
λ 4916 has no components.

$\lambda = 4359$.

+0.121 ($\frac{2}{3}$); +0.105 ($\frac{2}{3}$); +0.043 ($\frac{1}{4}$); +0.020 ($\frac{1}{5}$);
Principal line, (1); −0.023 (1); −0.052 ($\frac{1}{7}$); −0.097 ($\frac{2}{3}$);
−0.112 ($\frac{2}{5}$).

$\lambda = 4348$.

+0.083 ($\frac{1}{7}$); +0.053 ($\frac{1}{7}$); Principal line (1); −0.046 ($\frac{1}{5}$).

$\lambda = 4339$.

+0.06 ($\frac{1}{10}$); Principal line (1) −0.12 ($\frac{1}{10}$).

$\lambda = 4078$.

+0.074 ($\frac{1}{9}$); +0.049 ($\frac{1}{4}$); +0.032 ($\frac{1}{10}$);
Principal line (1); −0.046 ($\frac{1}{8}$); −0.076 ($\frac{1}{6}$).

$\lambda = 4047$.

+0.067 ($\frac{1}{2}$); Principal line (1); −0.051 ($\frac{3}{4}$); −0.111 ($\frac{1}{3}$)

A very interesting and peculiar phenomenon occurred with the radiations from the quartz lamps. During the first few minutes after excitation the lines gave the constitution as given above. When their intensity was increased by taking out some of the external resistance, the yellow lines $\lambda\lambda$ 5790, 5769 and the bright green line λ 5461 broke up into five equidistant hazy lines. The same phenomenon happened in the radiations from the mercury spark in air. The most remarkable thing was that the number of these hazy lines or bands was always five. No explanation for this is offered.

Cadmium.—Both Michelson's tubes and tubes with external electrodes were used. The results are as follows:

$\lambda = 6439$ has no components.
$\lambda = 6325$ is a hazy line. It is a double line in tubes with external electrodes.
$\lambda = 5155$ is also a single line.

$\lambda = 5086$.

+0.076 ($\frac{1}{5}$); Principal line (1); −0.026 ($\frac{1}{8}$?)

$\lambda = 4800$.

+0.059 ($\frac{1}{4}$); Principal line (1); −0.034 ($\frac{1}{6}$); −0.080 ($\frac{1}{5}$).

$\lambda = 4678$.

+0.030 ($\frac{1}{3}$); Principal line (1); −0.056 ($\frac{1}{6}$).

$\lambda = 4662$ is a single line.

Sodium.—Due to the high temperature in the cadmium tubes, sodium vapor was given off from the glass. The D lines were without components, but they easily reversed, giving the appearance of double lines.

Zinc.—A tube with external electrodes was employed. The lines $\lambda\lambda$ 6362, 5182, 4810, 4722, 4680, are all sharp and single.

Thallium.—The green line of thallium λ 5351 produced in a Hamy tube has one component of half the intensity of the principal line and of greater wave-length.

Hydrogen.—The red hydrogen line λ 6563 is double, the distance between the components is 0.14 Å. U. Both components show reversal.

The following are his conclusions. They cannot be considered as new. His results in general corroborate those of other investigators, and his remarks regarding the causes of the differences in the results have been cited before.

1. All the strong lines in the mercury spectrum are very complex, with the exception of λ 4916. Only some of the lines of cadmium have components. The D lines and the zinc lines are single. The red line of hydrogen is double.

2. No change in the wave-length of the components was observed. The relative intensities, however, often varied. This result is worthy of notice, because such changes give the appearance of a shifting of the lines if the apparatus has not sufficient resolving power.

3. "Ghosts" may be excluded from the observations. Even if the agreement of the above results with those of others is not good, the author has good reasons for the exclusion of ghosts. In all parts of the spectrum the echelon used showed different strong lines as single, while Lummer and Gehrcke with their spectroscope did not find a line without components. Further proof is given by the very different structure of the neighboring lines $\lambda\lambda$ 5790, 5769 of mercury, 5896, 5890 of sodium, and 5876 of helium. To this is added the good agreement of the measurement of the yellow helium line with that of Runge and Paschen, who used a large Rowland grating.

4. The larger the resolving power of a spectroscope, the easier can a reversal of a line be observed which as such is often impossible to recognize.

James Barnes.

Johns Hopkins University.

Atlas Stellarum Variabilium, Series V, containing, for all parts of the sky, the variable stars whose minimum light is above magnitude 7. By J. G. HAGEN, S.J., Director of Georgetown College Observatory. Berlin: Felix L. Dames, 1906. 10×12 inches, 21 charts, catalogue and index.

Series I, II, and III of this important work, already published, contain the well-known telescopic variables from declination $-25°$ to the north pole. Series IV, promised for the autumn of 1906, will comprise a list of the brighter telescopic variables for which a three-inch telescope will suffice. The approaching completion of this useful work thus adds to the value of each part. It is also of interest to note that this is probably the last work of the kind which will be done without the aid of photography.

From the nature of the subject-matter, the charts and catalogue sheets differ in numerous particulars from the former series. As most of the charts contain more than one variable, they are arranged by regions, and therefore on different scales, the radius of the sphere varying (Table I) between 160 and 650 mm. The stars are printed in black with disks of different shapes and carefully graded sizes from magnitudes 1 to 7. The lines and constellation names on the charts are in red, therefore indistinct by lamplight, in fact almost invisible by the red light which many observers prefer for recording observations of faint variables at the telescope.

The difficult problem of deciding just what stars to show on the charts has been solved as follows. Only stars to the fifth magnitude are shown, except in the vicinity of the variables. In the vicinity of each variable, fainter stars to magnitude 7 were selected *from the sky itself*, for the double purpose of furnishing a suitable series of comparison stars and making identifications easy and certain. As a further aid the sheet for Chart XIX has in one corner a small chart of the region around *η Carinae* on a threefold scale. While more of these auxiliary charts would be useful, it is evident that completeness would be an impossibility.

The catalogue sheets accompanying each chart are models. They contain the current number of each star shown on the chart, the constellation, Bayer's letter, the Flamsteed and *B. D.* numbers, and the positions for 1900. Then follow the most important columns, giving the photometric magnitudes from the Potsdam and Harvard catalogues, or for the southern stars from the *Uranometria Argentina*, a column of notes giving other designations of some stars and colors from Krueger's *Catalog der farbigen Sterne*. Last, and most important of all, is a table for each

variable showing the comparison stars which have been used by each of the best observers who have worked with this class of variable stars.

Only one variable of doubtful standing has been admitted to the list, that being *32 (RU) Cassiopeiae*, for which the evidence for and against a change in light is conflicting and very puzzling. But this does not detract from the high praise which must be accorded to this exceedingly valuable work which puts the astronomical world under great obligations to the distinguished author and his beneficent patroness, the late Catherine W. Bruce, by whose liberality the expenses of publication were met

J. A. PARKHURST.

PLATE I

SPECTROSCOPIC LABORATORY OF THE SOLAR OBSERVATORY

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXIV SEPTEMBER 1906 NUMBER 2

THE SPECTROSCOPIC LABORATORY OF THE SOLAR OBSERVATORY[1]

By GEORGE E. HALE

An important requirement in the programme of research of the Solar Observatory is met by the provision of a spectroscopic laboratory, adequately equipped for the investigation of such physical and chemical phenomena as may be encountered in connection with our solar and stellar observations. It will only occasionally happen that data required to explain the results obtained with the solar spectrograph and the spectroheliograph, are already available in spectroscopic literature. The Zeeman effect, for instance, has been recorded in the case of comparatively few lines, so that without additional observations it could not be determined whether certain lines which vary together in the Sun, also behave alike in a magnetic field. Displacements due to pressure must frequently be known in connection with measurements of lines in the solar spectrum, but the required values are only occasionally found in published papers. The same may be said of the effects of change of temperature, of potential, of self-induction, of current strength, of the influence of the gaseous atmosphere in which the luminous source is placed, etc. It is evident that what is essential, in case the results constantly encountered in both solar and stellar

[1] *Contributions from the Solar Observatory*, No. 10.

work are to be interpreted without delay, is a collection of light-sources so designed and arranged that the effect of any of the above mentioned variables can be observed with spectroscopes of any desired resolving power.[1] It is not so much a question of the saving of time, which the provision of these means undoubtedly offers, as it is of the greatly increased efficiency of the working programme thus rendered possible. The immediate imitation in the laboratory, under experimental conditions subject to easy control, of solar and stellar phenomena, not only tends to clear up obscure points, but prepares the way for the development along logical lines of the train of reasoning started by the astronomical work. It is a question, then, of equipping the laboratory in such a way that its various resources may be effectively utilized at any time, and without the delays ordinarily experienced when apparatus must be specially prepared for a certain investigation. In the desired plan the apparatus must be always ready, needing only the operation of a switch or the adjustment of a mirror to bring it into action.

In the spectroscopic laboratory of the Yerkes Observatory I first tried the principle which has been more fully developed in our spectroscopic laboratory on Mount Wilson. The various light-sources, including an ordinary arc in air, a spark in air, a spark in liquids, and a spark in compressed gases, were arranged on the circumference of a wooden table, having at its center a plane mirror capable of rotation about a vertical axis. By means of this mirror, set at the proper angle, light from any one of the sources was reflected to a concave mirror which, in its turn, formed an enlarged image of the light-source on the slit of a concave grating spectrograph.

At the Solar Observatory the apparatus is arranged as shown in Plate I. Instead of a circular wooden table, an annular concrete pier is employed, giving space on the inner wall for the various switches used to control the current supplied to the different sources, and also permitting the observer to inspect any light-source from the direction of the plane mirror at the center of the pier. Instead

[1] As an illustration of this it may be remarked that in our study of sun-spot spectra the following light-sources have been employed: ordinary arc, in air and in CO_2; rotating arc, at low and high pressures; synchronous arc; spark in air; spark in water; oxyhydrogen flame; electric furnace.

of a single plane mirror, two are provided, capable of rotation independently of one another, about the same vertical axis. By means of divided circles, the azimuth of either mirror can be read. When the Littrow spectrograph[1] is in use, only the lower plane mirror is employed. By setting this at the proper angle the light from any one of the sources can be sent to the concave mirror (seen near the the middle of Plate I), which forms an image of the source on the slit of the Littrow spectrograph. If the one-prism quartz spectrograph, the interferometer, or the echelon spectroscope is to be used in place of the Littrow spectrograph, for the study of the light-source, the concave mirror is tipped back at a small angle, so as to return the light to the upper plane mirror, from which it is reflected to the slit of one of these instruments. In Plate I the quartz spectrograph may be seen just above the concave mirror. To the right of this is a Hilger one-prism spectroscope, which provides a monochromatic beam for observation with the interferometer (shown in position near the spectroscope), or with the echelon spectroscope, which stands on the right of the pier that carries these instruments. This pier is separated from the annular pier by a space through which the observer may pass.

The various analyzing instruments, with which the light-sources are studied, may be briefly described as follows:

1. A direct-vision spectroscope by Jobin, for the preliminary visual examination of spectra.

2. A Fuess quartz spectrograph, with collimator of 3.8 cm aperture and 81.3 cm focal length, and camera of the same aperture and focal length. This instrument has a double Cornu prism of quartz, and a Zeiss prism of ultra-violet glass, to be used if somewhat higher dispersion is desired. The spectrograph is provided with means for photographing a series of narrow spectra on a single plate, and is used for preliminary and for qualitative studies, especially in the ultra-violet.

3. A Littrow spectrograph of 12.5 cm aperture and 3.72 m focal length. The combined camera and collimating lens is by Brashear, and is corrected for work in the visible spectrum. The

[1] The rectangular box which carries the slit and plate-holder of this instrument is shown on the pier at the left of Plate I.

plane grating, by Michelson,[1] has a ruled surface 8.3×4.4 cm, with 7,000 lines to the cm. It may be used in any order, by setting it at the proper azimuth. The slit is mounted immediately above the photographic plate. The plate-holder can be raised or lowered, so as to permit several spectra to be photographed on a single plate.[2]

4. A Michelson interferometer (not shown in the plate), for the analysis of spectral lines, the determination of absolute wave-lengths and the measurement of lengths. This instrument receives monochromatic light from a one-prism spectroscope by Hilger, provided with a prism of special form, from which the beam emerges at an angle of 90° with the axis of the collimator. Different parts of the spectrum can be brought to the slit of the interferometer collimator by rotating the prism. The same spectroscope can be used in connection with a Pérot-Fabry interferometer or with an echelon spectroscope.

5. A Pérot-Fabry interferometer by Jobin, exactly similar to the instrument used by Messrs. Pérot and Fabry for the determination of absolute wave-lengths and the analysis of spectral lines.

6. A 33-plate echelon spectroscope by Hilger, with plates 15 mm thick. This instrument is used in studies of the Zeeman effect, and other work requiring the highest obtainable resolving power. The slit can be opened or closed from the eye-end, or moved entirely out of the way when a spectral line is being picked out for observation. In the latter case the echelon is moved to one side and the image of the spectrum, formed by the auxiliary spectroscope in the focal plane of the collimator of the echelon spectroscope, is seen with the observing telescope. Provision is made for using the echelon at 90° from its ordinary position, as Nutting has done, for the purpose of separating the overlapping spectra. For spectra having but few lines it is sometimes advantageous to remove the prism from the auxiliary spectroscope and insert it beyond the echelon. When this is done the observing telescope stands at 90° with the axis of the echelon, as shown in Plate I.

[1] We are indebted to Professor Michelson for the use of this excellent grating, which is one of the first products of his new ruling engine.

[2] This temporary instrument will be replaced by a permanent one of 5.5 m focal length.

The following apparatus stands on the annular pier: The first instrument on the right is a DuBois magnet of Hartmann & Braun's larger model, for studies of the Zeeman effect. It is mounted on a base in such a way that it can be rotated through an angle of 90°, so that light from the source can be observed parallel or at right angles to the lines of force. A bismuth spiral is provided for measuring the strength of the field. In the illustration a mercury tube is hung between the poles of the magnet, and connected by heavy pressure tubing with the Geryk duplex vacuum pump shown on the wooden table at the right. The magnet requires a potential of 64 volts, which is supplied from a storage battery in an adjoining building. The vacuum pump, which is supplied with a McLeod gauge by Müller-Uri, is driven by a small electric motor. A transformer of 600 watts capacity, giving from 6,000 to 10,000 volts, is used to illuminate the mercury tube. This is supplied with an alternating current from the generator in the power house. In connection with the transformer there are provided a condenser of variable capacity, and a coil of variable inductance.

Just beyond the magnet may be seen the apparatus for the study of the spark spectrum in air, at atmospheric pressure. The metals to be investigated are held by a pair of clamps in the lower set of terminals. The upper terminals are in series with the lower ones and serve as an auxiliary spark-gap. An electric fan, blowing a strong current of air upon the two sparks, prevents arcing and undue heating of the poles.

To the left of the air spark stands the apparatus for the study of the spark spectrum in water and other liquids. This consists of a strong iron case, into which the horizontal terminals are led through long corrugated cylinders of vulcanite. Glass windows, in the front and rear of the case, permit the spark to be seen from either side. The metals to be investigated are turned or ground into the form of small rods, and are mounted in brass caps, which can be screwed as terminals to the rods passing through the vulcanite cylinders. These rods are threaded, permitting the distance between the terminals to be varied at will. As the poles are rapidly consumed during the passage of the spark, it is necessary to adjust the distance between them while the exposure is in progress. For

this purpose a gear is attached to the rod carrying one of the terminals (the left one in the illustration), and this can be turned by means of a pinion at the inner extremity of a long ebonite handle, mounted on a conical ebonite support attached to the iron case. By this means the distance between the poles can safely be controlled, even when a transformer giving 64,000 volts is employed. When water is used it is kept circulating through the tank during the exposure, entering by means of a rubber tube near the bottom of the tank, and passing out through a rubber tube at the top. An auxiliary air-spark, blown out by the electric fan, is always used in series with the water-spark. The voltage, self-induction, capacity, etc., of the discharge circuit can be varied as desired. A tight-fitting iron cover is provided for the tank, in case high liquid pressures are to be used.

To the left of the central plane mirrors may be seen the ordinary arc, for experiments at atmospheric pressure with carbon or metallic poles. This arc is provided with an automatic regulator, so constructed as to keep the poles well separated during the exposure. By means of a suitable rheostat, the current through the arc can be varied as desired. Important differences in metallic spectra are obtained by the use of currents of 2 amperes and 30 amperes respectively, the difference of potential between the poles remaining the same in each case. Direct current of any desired voltage, up to 110, is applied to this arc from a storage battery, and alternating current, of one, two, or three phase, from the generator in the power house.

The next light-source on the pier is a synchronous rotating arc designed by Professor Crew, and constructed for us under his kind supervision. It consists of a small alternating motor, with a device for bringing it into synchronism with the single phase alternating current from our generator. By means of a position circle the rotating metallic electrode can be set at any desired angle, so as to permit the arc to be observed at any phase from 0° to 90°. The fixed electrode is adjustable by means of a hand-screw. The interesting variations in arc spectra obtained by varying the phase have been described by Professor Crew in the *Astrophysical Journal*.[1]

[1] 22, 199–203, 1905.

Next to the synchronous arc stands the pressure arc, for the study of the low potential discharge (up to 110 volts, direct or alternating current) in various gases at low and high pressures. This apparatus was constructed by Gaertner. The rotating arc is viewed through a window in the front of the case. One of the electrodes can be adjusted, if necessary, during the exposure. Pressures up to 60 atmospheres are supplied from cylinders of liquid CO_2, obtained in Los Angeles. For higher pressures a special pump, kindly designed by Mr. Petavel of the University of Manchester, and constructed under his supervision by Charles W. Cook, the University instrument maker, will soon be installed. For low pressures the Geryk pump is used.

All investigation on the details of the solar image are made with the aid of the Snow telescope. Sunlight is nevertheless frequently required in the laboratory for comparison spectra, etc., and is supplied by the Fuess heliostat shown on the shelf outside the window, on the right of the plate.

The switch-board, with connections to the power house and storage battery, is shown on the left of the plate. Ammeters and voltmeters for direct and alternating current are provided. A large 5 K. W. transformer, specially built by the Peerless Electric Company, giving 1,000, 2,000, 4,000, 8,000, 16,000, 32,000 or 64,000 volts, will soon be installed.

A Ducretet coil, giving a 35-cm spark, with rotating mercury interrupter, has recently arrived. This will be used for various purposes, and with an X-ray tube for certain investigations on ionization. A sensitive radiometer, built by Gaertner, and fitted by Professor Nichols with a suspension system which he was kind enough to make for us, is also available.

Our recent study of sun-spot spectra has made it necessary to supplement the above equipment with a Moissan electric furnace, of 50 K. W. capacity. This is being installed, with a large Littrow spectrograph, in our Pasadena laboratory, since the power plant on Mount Wilson cannot supply sufficient current. The 5 K. W. high-potential transformer will also be used for the present in Pasadena.

It is not the purpose of this article to describe in detail the other rooms of the laboratory, which include a plate-measuring room,

with a stereocomparator and other measuring machines for the study of photographs of spectra and spectroheliograph plates; an enlarging room, with cameras for use with skylight or electric arc; two photographic dark rooms; a clock room, containing a Riefler mean time clock, with nickel-steel pendulum, which controls electrically a mean time clock in the Snow telescope house; a small chemical laboratory, etc. The laboratory is heated by steam and lighted by electricity.

SOLAR OBSERVATORY,
Mount Wilson, California
July 1906.

SUN-SPOT LINES IN THE SPECTRUM OF *ARCTURUS*[1]

BY WALTER S. ADAMS

The spectrum of *Arcturus* has always been of considerable interest to those investigating the subject of stellar classification, because of its intimate relationship to that of the Sun. Certain differences have, indeed, been recognized,[2] but since the study of its spectrum, as in the case of most other stars, has been confined almost wholly to a limited region in the blue portion photographed for the purpose of determinations of radial velocity, it has not been possible to establish any very definite relationship with other stars or with the Sun.

In connection with some investigations at this Observatory of the lines affected in sun-spots, a number of photographs in the blue and violet regions of the large sun-spot of the latter part of June were obtained, and a study of these showed the existence of a considerable number of spot lines, some of which occur as far to the violet as λ 3900. The possibility of applying these results to stellar spectra, and particularly to the case of *Arcturus*, at once suggested itself, and the comparison has proved of exceptional interest.

The material available for the study of the star was a plate obtained in July 1905 by Professor Hale and the writer in the course of some investigations on the practicability of using diffraction gratings for the photography of stellar spectra. This plate was taken in the first order of a 5-inch (12.7 cm) Rowland plane grating used in conjunction with two lenses of 5 inches aperture and 149 inches (3.78 m) focal length. The grating was set so that λ 4500 fell at about the center of the plate, and the plate itself was bent to a slight curve. At the time at which this photograph was taken the mirrors of the Snow telescope were considerably tarnished, and the exposure time was very long, amounting to about 23 hours, extended over portions of five nights. An accurate temperature control, however, resulted in a very satisfactory plate, giving an extent of spectrum

[1] *Contributions from the Solar Observatory*, No. 12.

[2] V. M. Slipher, "Observations of Standard Velocity Stars," *Astrophysical Journal*, **22**, 323, 1905.

of suitable strength and in good definition from λ 4350 to λ 4900. The linear scale of this plate is 1 mm = 4.3 t.-m., which is about two and one-half times as great as that of the more powerful spectroscopes used for determinations of radial velocity. The comparison spectrum employed was that of the Sun, three exposures on three of the intervening days having been made in all. This has proved of the greatest value in the study of the plate, since it has been possible to identify practically all of the stellar lines directly without the necessity of measurement, and at the same time the identical solar lines are available for comparison which were used in the estimation of the intensities of the sun-spot lines.

In the tables which follow, the comparison of the spectrum of *Arcturus* with that of spots for the region λ 4350 to λ 4900 is given in full. In view of the importance of sun-spot lines in stellar spectra, it has seemed best to publish a list of these lines for the violet region as well. The study of more sun-spot plates will, no doubt, result in the addition of other lines to this list, as well as in some modifications of the lines given. The intensities are in all cases referred to the intensities for the solar lines given in Rowland's table. The lower photographic resolution on the stellar plate has involved the combination of lines many of which are resolved on the spot plates, and in such cases the intensities of the lines combining to form the blends are given in full in the list of spot intensities. The abbreviation "n. c." indicates that the line is not affected.

The agreement of the results for spot and star in these tables is certainly very striking. Not only are the lines affected in spots similarly affected in the star, but the absolute intensities of the lines are remarkably close in the two cases. The conclusion seems to be unavoidable that the physical conditions existing in sun-spots and in the atmosphere of this star are nearly identical. Should it prove, as seems at present very probable, that the differences between the spectrum of the Sun and that of spots are to be accounted for on the basis of a lower temperature in the latter, we must also infer a lower temperature for *Arcturus* than for the Sun.

A study similar to this on the spectrum of *α Orionis*[1] has shown

[1] Hale and Adams, "Sun-spot Lines in the Spectra of Red Stars," *Astrophysical Journal*, **23**, 400, 1906.

λ	Element	Intensity in Sun	Intensity in Spot	Int'nsity in Star	Notes
3905.66	*Si*	12	10		
3910.98	*Fe–V*	4	5		
3913.61	*Ti*	5	4–5		
3921.56	*Ti*	1	2		
3924.67	*Ti*	4	5		
3930.02	*Ti*	2	3		
3964.32	..	1	4		
.42	*Ti*	2			
3982.63	*Ti*	2	7		
.74	*Y*	3			
3984.48	*Cr*	2	3		
3991.83	..	2	3		
3996.68	*Sc*	00	0		
3998.79	*Ti*	4	5		
4053.98	*Cr–Fe–Ti*	3	2–3		
4060.42	*Ti*	1	2		
4082.59	*Sc–Fe–Ti*	3	4		
4090.73	*V*	1	2		
4092.82	*V, Ca*	3 d?	4		
4095.09	*Ca?*	4	5		
4099.94	*V*	2	3–4		
4105.32	*V*	2	3–4		
4112.87	*Ti*	1	2–3		
4113.07	..	1	3–4		
.12	*Fe*	3			
4115.33	*V*	3	4		
4116.63	*V*	1	0		
.71	*V, Fe?*	0			
4123.54	*Cr*	0	3–4		
.66	*Ce, V–Mn*	1			
.71	*Ti*	000			
4147.84	*Fe*	4	5–6		
4159.35	..	5	4–5		
4186.28	*Ti*	1	2		
4190.87	*C, Co*	1 Nd?	2		
4200.15	*Fe*	2	3–4		
.26	*Cr*	00			
4209.98	*V*	1	2		
4216.35	*Fe*	3 d?	4		
4232.76	*V*	00	3		
.89	*Fe*	2			
4252.39	..	00	1–2		
.47	*Co*	0			
4258.48	*Fe*	2	3–4		
4283.17	*Ca*	4	5		
4295.91	*Cr, Ti*	2	3		
4299.15	*Ca*	3	4		
4299.80	*Fe, Ti*	2	3		
4300.21	*Ti*	3	2–3		
4300.73	*Ti*	2	2–3		
4302.69	*Ca*	4	5–6		
4330.19	*V*	0 N	2		
4332.99	*V*	0	1–2		
33.08	..	0			
4337.72	*Cr*	3	4		

λ	Element	Intensity in Sun	Intensity in Spot	Int'nsity in Star	Notes
4338.08	*Ti*	4	3		
4338.43	*Fe*	1	1–2		
4339.62	*Cr*	4	5		
4340.63	*H*	20 N	15		Weakened on red edge mainly
4341.17	*V*	0	2		
4347.40	*Fe*	1	2		
4351.00	*Ti*	1	0		
4351.22	*Cr*	3	4		
4352.91	*Fe*	4	6–7	7–8	Widened to red
53.04	*V*	0			
4355.26	*Ca* ?	2	2–3	3	
4356.06	..	0	2	2	
.16	*Ni*	0			
4363.63	..	0	1–2	2	
.77	..	0			
4367.75	*Fe*	5	6	n.c.	
.84	*Ti*	2			
4371.14	*Zr*	1	n.c.	6	
.22	..	1	n.c.		
.32	..	00	n.c.		
.44	*Cr*	2	3–4		
4372.50	..	0 d ?	1	1	
4373.42	*Cr*	1	2	2	In star this line is covered by shading of λ 4383.72
4382.85	*Mn*	0	1	..	
.93	–*Fe*	2			
4387.01	*Ti* ?	1	0	..	
4390.15	*V*	2	3–4	3–4	
4391.82	*Co*	0			
91.92	*Cr*	1	3	5	
92.03	*Co*	0			
92.24	*V* ?	1 N	2		
4393.20	*V* ?	0	1	0–1	
4395.20	*Ti*	3	3–4	7	
.41	*V*, *Zr*	2	3		
4400.74	*V*	1	2–3	3	
4406.81	*V*–	2	4	5	
4407.81	*V*	2	7–8	7	
.87	*Fe*	4			
4408.36	*V*	2	3	3	
4408.58	*Fe*	3	6	6	
.68	*V*	2			
4412.30	*V*	00	2–3	4	
.42	*Cr*	0			
4416.64	*V*	0	2–3	3	
4416.98	..	2	1–2	1	
4417.45	*Ti*	0	2	3	
.58	*Co*	00			
4419.04	*Mn*	00 N	1	1	
20.10	*V*	00 N			
4421.73	*V*	0	2	2	
4423.30	*Fe*	1	2	2–3	
.43	*Cr*	0 N			
4426.20	*Ti*	0 Nd ?	2	2	
4427.27	*Ti*	2	3	8	
.48	*Fe*	5	n.c.		

λ	Element	Intensity in Sun	Intensity in Spot	Int'nsity in Star	Notes
4428.71	*V–Cr*	1 d?	2	2	
4429.96	*V*	00	1	1–2	
4433.95	*Fe*	1	n.c.	2	
34.17	*Ti*	0Nd?	1		
4435.13	*Ca*	5	6	8	
.32	*Fe*	2	n.c.		
4436.31	*V*	0	2	3–4	
.52	*Mn*	2	n.c.		
4437.73	..	0	n.c.		
37.86	..	00	n.c.	4	
38.01	*V*	0	2		
4441.88	*V*–	3Nd?	5	4–5	
4443.98	*Ti*	5	4–5	4–5	
4444.57	*V–Ti*	00	2	4	
.73	*Fe, Ti*	2	n.c.		
4445.64	*Fe*	1	2–3	2–3	
4449.31	*Ti*	2	2–3	n.c.	
4452.17	*V*	0N	2	2	
4453.88	*Ti*	1	1–2	1–2	
4455.98	*Mn*	2	6–7	6	
56.06	*Ca*	3			
4456.79	*Ca*	2	3–4	3	
4459.92	*V*	1	3	3	
4460.39	*V*	0			
.46	*Mn*	1	4	4	
.52	..	0			
4462.16	*Fe–Mn*	3Nd?	3–4	4	
4465.98	*Ti*	1	2	1–2	
4468.66	*Ti*–	5	4–5	n.c.	
4469.87	*V*	00	0	..	Blended in star with lines to violet
4471.41	*Ti*	0	2	1–2	
4471.72	..	00N			
.85	..	0	1–2	2–3	
.97	..	00			
4475.03	*Ti*	0	1–2	2	
4488.49	..	1	0–1	n.c.	Enhanced line of *Ti*
4496.12	..	1	n.c.	4	
.32	*Ti*	1	2		
4497.84	*Ti*	0N	1–2	1	
4501.44	*Ti*, –	5	4	4–5	
4503.03	..	00	1	n.c.	
04.04	*Mn*	00			
4508.46	*Fe?*, –	4	3	2–3	
4512.91	*Ti*	3	4–5	4	
4518.20	*Ti*	3	4–5	4	
4518.75	..	0	2	2–3	
.87	*Ti*	0			
4522.69	*Fe?*	0	n.c.		
.80	..	3	2	7	4522.80 is an enhanced line of *Fe*
.97	*Ti*	2	4		
4527.49	*Ti*	3	5	5	
.63	..	0			
4533.42	*Ti*	4	5	5	
4534.95	*Ti*	4	5	5	

λ	Element	Intensity in Sun	Intensity in Spot	Int'nsity in Star	Notes
4535.74	*Ti*	3			
.88	*Cr*	1	5–6	6	
.91	*Zr*	0			
4536.09	*Ti*	2	5–6	6	
.22	*Ti*	2			
4540.67	*Cr*	2	3	n.c.	
4544.79	*Cr*	1	5	5	
.86	*Ti*	3			
4545.51	*Cr–V*	0	1	1	
4546.13	*Cr*	3	4	4	
4548.94	*Ti*	2	3	3–4	
4549.64	*Fe*	2	7	6	
.81	*Ti–Co*	6d ?			
4552.63	*Ti*	2	4–5	4	
.72	*Fe*	1			
4555.66	*Ti*	3	4–5	4	
4556.06	..	3	2	5	
.31	*Fe–Cr*	4	n.c.		
4558.83	*Cr ?*	3	2	1	Enhanced line of *Cr*
4560.10	*Ni, Ti*	0	1	2	
4562.81	*Ti*	00	1–2	2	
4563.94	*Ti*	4	3–4	3	
4569.69	*Cr*	00	1–2	1–2	
.79	*Cr*	0			
4571.28	*Mg*	5	6	7	
4571.85	*Cr*	1	1–2	1–2	
4572.16	*Ti–*	6	5	4–5	
4574.90	*Fe*	2	3	3–4	Widened to red
75.08	..	00 Nd ?			
4575.73	*Zr*	00 N	0–1	1	
4576.51	..	2	1	1	
4577.36	*V*	0	2–3	3	
4578.73	*Ca*	3	5	5	
.91	*V*	00 N			
4580.23	*Cr*	3	4–5	4–5	
4580.59	*V*	1	2–3	3	
4581.58	*Ca*	4	10	10	
.69	*Co, Fe*	4			
4584.02	*Fe–*	4	3	3	Enhanced line of *Fe*
4586.05	*Ca*	4	6	5–6	
.16	..	0			
4586.55	*V*	1	3	3–4	
4588.38	..	3	2	1	Enhanced line of *Cr*
4594.30	*V*	2 N	4–5	4	
4600.03	*Cr*	3	5		
01.11	..	0	n.c.	6	
01.21	*Cr*	0	0–1		
4603.13	*Fe*	6	7–8	7	
4606.40	*Ni, C*	2	3	2–3	
4607.51	*Sr*	1	2–3	2	
4616.30	*Cr*	4	6	5	
4617.45	*Ti*	3	4–5	4	
4619.71	*Cr*	1	1–2	3–4	
.96	..	00	1–2		
4620.69	*Fe*	1	0	0	

λ	Element	Intensity in Sun	Intensity in Spot	Int'nsity in Star	Notes
4622.06	*Cr*	0	3	3	
.13	*Cr*	1			
4623.28	*Ti*	2	3	3	
4626.36	*Cr*	5	7	6	
4630.31	*Fe*	4	5	5	
4635.35	*V*	00 N	1–2	1–2	
4639.54	*Ti*	2	4	4–5	
.68	*Cr*	0			
4639.85	*Ti*	2	3–4	7	
40.12	*Ti*	1	3–4		
4645.37	*Ti*	0	2	2–3	
4646.35	*Cr*	5	6	6	
4650.19	*Ti*	0	1	1–2	
4651.46	*Cr*	4	6	5	
4652.34	*Cr*	5	7	7	
4653.55	..	00	1–2	1	
.68	..	00			
4656.23	*Ti*	0	1–2	1–2	
.36	*Cr*	0			
4656.64	*Ti*	3	5	5	
4657.38	*Ti?*	2	1	1	
4664.96	*Cr*	3	4	n.c.	
4668.75	..	1 N	2–3	2	
4670.59	..	2	3	4	
4675.29	*Ti*	1 N	2	2	
4680.48	..	1	2	2	
4682.09	*Ti*	3	5	5	
4685.45	*Ca*	2 N	3	3	
4687.85	..	00	1–2	2	
.98	*Zr*	0			
5688.36	*Fe*	2	1	2–3	
.55	..	0	1		
4690.98	*Ti*	00	0–1	1	
4693.85	*Ti*	0	1–2	2	
4698.58	*Co*	0	n.c.	5	
.64	*Cr*	1	n.c.		
.80	*Cr*	1	n.c.		
.95	*Ti*	1	2		
4710.37	*Ti*	00	5	5	
.47	*Fe*	3			
4715.47	*C?*	00	2	2–3	
4717.76	..	0	1	2	
4718.60	*Cr*	3	4	n.c.	
4722.80	*Ti*	0	2	2–3	
4723.29	*Cr*	00	3	4	
.36	*Ti*	00			
4729.86	*Fe? Cr*	1	1–2	1–2	
4733.78	*Fe*	4	6	6	
4742.98	*Ti*	1	2	2	
4744.01	..	000	1	..	
4744.57	*Fe?*	3	2	2	
4748.32	*Fe?*	4	3	2	
4752.01	*Na?*	00	0–1	0	
4755.34	..	00	0	..	
4756.30	*Cr*	2	3–4	3	

λ	Element	Intensity in Sun	Intensity in Spot	Int'nsity in Star	Notes
4758.31	*Ti*	1	2	2	
4762.57	*Mn*	5	6	7	
4771.28	*Ti–Co*	00	1	2	
4778.44	*Ti*	00	0–1	0	
4779.63	*Fe*	1	2	2	
4781.91	..	00	1–2	2–3	
4789.53	*Cr*	2	3	3	
4792.70	*Ti–Cr*	2	3	3–4	
4799.98	*Ti*	1	2	2	
4805.61	*Ti*	0	1	2	
4807.18	*Ni*	2	1–2	1–2	
4820.59	*Ti*	1	3	3	
4821.31	..	0	000	00	
4823.70	*Mn*	5	6–7	6	
4825.53	..	00	1	1	
.67	*Ti*	000			
4827.64	*V*	000	2	2–3	
.80	*Ti*	00			
4829.55	*Cr*	2	3	3	
4831.36	*Ni*	3	2–3	2	
4831.83	*V*	00	2–3	3	
4832.62	*V*	00	2	3	
4832.90	*Fe*	3	2	2	
4836.42	..	0	00	00	
4841.07	*Ti*	3	4–5	4	
4847.50	*Ca*	0	2	1–2	
4848.44	..	2	n.c.	3	
.60	*Ti*	0000	0–1		
4849.08	*Fe*	1	1–2	1–2	
4851.69	*Ca, V*	1	3–4	4–5	
4856.20	*Ti*	1	3	3	
4861.53	*H*	30	25	25	
4862.03	*Cr*	0	2	..	In star this line is lost in shade of $H\beta$
4864.92	*V*	0	3	3	
4868.45	*Ti*	0	1–2	3	
.60	..	00 d?	n.c.		
4870.32	*Ti*	1	2	2	
4875.67	*V*	1	3	3	
4876.59	..	1	0	0	
4881.74	*V*	1 N	3	5	
.90	*Fe*	2	n.c.		
4885.26	*Ti*	2	3–4	3	
4885.96	*Cr*	0	1–2	n.c.	
86.13	*Cr*	00			
4888.71	*Cr*	00	4	3–4	
.82	*Fe*	2			
4893.03	*Fe*	1	0–1	0	
4900.10	*Ti, La*	2	3–4	4	

that sun-spot lines are also present in the spectrum of that star. The amount by which they are affected, however, is, as a rule, considerably greater than in the case of spots or of *Arcturus*. We have, accordingly, in Sun, *Arcturus*, and *a Orionis*, a series arranged in

the order of increasing differences for the characteristic spot lines. In the event referred to above—namely, that these differences can be accounted for on a temperature basis—this sequence would represent a scale of descending temperatures.

In a recent letter to Professor Hale received while this investigation was in progress, Mr. Baxandall, of the Solar Physics Observatory at South Kensington, refers to some studies of the spectrum of *Arcturus* made by himself in the blue region of the spectrum. The absence of a list of sun-spot lines in this region prevented him from making a direct comparison, but his conclusion, that most of the titanium and vanadium lines in the spectrum of *Arcturus* are considerably stronger than in the Sun, is in agreement with the results found in this discussion.

I am indebted to Messrs. Gale and Ellerman for the sun-spot plates used in this investigation.

Solar Observatory, Mount Wilson,
August 1906.

RESULTS OF SOLAR OBSERVATIONS AT PRINCETON, 1905–1906

By WALTER M. MITCHELL

The preliminary account of the observations of reversed lines in the spectra of sun-spots at Princeton was published in this *Journal* for June 1904. A more detailed account of the observations of the sun-spot spectrum appearing about a year later, in the *Astrophysical Journal* for July 1905, gave a complete table of the wave-lengths recorded. It had been found, in looking over unpublished observations of Professor Young, that some eighteen of these reversed lines had already been observed by him during the early summer of 1892, while working with the same instrument.

Observations have been continued during the period from October 1905 to May 1906; the whole region of the spectrum F–*a* (λλ 4861–7160) has been gone over some twelve times since the observations were begun in March 1904. All observations have been made in the same manner and with the same instrument.

In comparing the results of the observations made since October 1905 with those previous to that date, it was found that the selection of widened lines in the two periods was so similar that it would not be necessary to publish these lines a second time. But this is not the case with the reversed and weakened lines, many new ones of both these classes, particularly the latter, having been observed. The table at the end of this paper gives only the reversed and the weakened lines; widened and intensified lines will be found in the table published last year.[1]

During the more recent observations of the sun-spot spectrum it has been noticed that the number of weakened lines recorded has greatly increased. Many lines that were formerly reversed were now found weakened; also many new weakened lines were found, the lines not being affected in previous observations. This change may be an indication of alteration in the general constitution of a sun-spot in connection with the passing of the sun-spot maximum,

[1] *Astrophysical Journal*, **22**, 11, 1905.

but the period of observation is too short for any definite conclusion to be drawn. Scattered light in our atmosphere often causes the

FIG. 1

FIG. 2

FIG 3

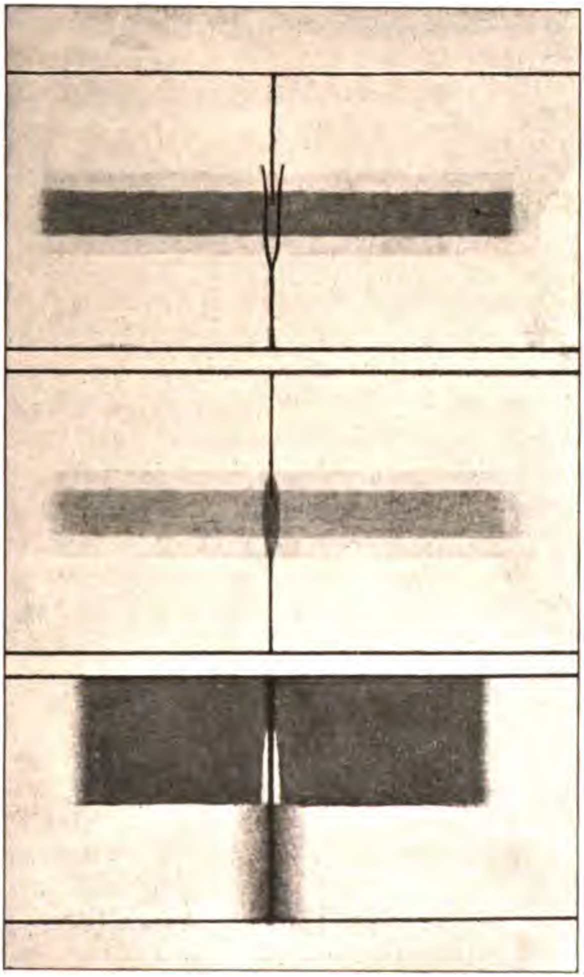


FIG. 1.— Appearance of reversed lines on Oct 31, 1905

FIG. 2.— Weakened lines on April 3, 1906.

FIG. 3.— *b* lines in chromosphere.

reversed lines to be very indistinct, and it is probable that in several cases the indistinct reversed line has been mistaken for a weakened line. These cases are indicated in the table. It has been observed

that generally the more reversals there are in the spectrum of a spot, the fewer are the weakened lines and vice versa.

The existence of "veils" across the spot umbra may also cause the reversals to be less distinct. This would seem to be the case in one instance recorded on May 11, 1906. My notebook states:

Noticed that the *Fe* lines $\lambda 5250.39$ and $\lambda 5712.36$ are *not* reversed where on last observation, three days ago, they were reversed strongly. Also other lines in the red are not so widely reversed as on previous date. . . . . Spot has become overgrown with veils, bridges, etc. Many red reversed lines show the weakened appearance and have dark lines partly through the weakened part.

Unfortunately, the sky clouded over very soon after observations were begun, so that other lines could not be examined in detail.

There have been several instances, in addition to the one just mentioned, in which the reversals presented a very peculiar appearance. A notable one occurred on October 31, 1905, in a large spot of the minimum type. The reversal extended considerably below the spot umbra and penumbra, while above was the very curious tripling of the spectrum line, the two halves of the reversal diverging and extending slightly beyond the penumbra, the ordinary absorption line continuing into the umbra spectrum and ending abruptly (Fig. 1). This appearance was not confined to one line, but was distinctly seen in all the more widely reversed *Fe* lines and one *Ni* line, in various parts of the spectrum. Several instances were recorded where the reversed part of the line was displaced slightly to one side, instead of being central. Another curious case was recorded on April 3, 1906, where on one side of the spot the lines were distinctly reversed, while on the other side the same lines were weakened, with a continuation of the dark line extending into the weakened part (Fig. 2). This was observed in all the reversed lines on that date. The probable cause of these curious weakened lines seems to be condensation of the gases of the spot, forming a thin photosphere over portions of the penumbra and umbra in the shape of veils. The thin photosphere, making a brighter background than the dark spot umbra, tends to bring out the ordinary absorption line.

With the more widely reversed lines it was found possible to measure the distance between the two components. The mean distance between the components of the various reversed lines thus measured

was 0.189 t.-m. The probable error of the mean of five measures of the width of the reversed line λ 6173.55 was found to be 0.0077 t.-m. The detailed results of the measures are given in the table.

It was thought that possibly the gas masses causing the reversed lines (of iron) might be similar to the iron flocculi photographed by Hale and Adams at Mount Wilson,[1] but careful examinations of the whole disk have so far failed to reveal reversals anywhere but over the spot umbra and occasionally in the immediate vicinity of the spot. Hence it seems safe to say that the reversals of such lines as I have recorded exist only in the regions mentioned, indicating most probably that the gases causing them are essentially different from the flocculi, since these are well distributed over the whole disk.

If a curve is drawn the ordinates of which are the percentage of lines reversed to total number affected in a given region, while the abscissæ are the wave-lengths of these regions, it will be found that there is a very decided maximum at λ 6200. From Wien's law, λT = const., we find 4700° for the temperature of the gases producing the reversed lines (using 2940 for the constant, the value found by Lummer for a black body). Adopting 6000° as the photospheric temperature, we find from Stephan's law the value 0.38 for the ratio of sun-spot radiation to photosphere radiation, a value in very close agreement with 0.41 deduced by Wilson from direct observation.[2]

Observations of the band-lines have been made whenever possible. On three occasions they have been observed below B extending to approximately λ 7100. The regions in which I have found that they are seen are:

λλ 7100–6760. From λλ 6760–6450 the spectrum is only occasionally resolved and then generally indistinct and blurred.

λλ 6450–6360. In this region I remarked[3] that the fine lines of the spot spectrum were arranged into groups. Several of the large spots visible in the fall of 1905 showed that two of these groups had the appearance of short flutings, the wave-lengths being λ 6389

[1] *Astrophysical Journal*, **23**, 62, 1906.

[2] *Monthly Notices*, **55**, 461, 1895.

[3] *Astrophysical Journal*, **19**, 359, 1904.

and λ 6381. Fowler also records[1] two "well-developed flutings" at these wave-lengths. From $\lambda\lambda$ 6360–5700 the band-lines are generally faint and indistinct.

$\lambda\lambda$ 5700–5200. In this region they become fairly prominent.

$\lambda\lambda$ 5200–5000. The band-lines here are very conspicuous.

Hence it can be said that the resolution of the spot spectrum into fine lines is least likely to be seen in the neighborhood of C and the D lines, while above λ 5000 they have as yet not been observed. In the vicinity of B the appearance is similar to that in other regions. As visual observations are not well adapted for the purpose, I have made no attempt to determine the wave-lengths of these lines.

THE CHROMOSPHERIC SPECTRUM

It has been found that generally the visibility of the chromospheric lines depends more upon the state of the Earth's atmosphere—i. e., the "seeing"—than on the state of the chromosphere. Many lines have been seen when the slit was set on parts of the chromosphere that were absolutely without prominences or eruptions of any kind. It thus seems that an eruption or prominence is not essentially necessary to render the chromospheric lines visible; a sufficiently large solar image and steady seeing should render the majority visible whether there is a chromospheric disturbance or not. It was frequently noticed during the moments of good seeing that many lines flashed out, only to disappear again with the return of atmospheric unsteadiness.

Several lines have been found to be always doubly reversed. The most prominent of these are the b lines. b_1, b_2, the lower (enhanced iron) component of b_3, are usually quite bright, while the upper (magnesium) component of b_4 is fainter and more difficult to see. The appearance of b_1 and b_2 in the chromosphere gives one the impression that the usual dark wings have been replaced by narrow bright ones, the central dark absorption line retaining its usual width and darkness, the reversals being, as one might say, outside the absorption line (Fig. 3). b_3 and b_4 are reproductions on a smaller scale, and are best seen with a very high dispersion fourth-order spectrum of a 20,000 line grating.

One very notable instance of chromospheric activity was recorded

[1] *Monthly Notices*, **65**, 217, 1905.

on May 13, 1906, at 8^h 15^m G. M. T. One of a pair of medium-sized spots was just going off the limb, the other being inside of it and nearer the Sun's equator. A very hasty examination showed that practically every dark line, as nearly as could be estimated, was bright over the region of the departing sun-spot. Owing to the short duration of the outburst, about 25 minutes, it was possible to record only a few of these bright lines. Out of a total of 83 lines recorded 62 were doubly reversed, the appearance being similar to that of the *b*'s described above. The majority of these doubly reversed lines were heavy iron lines which are winged in the spot spectrum. The usual chromospheric lines (those in Young's list with a fairly high frequency) were in the majority of instances *not* doubly reversed, but simply much brighter than under ordinary circumstances. When the slit was set making a small angle of 5° or 10° with the Sun's limb, which position I have found best for chromospheric observation, it was seen that the bright chromospheric lines made a small angle toward the red with the ordinary Fraunhofer line, apparently indicating that with increasing distance from the Sun's center the chromospheric gases were receding with increasing velocity.

By a suitable adjustment of the spectroscope the chromospheric spectrum was obtained simultaneously with that of the remaining sun-spot: the difference between the two spectra was most striking and showed clearly the effects of the widely differing physical conditions in the chromosphere and the spot. From the great difference between the two classes of chromospheric lines it seemed as if there were different forces at work producing them. The doubly reversed lines might perhaps be the result of anomalous dispersion, as has been suggested by Julius.[1]

In a recent paper[2] Fowler calls attention to the fact that the enhanced lines of iron, titanium, and chromium appear as high-level lines in the chromosphere, while in the spot spectrum the corresponding Fraunhofer lines are enfeebled or weakened. My own observations agree in the main with Fowler's. However, the enhanced *Ti* line at λ 5154.24 has not been seen, nor is it given in Young's revised list. Some investigations which I made during

[1] *Astrophysical Journal*, **15**, 28, 1902.

[2] *Monthly Notices*, **61**, 361, 1906.

the last winter on the spark spectrum of iron and titanium confirm the results given by Fowler, with the addition that the line λ 6247.77 is probably an enhanced line of iron.

The question arises what may be the possible explanation of the prevalence of enhanced lines in the chromospheric spectrum. Lockyer has attempted to explain this on the supposition that the temperature of the chromosphere is that of the electric spark.[1] In an analogous manner, Scheiner[2] suggested that the temperature of certain stars might be determined by presence of certain spark lines in their spectra, the *Mg* line λ 4482 in particular. But Hartmann and Eberhard have shown[3] that these conclusions are unjustifiable, and since an atmosphere of hydrogen will produce the spark lines in the arc spectrum, they suggest that the prominence of λ 4482 is due to the abundance of hydrogen in the atmospheres of the stars of the particular type. It therefore seems possible that the prevalence of hydrogen in the chromosphere may cause certain lines to be suppressed, at least to such an extent that they become invisible against the background of the sky spectrum. This would be borne out by the observations of Fowler mentioned above, for he states that the high-level lines are enhanced lines, as would be supposed, since it is probable that high in the chromosphere the proportion of hydrogen is greatest, while low in the reversing layer, where the proportion of hydrogen is probably much less, other lines beside those found in the spark spectrum make their appearance. The emission coefficient of these high-level lines being greater than that of the lines caused by lower-level gases, the absorption of the lines would not be so great, hence across the dark umbra of the spot the line would have an enfeebled or weakened appearance.

But these views cannot be fully confirmed until additional investigations have been made of arc spectra in hydrogen, since up to this time investigators have confined themselves to the violet regions of the spectrum alone. Nevertheless it seems probable, should the investigations be extended to the longer wave-lengths, that a state of affairs will be found altogether similar to that found in the more refrangible regions.

[1] *Proc. R. S.*, **68**, 178, 1901.

[2] *Astronomy and Astro-Physics*, **13**, 569, 1903.

[3] *Astronomische Nachrichten*, **161**, 314, 1903.

TABLE I

Wave-Length	Element	Spot Spectrum: Total No. Obs.	Spot Spectrum: Reversed: No. Obs.	Spot Spectrum: Reversed: Inten.	Spot Spectrum: Weakened: No. Obs.	Spot Spectrum: Weakened: Inten.	Chromosphere: No. Obs.	Chromosphere: Inten.	Remarks
4876.59		1			1	−5			
4918.19	*Fe*	1			1	−1			Hazy.
4921.96	*La–Ti*	7					3	7	Always darkened in spot.
4924.11	p*Fe*						3	15	Double reversal twice in chromosphere.
4925.75	*Ni*	2			2	−1			
4934.4	*Ba ?*						3	5	Double reversal once, w.-l. seems to be somewhat longer than that of *Ba* line.
4952.46	*Fe*	2			1	−1			
4952.82	*Fe*	4			2	−2			
4953.39	*Ni*	2			1	−1			
4970.09	*Fe*	3			2	−1			Generally hazy.
4993.53							1	5	
4998.41	*Ni*	2			2	−3			Obliterated once.
4999.69	*Ti–La*	2			2	−2			
5015.80	*He*						3	7	Probably a high level line.
5018.63	p*Fe*						3	10	Double reversal twice. The red component.
5027.94	*Fe*	4			2	−4			
5039.54	*Ni*	6			3	−4			Generally weakened and hazy.
5061.70		1	1	10					
5061.88		1	1	10					
5072.85	*Fe*	1			1	−4			
5080.71	*Ni*	4			4	−3			
5081.29	*Ni*	1			1	−2			
5082.53	*Ni*	7			7	−4			Obliterated twice, other times weakened.
5087.60	*Y*						1	10	
5112.46							2	5	Wave-length longer than given by Young.
5114.43							1	2	
5120.59	*Ti*	6			2	−2	1	2	
5123.39	*Y*						2	3	
5129.81	*Fe*	6			6	−4			Obliterated four times.
5130.54	*Ni*						1	2	Probably high level.
5131.64	*Fe*	2	1	5	1	−3			Hazy both times.
5145.27	*Fe*	6			1	−2			Generally widened and hazy in spot spectrum.
5159.23	*Fe*	1			1	−4			
5164.72	*Fe ?*	1			1	−3			
5165.42	*C*						1	1	Very faint.
5165.59	*Fe*	5			4	−3			
5167.50	*Mg*						5	10	The upper component of b_4 generally doubly reversed.
5168.83	*Ni*	2			2	−2			
5169.22	p*Fe*	7			7	−3	5	25	The lower component of b_3, always doubly reversed in chromosphere; weakened in spot.
5170.94	*Fe*	2			2	−3			
5171.78	*Fe*	1			1	−5			
5172.86	*Mg*						5	30	b_2. Always double reversal.
5173.92	*Ti*						1	1	
5183.79	*Mg*						5	30	b_1. Always double reversal.
5186.07	p*Ti*	2			2	−3			
5187.80							1	5	Is not the *Fe* line λ 5188.08.

TABLE I—*Continued*

Wave-Length	Element	Spot Spectrum: Total No. Obs.	Spot Spectrum: Reversed: No. Obs.	Spot Spectrum: Reversed: Inten.	Spot Spectrum: Weakened: No. Obs.	Spot Spectrum: Weakened: Inten.	Chromosphere: No. Obs.	Chromosphere: Inten.	Remarks
5188.08	*Fe*	2			2	−2			
5189.0	*Ti–Ca*						4	5	Not sure which component rever-
5191.63	*Fe*	2	1	?					ses. Fowler gives origin of λ
5196.23	*Fe*	6			5	−3			5188.86 as p *Ti*.
5197.74	p*Fe*	2			1	−4	5	15	High level line, double reversal
5200.36	*Cr*						1	5	once.
5202.52	*Fe*	4			1	−1	1	7	Double reversal in chromosphere.
5204.70	*Cr–Fe*	5			2	−1	1	10	Double reversal in chromosphere.
5205.90	*Y*	1			1	−4	2	3	
5206.22	*Cr–Ti*	5			2	−2	2	9	Double reversal in chromosphere.
									Generally winged in spot.
5208.78	*Fe*						1	10	Double reversal in chromosphere.
5225.70	*Fe*	6	5	4					
5225.97	*Cr*	4	3	3					
5226.71	p*Ti*	2	1	?	1	−4	3	6	
5227.04	*Fe–Cr*	8	3	2	3	−3	1	4	Double reversal in chromosphere.
5227.36	*Fe*	1			1	−1	1	4	Double reversal in chromosphere.
5234.79	p*Fe*	2			2	−3	5	15	High level line. Double reversal
5237.40	p*Cr*	4			4	−3	1	5	once.
5239.99		1			1	−3	2	5	
5247.23	*Fe*	5	4	2					
5247.74	*Cr*	8	6	3					Young's chromosphere line.
5249.8							2	2	
5250.39	*Fe*	12	9	10					Frequently reversed outside of
3253.63	*Fe*	1	1	1					spot umbra, width of reversal
5255.30	*Cr*	5	2	2					0.170 t.-m.
5255.8							2	1	Not sure of this line.
5260.0							2	2	
5260.56	*Ca*	11	4	5					Young's chromosphere line.
5264.98							1	7	
5269.72	*Fe*						3	7	E_2, Double reversal twice.
5270.50	*Ca–Fe*						2	5	E_1, Double reversal once. Not
									sure which component.
5273.34	*Fe*						2	4	Double reversal once.
5274.58							2	2	
5275.93	*Cr*	10	5	5					
5276.17	p*Fe*						5	8	Not sure of wave-length.
5284.28	*Ti*	1			1	−5	5	6	
5284.79		2			1	−4			
5293.21	Awv?						1	5	
5298.19	*Cr*	4	1	2	1	−2			
5298.46	*Cr*	4	1	?	1	−2			
5298.67	*Ti*	5	2	2	1	−2			
5298.96	*Fe*	2	1	1	1	−3			
5300.93	*Cr*	7	2	3					
5304.36	*Cr*	6	1	2					

TABLE I—*Continued*

Wave-Length	Element	Spot Spectrum: Total No. Obs.	Spot Spectrum: Reversed: No. Obs.	Spot Spectrum: Reversed: Inten.	Spot Spectrum: Weakened: No. Obs.	Spot Spectrum: Weakened: Inten.	Chromosphere: No. Obs.	Chromosphere: Inten.	Remarks
5306.04							1	5	
5307.54	*Fe*						1	4	Double reversal.
5308.60							1	5	
5313.03	*Cr*	2	1	2					
5316.79	p*Fe*						5	20	Double reversal twice.
5320.0?	*Fe*						2	7	Not sure of wave-length.
5321.0?							1	2	Not sure of wave-length.
5324.37	*Fe*						1	3	Double reversal. Winged in spot.
5325.74							2	10	Wave-length seems to be greater than that given by Young. One observation by Young records "disappears in spot spectrum."
5328.24	*Fe*						1	15	Winged in spot. Double reversal in chromosphere.
5328.75	*Fe*						1	10	Double reversal.
5329.33	*Cr*	4	1	1			1	4	Double reversal.
5329.97	*Cr*	7	1	?	1	−4			
5330.18	*Fe*						1	4	Double reversal.
5335.05	*Co*						1	5	
5336.97	p*Ti*						1	20	Double reversal.
5337.91							1	5	
5340.12	*Fe*	1			1	−2	1	3	Double reversal.
5341.21	*Fe*						1	3	Double reversal.
5345.99	*Cr*	4			2	−2			Thinned and winged.
5348.51	*Cr*	5			2	−2			Thinned and winged.
5349.93	*Fe*	2	1	?	1	−2			
5353.70	*Co*						2	4	Double reversal.
5361.81		2			1	−2	3	5	Double reversal.
5363.06	p*Fe*						3	8	
5365.07	*Fe*						1	3	Double reversal.
5365.60	*Fe*						1	3	Double reversal.
5367.67	*Fe*						1	3	Double reversal.
5370.17	*Fe*						1	2	Double reversal.
5371.66	*Cr ?*						1	10	Double reversal.
5373.91	*Fe,Cr*	5	1	?	3	−3			
5377.80	*Mn*	2			2	−4			
5379.77	*Fe*						1	10	Double reversal.
5381.22	*Fe*						1	3	Double reversal. There is a p*Ti* line at λ 5381.204. This may be it and not the *Fe* line.
5382.47		1			1	−3			
5389.68	*Fe*						1	3	Double reversal.
5394.85	*Mn*	11	2	2					Generally widened and darkened.
5399.67	*Mn*	6	3	5					
5401.47		4	1	?	3	−4			Bright over spot on one occasion.
5405.99	*Fe*						1	3	Double reversal.

TABLE I—*Continued*

Wave-Length	Element	Spot Spectrum: Total No. Obs.	Spot Spectrum: Reversed: No. Obs.	Spot Spectrum: Reversed: Inten.	Spot Spectrum: Weakened: No. Obs.	Spot Spectrum: Weakened: Inten.	Chromosphere: No. Obs.	Chromosphere: Inten.	Remarks
5409.34	*Fe*	2	1	2	1	−2			
5410.00	*Cr*						1	5	
5411.12	*Fe*						1	4	Double reversal.
5413.89	*Mn*						1	10	Hazy in spot.
5415.42	*Fe–V*						1	4	Double reversal.
5418.98	*Ti?*						1	3	Double reversal.
5425.46		3			3	−3	2	7	Young observes "very much weakened in spot."
5429.91	*Fe*						1	2	Double reversal.
5432.75	*Mn*	11	3	7					Generally much widened and darkened.
5433.16	*Fe*						1	5	Double reversal.
5434.74	*Fe*						1	10	Double reversal. Winged in spot.
5436.51	*Fe*	6	1	2	3	−2			
5436.80	*Fe*	9	4	5	2	−2			
5445.26	*Fe*	3			2	−2	1	5	Double reversal. Thinned in spot.
5447.13	*Fe*						1	3	Double reversal.
5455.83	*Fe*						1	3	Double reversal.
5460.72		12	1	?					Always strongly darkened.
5461.76		9	7	6					
5462.71	*Ni*	8	7	7	1	−2			Weakened observation may be mistake for reversal.
5467.20	*Fe*	4	1	?	1	−2			
5470.84	*Mn*	10	2	10					
5483.31	*Fe*	5	4	3	1	−2			Weakened observation may be mistake for reversal.
5493.71	*Fe*	3	2	4	1	−3			
5494.68	*Fe*	5	3	4	1	−1			
5495.10	*Ni*	4	4	9					
5497.74	*Fe*	9	1	?			1	4	Double reversal. Winged in spot.
5501.68	*Fe*	5					1	5	Double reversal. Winged in spot
5506.10	*Mn*	8	1	5					
5511.87	*Fe*	1	1	8					
5527.03	*Si*						2	10	Double reversal once. Fowler gives origin as p*Sc*.
5528.64	*Mg*	3					1	10	Winged in spot.
5535.06	*Fe*	1			1	−3	4	12	Fowler gives origin as p*Fe?*
5537.97	*Mn*	11	5	10					
5538.74	*Fe*	10	8	9	1	−2			
5544.16	*Fe*	3	2	1					
5546.73	*Fe*	6	2	4	1	−3			
5569.85	*Fe*						1	4	Double reversal.
5573.07	*Fe*	2					1	3	Double reversal. Winged in spot.
5576.32	*Fe*						1	4	Double reversal.
5584.53	*V*	7	1	6					
5586.09	*Fe*	3					1	3	Double reversal. Winged in spot.
5588.98	*Ca*	4					1	3	Winged in spot.

TABLE I—*Continued*

Wave-Length	Element	Spot Spectrum					Chromosphere		Remarks
		Total No. Obs.	Reversed		Weakened				
			No. Obs.	Inten.	No. Obs.	Inten.	No. Obs.	Inten.	
5594.69	*Ca*						1	3	Not sure which component reverses.
5594.88	*Fe*	2			2	−3			
5615.88	*Fe*						1	3	Double reversal.
5619.82		6	2	2	1	−3			
5620.72	*Fe*	5	2	?	3	−4			
5625.54	*Ni*	1			1	−4			
5627.9							1	2	Do not think that this is the *V* line.
5636.93	*Fe*	2	1	4					
5637.63	*Fe*	2	1	4					
5641.21		3	2	5	1	−3			
5645.83	*Si*	3			3	−4			Obliterated once.
5648.00							1	4	Wave-length uncertain.
5654.09	*Fe*	2	1	2					
5657.66	*V*	11	1	6			1	8	Reversed? twice in spot.
5658.10	*Y*,–	4	4	3					
5663.16	*Ti-Fe-Y*	7					1	10	Widened and darkened in spot.
5667.37		2	1	10					
5667.74	*Fe*	2	1	9					
5668.59	*V*	11	2	6					
5669.26	*Sc?*	3	1	1	2	−3			
5684.71	*Si*	4			4	−3			
5690.65	*Si*	5			5	−3			
5701.32	*Si*	2			2	−3			
5708.62	*Si*	3			3	−2			Obliterated once.
5712.36	*Fe*	8	8	7					
5722.10							1	1	Very faint.
5727.87	*Cr?*	10	1	8					
5731.44	*V*	10	1	9					Reversed? four times.
5731.98	*Fe*	1			1	−2			
5739.87		8	1	6					
5742.07	*Fe*	1	1	3					
5746.64	A,–	2							Reversed? twice.
5748.17	*Fe*	5	1	6					Reversed? once.
5748.57	*Ni*	8	6	4					
5781.13	*Cr–Ti*	5	3	2					
5781.40	*Cr*	8	8	8					
5781.97	*Cr*	8	8	10					Width of reversal 0.107 t.-m.
5783.29	*Cr*	7	7	5					
5784.08	*Cr*	7	6	3					
5784.88	*Fe*	6	6	4					
5785.19	*Cr*	7	6	5					
5785.50	*Fe*	6	4	4					
5786.19	*Ti Cr*	6	2	2					
5798.08		7	1	?	2	−3			
5847.22	*Ni*	5	1	?					
5853.90	*Ba?*						1	5	

TABLE I—*Continued*

Wave-Length	Element	Spot Spectrum					Chromosphere		Remarks
		Total No. Obs.	Reversed		Weakened				
			No. Obs.	Inten.	No. Obs.	Inten.	No. Obs.	Inten.	
5873.44		2	1	8					
5875.92	*He*						...	90	
5880.49		10	10	10					
5890.19	*Na*	5			1	−1	5	10	Generally doubly reversed in chromosphere.
5896.36	*Na*	5			1	−1	5	10	Generally doubly reversed in chromosphere.
5902.69	*Fe*	1	1	8					
5910.20	*Fe*	5	5	6					
5916.47	*Fe*	5	3	2					
5918.77	*Ti*	6	2	3					
5938.04		11	5	5					
5958.46	*Fe*	1	1	8					
5999.92	*Ti*	9	2	2					
6002.97		5	2	4					
6005.77	*Fe*	9	9	5					
6008.19	*Fe*	11	11	6					
6008.79	*Fe*	1	1	2					
6012.45	*Ni*	9	9	6					
6013.72	*Mn*	11	8	4					
6016.86	*Mn*	11	8	4					
6022.02	*Mn*	6	1	?					
6030.95	*V*	10	6	3					
6053.01	*Ni*	3	1	5					
6064.85	*Ti*	10	10	9					Width of reversal 0.148 t.-m.
6079.23	*Fe*	11	11	8					
6081.67	*V*	10	6	8					
6082.93	*Fe*	12	12	8					
6085.47	*Ti–Fe*	9	2	4					
6089.79	*Fe*	7	4	3					
6090.43	*Ti-V*	11	3	6					
6091.40	*Ti*	8	2	3					
6093.03	*Ti?*	9	4	6					
6093.37	*Mn?*	8	2	3					
6096.88	*Fe*	4	3	3					
6098.46		3	2	2					
6102.39	*Fe*	2			2	−3			Young's chromosphere line.
6102.94	*Ca*	11	6	4					Young's chromosphere line.
6103.40	*Fe*	3			1	−4			Young's chromosphere line.
6111.29	*Ni*	3			2	−4			Obliterated once.
6111.87	*V*	11	3	4					
6122.43	*Ca*	8	1	2					Generally much winged. Young's chromosphere line.
6125.24		2			2	−5			Obliterated twice.
6126.44	*Ti*	10	3	3					
6127.85		5	2	7					
6129.19	*Ni*	11	4	3					

TABLE I—*Continued*

Wave-Length	Element	Spot Spectrum: Total No. Obs.	Reversed: No. Obs.	Reversed: Inten.	Weakened: No. Obs.	Weakened: Inten.	Chromosphere: No. Obs.	Chromosphere: Inten.	Remarks
6131.79		1	1	3					
6132.07		1	1	4					
6137.21	*Fe*	12	12	10					Width of reversal 0.189 t.-m.
6141.93	*Ba-Fe*						1	5	
6142.70		2			2	−4			
6145.23		1			1	−3			
6149.46		3			3	−4			Obliterated twice. Young's chromosphere line.
6150.36	*V*	11	1	5					
6151.83	*Fe*	9	9	7					Frequently reversed outside of spot umbra.
6154.44	*Na*	11	6	6					Young's chromosphere line.
6156.24		4	2	7					
6163.97	*Ca*	8	2	?					
6170.73	*Fe-Ni*	2	2	4					
6173.55	*Fe*	12	12	10					Frequently reversed outside of spot. Average width of reversal, 0.210 t.-m. Young's chromosphere line.
6188.21	*Fe*	7	6	7					
6191.39	*Ni*	11	7	5					
6199.38	*V*	12	3	7					
6200.53	*Fe*	6	3	2					Young's chromosphere line.
6204.83	*Ni*	2	1	4	1	−3			
6210.90		10	1	4					
6213.64	*Fe*	11	11	8					
6214.08	*V*	11	3	7					
6216.57	*V*	10	5	8					Young's chromosphere line.
6219.49	*Fe*	8	1	4					Young's chromosphere line.
6224.71	*V*	11	3	7					
6226.95	*Fe*	4	3	1					
6232.86	*Fe*	11	11	9					Young's chromosphere line.
6233.08		8	3	7					Apparently no dark line at this point.
6237.53		3			3	−3			
6238.60		5			5	−4			Young's chromosphere line.
6240.86	*Fe*	7							Reversed? twice.
6243.06	*V*	10	3	6					
6243.32	*V*	11	3	7					
6244.03		5			3	−3			
6244.69		5			5	−3			Obliterated once.
6245.83		5			5	−4			Obliterated once.
6246.54	*Fe*	3	2	4					
6247.77	p*Fe*	6			6	−4			Young's chromosphere line. Enhanced iron.
6252.05	*V*	12	4	7					
6265.35	*Fe*	4	2	2					
6266.55	*V*	10	3	6					
6269.08	*V*	11	4	6					
6274.87	*V*	11	4	6					
6280.83	*Fe*	6	1	2					Generally hazy.
6285.38	*V*	11	3	5					

TABLE I—*Continued*

Wave Length	Element	Spot Spectrum: Total No. Obs.	Spot Spectrum: Reversed No. Obs.	Spot Spectrum: Reversed Inten.	Spot Spectrum: Weakened No. Obs.	Spot Spectrum: Weakened Inten.	Chromosphere: No. Obs.	Chromosphere: Inten.	Remarks
6291.18	*Fe*	1	1	2					
6296.58	*V*	11	6	8					
6301.72	*Fe*	6	1	1					Young's chromosphere line.
6302.71	*Fe*	10	7	10					Young's chromosphere line.
6318.24	*Fe*	1			1	−2			Young's chromosphere line.
6319.46		2			2	−3			
6322.91	*Fe*	2	1	4					
6327.82	*Ni*	4	1	5					
6330.32	*Cr*	11	8	9					
6331.06	*Fe*	8	2	4	1	−2			
6337.05	*Fe*	10	7	5					Young's chromosphere line.
6347.31		5			5	−4			Obliterated once. Young's chromosphere line.
6358.90	*Fe*	7	4	4					
6363.09	*Cr-Fe*	11	5	9					
6369.68	*Fe*	1			1	−4			Young's chromosphere line.
6371.57	*Fe*	2			1	−4			Young's chromosphere line.
6400.54	*Fe*	8	4	6					
6415.20		3			3	−4			
6417.13	*Fe?*	1			1	−4			Young's chromosphere line.
6420.17	*Fe*	3	1	5	2	−2			
6432.89	*Fe*	7			7	−4	1	5	Generally obliterated in spot.
6450.03	*Ca*	2	1	5					
6455.82	*Ca*	9	1	7					
6456.60	p*Fe*	7			7	−4	1	5	
6475.85		5	1	?	1	−5			
6482.10		5	2	4					
6496.69	*Fe*	6			2	−3			
6499.17	*Fe*	10	5	7					
6516.31		1			1	−1	1	8	
6532.0	*V?*	11	1	5					
6546.48	*Ti-Fe*	1			1	−3			
6563.05	*H*						...	100	
6574.47		12	3	5					
6581.45		6	1	4					
6633.09	*Fe*	3	1	6					
6678.37	*He*						2	10	
6705.35		6	1	4					
6752.97	*Fe*	4	1	?	1	−2			
6842.29		1	1	4					
6842.95		3	3	6					
6881.98	*Cr*	5	1	5					
6882.78	*Cr*	5	1	5					
6883.33	*Cr*	5	1	4					

The table, as already stated, contains the results of both sun-spot and chromospheric observations.

Columns 1 and 2 give the wave-lengths and probable origin of the various lines. Enhanced lines are indicated by "p," in accordance with Lockyer's and Fowler's notation.

Column 3 gives the total number of observations of the line in the sun-spot spectrum.

Column 4 gives the number of times that the line was seen reversed; while column 5 indicates the mean intensity or degree of the reversal, 10 being the maximum.

Column 6 gives the number of times that the line was recorded weakened or diminished in intensity. Column 7 gives the amount of this diminution, -5 indicating that the line is obliterated.

Columns 8 and 9 give the number of chromospheric observations of the line, and the mean intensity; the latter being based on Professor Young's scale as nearly as possible.

Column 10 is devoted to general remarks.

For convenience this table can be summarized as follows:

TABLE II

Element	Reversed	Weakened	Chromosphere*	Reversed and Chromosphere	Reversed and Weakened	Weakened and Chromosphere	Reversed, Weakened and Chromosphere	Totals
Iron	51	25	34	..	11	8	1	130
p Iron	..	..	5	..	..	4	..	9
Chromium	20	4	4	1	1	3	1	34
Titanium	10	2	3	..	1	3	..	19
p Titanium	..	1	2	..	..	1	..	4
Vanadium	21	..	1	1	..	..	..	23
Manganese	10	1	1	..	..	..	..	12
Nickel	8	10	1	..	2	..	..	21
Silicon	..	5	1	..	..	..	..	6
Calcium	3	..	4	..	..	..	..	7
Magnesium	..	..	4	..	..	..	..	4
Cobalt	..	..	2	..	..	..	..	2
Lanthanum	..	1	1	..	..	..	..	2
Yttrium	1	..	3	..	..	1	..	5
Helium	..	..	3	..	..	..	..	3
Barium	..	..	3	..	..	..	..	3
Sodium	1	..	..	..	..	2	..	3
Hydrogen	..	..	1	..	..	..	.	1
Unknown	21	18	18	..	3	4	..	64
Totals	146	67	91	2	18	26	2	

* Young's chromospheric lines are not included in this table

The columns in this table are mutually exclusive, hence, for example, to find the total number of chromospheric lines due to any element, it is necessary to add the figures standing opposite to that element in columns 4, 5, 7, and 8.

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THE CHEMICAL REACTIVITY OF THE CARBONYL GROUP AS MEASURED BY ITS ABSORPTION SPECTRUM

By A. W. STEWART[1] AND E. C. C. BALY

The question as to how far the space relations of atoms within a molecule could affect the properties and reactions of certain compounds was first raised twenty years ago by Wislicenus, and since that time the problem has attracted the attention of many chemists and physicists. Yet, in spite of the great interest which it has aroused, it cannot be said that very great progress has been made in some branches of the subject. An example may be chosen from the work of Menschutkin upon the esterification of aliphatic acids.

By a series of measurements he was able to show that, while a simple acid esterified with ease, the derivatives of the same acid, which were formed by the substitution of methyl groups for hydrogen atoms in the original compound, were much more difficult to esterify. The velocity of esterification was proved to be quite independent of the affinity constant of the acid used, as the following table shows:

Name of Acid	Formula	Velocity of Esterification	Affinity Constant
Acetic	$CH_3.COOH$	3.661	0.00180
Propionic	$CH_3.CH_2.COOH$	3.044	0.00134
Isobutyric	$(CH_3)_2CH.COOH$	1.0196	0.00144
Trimethyl-acetic	$(CH_3)_3C.COOH$	0.0909	0.00098

The most obvious explanation for this phenomenon appeared to necessitate the assumption that the volumes of atoms were not negligibly small when compared with their intramolecular paths of vibration. If this assumption be made, the phenomenon of hindrance can be understood and the cause explained in the following manner. The esterification process requires that the alcohol molecule involved in the reaction should approach very closely to the carboxyl radicle of the acid used. Such an approach will be

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easy, so long as the groups attached to the carboxyl radicle are not bulky. This is the case in acetic acid. But if for the hydrogen atoms in the acid nucleus we substitute methyl radicles, which are much greater in volume, the alcohol molecule will find much greater difficulty in forcing its way into the neighborhood of the carboxyl group, just as a person would find greater difficulty in walking into a crowded room than into a sparsely filled one.

There can be no doubt that, if atoms have any size at all, this theory of "steric hindrance," as it is called, will hold good; but it cannot be proved that the effects attributed to this cause play any very considerable part in the reactions in question. It seems more probable that the free paths of the atoms in their intra-molecular vibrations are so large in comparison to the size of the atoms themselves that this heaping up of substituents in the neighborhood of the reactive group would have no very marked effect.

Stewart[1] has shown that, when a hydrogen atom near the carbonyl group of a ketone is replaced by a methyl radicle, the result is a decrease in the additive capacity of the carbonyl group. This might have been anticipated from the hypothesis of steric hindrance, since the volume of the methyl radicle must be greater than that of a hydrogen atom. A contradiction between theory and experiment is found in the case where, instead of a methyl radicle, a $-COOEt$ group is introduced into the molecule. In the case of the latter group it is found that, instead of decreasing the velocity of addition of sodium hydrogen sulphite, as its bulk might lead us to predict, it has the contrary effect; for some of these ketones which contain a carboxyl group are much more reactive than the corresponding simple ketone, and, *a fortiori*, than the methyl substituted ketone. The figures found for acetoacetic ester, acetone, and methyl ethyl ketone show this clearly:

		10	20	30	40 Min.
Acetoacetic ester	CH_3COCH_2COOEt	37.4	47.0	56.0	60.0
Acetone	CH_3COCH_3	28.5	39.7	47.0	53.6
Methyl-ethyl ketone	$CH_3COCH_2CH_3$	14.5	22.5	25.1	29.1

The figures give the percentage of bisulphite compound formed

[1] *Chem. Soc. Trans.*, **87**, 185, 1905; *Proc.*, **21**, 78.

by each ketone during the time indicated. It is thus made evident that some new influence has come into play, which tends to mask or modify the steric hindrance due to the more voluminous group.

The carboxyl group in itself, however, is not sufficient to produce this increased reactivity of the carbonyl radicle, as the rate of addition of sodium hydrogen sulphite to ethyl laevulate $CH_3.CO.CH_2CH_2.COOEt$ was found to be slightly lower than that found in the case of methyl propyl ketone $CH_3.CO.CH_2CH_2.CH_3$, which contains a carbon chain of the same length; and a like result was observed in the case of the diketone acetonyl acetone $CH_3.CO.CH_2.CH_2.CO.CH_3$. On the other hand, acetone dicarboxylic ester $COOEt.CH_2.CO.CH_2.COOEt$ has an additive capacity even greater than that of ethyl aceto-acetate. Acetone shows very little sign of tautomeric change; while, on the contrary, acetoacetic ester and acetone dicarboxylic ester are tautomeric compounds. Thus here again theory and experiment appear to be opposed to one another, the true carbonyl compound having much less reactive power than the semi-enolised body. It occurred to us that in this fact was to be found the key of the problem, and that the exceptionally great reactivity of the carbonyl group in tautomeric compounds was due to the actual process of tautomeric change. Acetoacetic ester, under conditions, exists as an equilibrium mixture of the two bodies (I) and (II); and the conversion of the first into the second, and vice versa, is going on continuously.

$$\text{(I)}\quad CH_3-\underset{\displaystyle OH}{\underset{|}{C}}=CH-COOEt \qquad \text{(II)}\quad CH_3-\underset{\displaystyle O}{\underset{\|}{C}}-CH-COOEt.$$

Now, when a molecule of (I) is converted into a molecule of (II), the result is the formation of a carbonyl group from a hydroxyl group. From analogy with the behavior of atoms in the nascent condition, we must suppose that this "nascent carbonyl group" is endowed with a much greater reactivity than that possessed by the ordinary non-nascent carbonyl radicle. This activity need not, however, be occasioned purely by the actual wandering of the hydrogen atom from the oxygen to the carbon; it may be due to some finer play of forces within the molecule which manifests itself in the production of the characteristic absorption of the acet-

oacetic ester spectrum. The condition into which the hydrogen atom is thrown as a result of this play of forces may be termed a condition of "potential tautomerism," and in it the hydrogen atom will possess a reactive power more or less analogous to that required by an atom as a consequence of the ionisation process.[1]

If we now apply this idea to several cases which have hitherto been classed under the head of steric hindrance, it will be found that they can be satisfactorily explained. Taking the case of the ketones which have already been dealt with by one of us,[2] a marked decrease in the reactivity of the carbonyl group is shown when the hydrogen atoms of acetone are successively replaced by methyl radicles.

In the course of their investigations of the spectra of derivatives of acetoacetic ester, Baly and Desch[3] proved that the equilibrium between the enolic and the ketonic forms produces an absorption band in the ultra-violet region of the spectrum; and they also showed that this band is not due to the shifting of a hydrogen or metallic atom, but is rather to be considered as the result of some intra-atomic change. In the hope of finding some analogous process in the simple ketones, we examined the absorption spectra of several; and we found that a similar absorption band exists there as well. We further noticed that *the persistence of this band decreases proportionately to the diminution in the reactivity of the ketone's carbonyl group.*[4]

For instance, the following figures show the percentages of oxime formed by various ketones in twenty minutes;[5] and on comparing these amounts with the curves of the absorption spectra shown in Fig. 1, the relation between the two will be evident.

	% Oxime Formed
Acetone, CH_3COCH_3	49.7
Methyl ethyl ketone, $CH_3COCH_2CH_3$	39.2
Methyl propyl ketone, $CH_3COCH_2CH_2CH_3$	37.3
Methyl iso-propyl ketone, $CH_3COCH(CH_3)_2$	31.5
Pinacoline, $CH_3.CO.C:(CH_3)_3$	17.0

[1] Baly and Desch, *Astrophysical Journal*, **23**, 110, 1906.

[2] Stewart, *loc. cit.*

[3] *Loc. cit.*

[4] The method of observing and plotting the curves of absorption spectra was described in the previous paper (*Astrophysical Journal*, **23**, 110, 1906).

[5] Stewart, *Chem. Soc. Trans.*, **87**, 410, 1905.

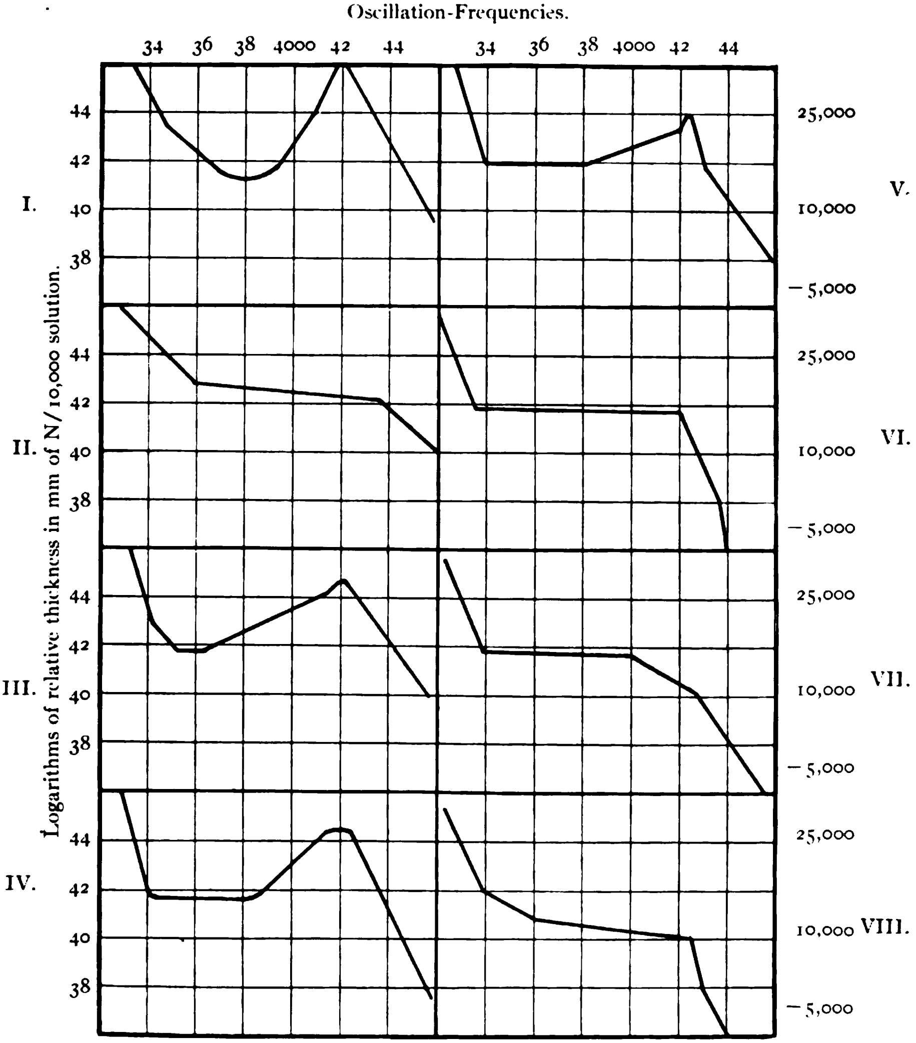


FIG. 1

I. Acetone.	V. Methyl iso propyl Ketone.
II. Acetone (in water).	VI. Pinacoline.
III. Methylethyl Ketone.	VII. Methyl hexyl Ketone.
IV. Methyl propyl Ketone	VIII. Methyl nonyl Ketone.

Lapworth[1] showed that the action of halogens upon acetone was preceded by the production of the enolic form of the ketone; and he found, further, that the presence of acids hastened the reaction. Now, he had already shown[2] that the presence of acids brings about a rapid attainment of equilibrium between the tautomeric forms of carbonyl compounds; or, in other words, the addition of acid has a tendency to produce a "nascent carbonyl group." Hence, in the case of acetone itself, not only is there direct spectroscopic evidence in favor of tautomeric change, but the chemical evidence at our disposal is also favorable. Instead of attributing Lapworth's results to the actual formation of the enolic form and an immediate addition of halogen, we prefer to look at them from another point of view. It is obvious, if we consider the change of the group $-CH=CH(O\overset{*}{H})-$ into $-CH\overset{*}{H}-CO-$, that the hydrogen atom marked with an asterisk must become "pseudo-nascent" in the process of change. It would therefore be peculiarly liable to chemical action, and would easily be replaced by halogens. The very great ease with which the methylene hydrogen atoms in acetylacetone are replaceable by halogens lends further support to our hypothesis.

In their paper Baly and Desch (*loc. cit.*) stated that acetonylacetone and ethyl laevulate were pure ketonic substances; but on examining the spectra of these substances at greater concentrations than were previously employed, we have been able to detect at one point a rapid extension of the spectrum which corresponds to a very shallow absorption band (Fig. 2). The shallowness of the band indicates that the tautomerism in these two compounds is very weak, which agrees with what has been found with regard to the reactivities of their carbonyl groups. The close agreement between theory and experiment in these cases is very noteworthy.

Now Petrenko-Kritschenko has shown[3] that the speed of phenylhydrazone formation is greatly influenced by the nature of the solvent in which the reaction is carried out. It appeared probable to us that this might be due to the influence of the solvent upon the

[1] *Chem. Soc. Trans.*, **85**, 32, 1904.

[2] *Ibid.*, **81**, 1503, 1902 and **83**, 1121, 1903.

[3] *Journ. Russ. phys.-chem. Soc.*, **35**, 404, 1903.

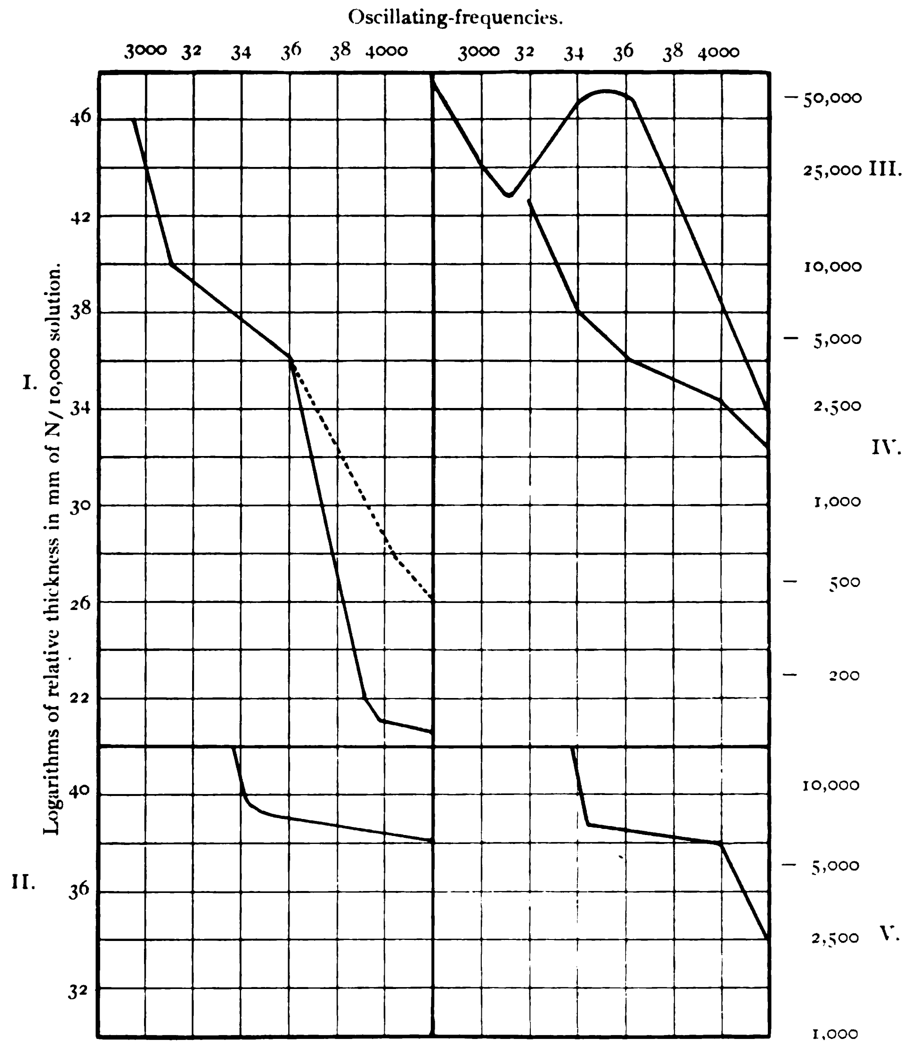


FIG. 2

I. Acetoacetic ester in alcohol (full curve).
The same in water (dotted curve).
II. Ethyl laevulate.
III. Ethyl pyruvate.
IV. Ethyl diethyl aceto acetate.
V. Acetonyl acetone.

tautomerism process; and to test the matter we examined the spectra of acetone and acetoacetic ester in aqueous solution, using as a control in the latter case the spectrum of diethylacetoacetic ester, which is much less tautomeric than the parent substance. From the curves for acetone in alcoholic and aqueous solution it will be seen that the influence of water is very marked, the band in the latter case being much shallower than in the former. The three curves shown in Fig. 2 give the absorption spectra of the acetoacetic ester series; and it is obvious from them that the water has reduced the tautomerism very considerably. It is probable that the greater the unsaturation of the solvent, the less reactivity will be shown by the carbonyl group of the dissolved ketone.

The evidence from simple ketones being so far favorable, we must now examine the case of ketones containing an ethyl carboxylate group. If tautomeric change alone were the cause of the reactivity of the carbonyl radicle, compounds containing the group $-CO-CH_2-CO-$ should be more reactive than those which do not contain it, and the reactivity of the carbonyl group in pyruvic ester (CH_3-$COCOOEt$) should not be at all abnormal, since the group in question does not occur in it; if, however, the reactivity were found to be great in this case, we hoped that some light might be thrown upon the problem by the study of the compound's spectrum.

We therefore decided to compare the rates of addition of potassium hydrogen sulphite to acetone, acetoacetic ester, acetonedicarboxylic ester, and pyruvic ester. The results obtained by the method employed are given in the table below:

		5	10	15 Min.
Acetone	CH_3COCH_3	5	7	9%
Acetoacetic ester	CH_3COCH_2COOEt	12	18	24
Acetonedicarboxylic ester	$CO:(CH_2COOEt)_2$	30	36	42
Pyruvic ester	$CH_3COCOOEt$	52	64	76

In spite of the precautions taken, these numbers are probably not quite accurate, owing to various causes which cannot be controlled; but the differences between the numbers themselves are very much larger than any possible experimental error under the conditions employed.

An examination of the figures shows that the introduction of a

$-COOEt$ group into acetone increases the additive capacity of the carbonyl group; the introduction of two $-COOEt$ groups still further enhances the reactivity of the carbonyl group; but the most striking effect is produced when, as in the case of pyruvic ester, the carbonyl and carboxyl groups are brought into juxtaposition in the chain. Now, in the case of pyruvic ester, though the compound does sometimes react in the enolic form, it is most improbable that the change of the enolic into the ketonic form is going on at a rate at all comparable to that in which it is occurring in acetoacetic ester or acetonedicarboxylic ester, so that it is not likely that the exceptional reactivity of the carbonyl group in pyruvic ester is due to this kind of tautomerism.

We thought it advisable to examine the spectrum of pyruvic ester, in the hope that some light might thus be thrown on the problem of the activity of the carbonyl group. We found that pyruvic ester gives an absorption band which lies much nearer to the red end of the spectrum than the band given by acetoacetic ester. The origin of the band in the pyruvic ester spectrum might be looked for in two phenomena; either in the enol-keto change of the group CH_3-CO-, or in the interaction of the carbonyl and carboxyl groups of the radicle $-CO-COOEt$. The first explanation is impossible, since, if the band were produced by a similar state of intra-atomic vibration in both instances, it would occur in nearly the same place in the spectrum, while actually the new band has its head with a frequency of 3100, while that of the acetoacetic ester band lies at 3700. Further, since the molecule of pyruvic ester is lighter than that of acetoacetic ester, we should expect to find the band in the latter case nearer to the red end of the spectrum than in the former; while actually the reverse of this is observed.

In order to make certain that the band in question was actually produced by the proximity of two true carbonyl groups in the chain—i. e., that it was not due to the $-COOEt$ group of the carboxylate radicle—we examined the spectra of several α-diketones and found a similar band in all of them, though in them it was situated nearer the red end of the spectrum. For example, in the case of camphorquinone $C_8H_{14}\left\langle\begin{array}{l}C=O\\ |\\ C=O\end{array}\right.$ (Fig. 3) it will be seen that an absorption

band is shown which has a very long persistence. Its head lies at a frequency of 2070.

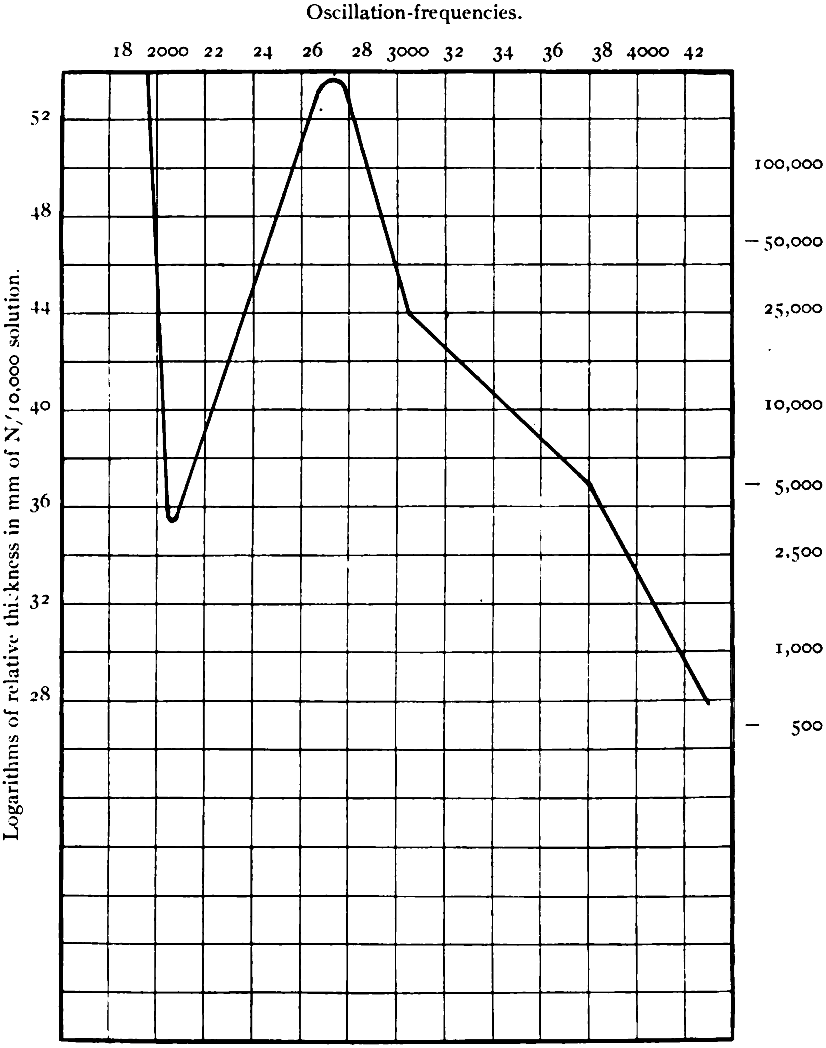


FIG. 3.—Camphor quinone.

It was thus proved that the type of band in question was due to the two carbonyl groups in the *a*-position to one another. It has already been pointed out that Baly and Desch concluded that,

though the keto-enol absorption band was produced by intra-atomic vibration, it was caused by the change of linkage brought about by the oscillation of the hydrogen atom during the change from the ketonic to the enolic form. From analogy we should expect to find a somewhat similar state of things in the present case. Now, the only possible way in which such a change of linkage can be supposed to occur in pyruvic ester is by imagining that, like acetoacetic ester, it occurs in two forms:

$$\text{(I)}\quad \begin{array}{l} CH_3-\underset{\displaystyle O}{\underset{\|}{C}}-\underset{\displaystyle O}{\underset{\|}{C}}-OEt \end{array} \qquad\qquad \text{(II)}\quad \begin{array}{l} CH_3-\underset{\displaystyle O}{\underset{|}{C}}=\underset{\displaystyle -O}{\underset{|}{C}}-OEt \end{array}$$

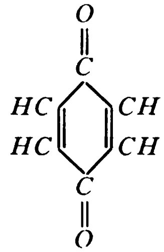
It is very hard to indicate exactly what is meant by the aid of the usual structural formulæ, as they only indicate a static condition of the molecule, while what we wish to suggest is essentially a dynamic state. We wish to make it perfectly clear that we do not suppose these two forms necessarily to exist statically; but, owing to the defect of ordinary structural formulæ, it is impossible to write them otherwise if the usual symbols be employed. Our conception can best be comprehended if it be clearly borne in mind that the two formulæ are not intended to represent actual compounds, but merely two phases of the same compound. If this conception of phases be understood, it will be apparent that the change of linkage is continually going on, and that this change will affect the intra-atomic relations of the molecule very much in the same way as they are affected by the phenomena of tautomerism.

At the same time, it should be noticed that the change of linkage from (II) to (I) would produce what we have already defined as a "nascent carbonyl group" which would have great reactivity. Thus we are led to conclude that substances which show these peculiar absorption bands will in general be more active chemically than other compounds which do not exhibit such selective absorption.

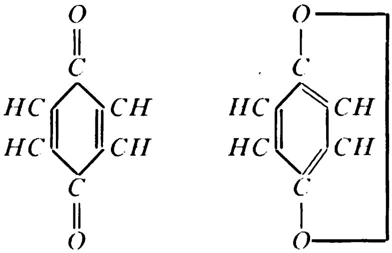
The idea which we have put forward cannot be considered as part of the theory of tautomerism, as, owing to its associations, the name "tautomerism" will always suggest the wandering of a hydrogen atom. It is unfortunate that the name "Desmotropism" has already been employed to denote tautomerism, as it seems well fitted to describe the phenomenon with which we have dealt. We

therefore wish to propose the word "Isorropesis" (*ἰσορροπεῖα*, "equipoise") to describe the process.

The arguments in favor of this theory appeared to us to warrant its application to other classes of compounds, and we proceeded to make a further series of investigations, some of which will be dealt with in a later paper. For the present, quinone



is the only compound which need be described. The known close relation to one another in which the two para-positions in benzene stand, seemed to lend probability to the idea that a band somewhat similar to those observed in compounds containing the group $-CO-$ $-CO-$ might be found in the spectra of the para-quinones. Our anticipations were again justified, as the quinone spectrum has a band almost identical with that of camphor quinone, its head lying at 2150 (see Fig. 4). Now, it is known that quinone can exist in two forms, for both of which chemical evidence has been adduced:



It appears not unwarrantable to assume that in this case also the absorption band is caused by the "make and break of contact" between the two oxygen atoms, as already suggested for pyruvic ester. Thus it may be concluded that the actual wandering of a hydrogen atom is not necessary for the production of these absorption bands. Again, since the result of this "make and break"

would be the production of two nascent carbonyl groups, an explanation is thus given of the great chemical activity of the carbonyl radicles in quinone.

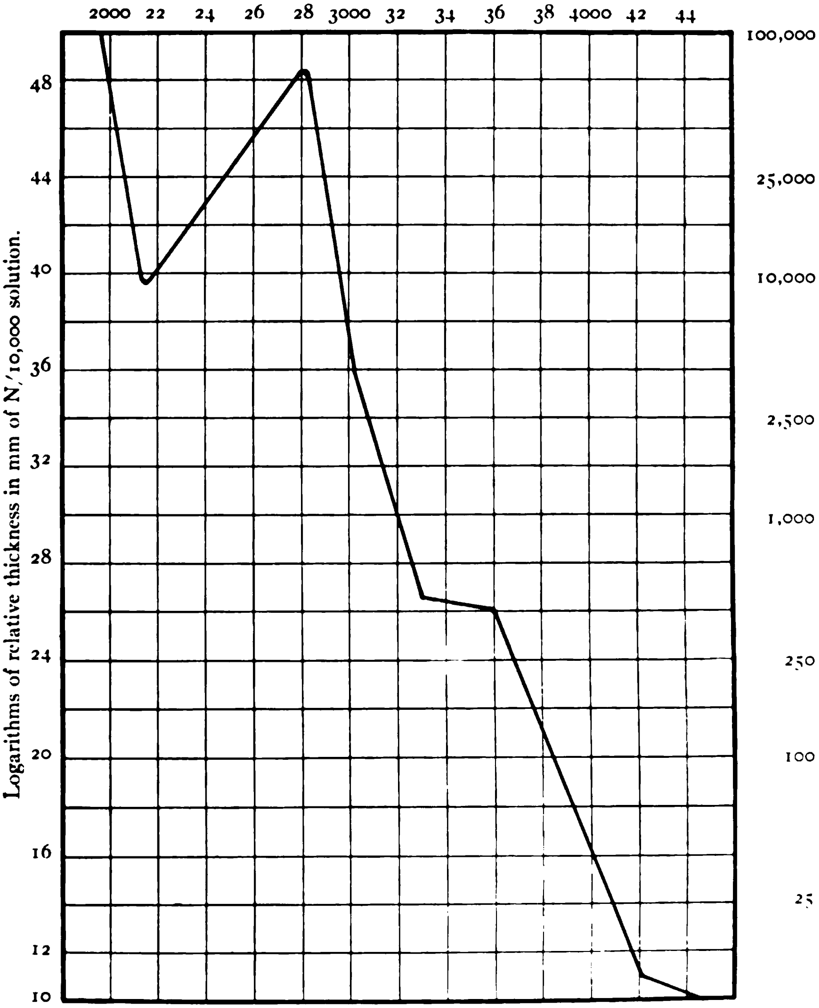


FIG. 4.—Quinone.

It is thus proved that the two carbonyl groups in the *a* position to one another give rise to a new type of absorption band. From

analogy with Baly and Desch's observations that the ultra-violet absorption bands are produced by the change of linkage brought about by the oscillation of the hydrogen atom in the process of enol-keto tautomerism, we should expect to find a somewhat similar change of linkage in the α-diketones. We propose to reserve a full discussion of this process to a further paper in connection with its relation to color, but it may be stated that we are undoubtedly concerned with the residual affinity upon the two oxygen atoms. The free period giving rise to the absorption band is due to a mutual disturbance or an oscillation between these residual affinities.

It would seem that the results described above are of considerable importance, for they show that the chemical reactivity of a ketonic compound can be measured directly by means of the spectroscope. We find that the persistence of the absorption bands given by these compounds is directly proportional to their chemical reactivity. It is a logical conclusion that the chemical reactivity of these compounds is due to a change of linkage within the carbonyl group of the ketone. This change of linkage may either be produced by a wandering hydrogen atom such as occurs in the reversible equilibrium

$$\begin{array}{c} -CH_2-\underset{\underset{O}{\|}}{C}- \end{array} \qquad\qquad \begin{array}{c} -CH=\underset{\underset{OH}{|}}{C}- \end{array}$$

which occurs or tends to occur in the monoketones and the β diketones containing the $-\underset{\underset{O}{\|}}{C}-CH_2-\underset{\underset{O}{\|}}{C}-$ group, or it may be produced by isorropesis or the oscillation between the residual affinities of two carbonyl oxygen atoms in juxtaposition, thus

$$-\underset{\underset{O}{\|}}{C}-\underset{\underset{O}{\|}}{C}- \qquad\qquad \begin{array}{c} -C=C- \\ |\quad\ | \\ O-O \end{array}$$

Furthermore, it is evident that all measurements of the chemical reactivity of a ketonic compound are simply measurements of the amount to which tautomerism is present in the compound. A very great deal of importance has been attached to these measurements in connection with the theory of steric hindrance, but our results, we are sure, will go far to discountenance this theory. There

is no doubt that the decrease in chemical reactivity caused by the substitution of hydrogen atoms by groups of atoms is caused, not by the bulk of these atoms, but simply by the fact that fewer hydrogen atoms are available for the tautomeric processes. It is unnecessary to cite in this place the many reactions of ketones which have been measured in support of the theory of steric hindrance, but there is no doubt that they can all be explained more easily and more rationally by the conception of the nascent carbonyl group set forth above.

The method employed for the photography of the absorption spectra of the substances, as mentioned above, has already been described;[1] the compounds were in each case prepared most carefully and satisfactorily answered every test of their purity. Except where especially mentioned to the contrary, the absorption spectra of the substances were observed in alcoholic solution.

CONCLUSIONS

1. The reactivity of any carbonyl group is not inherent in the group itself, but is produced by the action of neighboring atoms upon the carbonyl group rendering it "nascent."

2. Such action may take the form of tautomerism, or of a modification of tautomerism which does not require the actual transfer of a hydrogen atom from one atom to another, but merely some intra-atomic disturbance in the system $-CH_2-CO-$.

3. The action may also take the form of the process which we have termed "isorropesis," in which no actual wandering of the atoms occurs, but in which some oscillation between the residual affinities of the oxygen atoms of the two carbonyl groups is involved.

4. Many cases which at present are accounted for on the hypothesis of steric hindrance can be better accounted for either by tautomerism or isorropesis; and some cases which are in direct contradiction to the steric theory can also be explained. It is therefore claimed that the hypothesis of the "nascent carbonyl group" accounts more satisfactorily for the facts and is superior to explanations based upon the idea of steric hindrance.

5. When the possibility of the formation of a nascent carbonyl

[1] Baly and Desch, *loc. cit.*

group is excluded, neither the usual ketonic reactions nor an absorption band is observed. The carbonyl radicle may then be considered an "inactive" carbonyl group in contradistinction to a "nascent" one.

6. Inasmuch as both the processes of tautomerism and isorropesis result in the appearance of an absorption band whose persistence is a measure of the amount to which these processes are taking place, it is possible to determine the chemical reactivity of a ketonic compound by observing the persistence of the absorption band.

In conclusion we wish to express our thanks to the Chemical Society of London for a grant toward the expenses of this research; to Professor Collie and Dr. Smiles, for the great interest they have taken in the work; and to Mr. W. B. Tuck, B. Sc., for assistance during the course of the investigation.

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LINE STRUCTURE, III.

RELATION OF WIDTH TO INTENSITY OF NORMAL LINES

By P. G. NUTTING

That the spectral width of normal spectrum lines varies with the nature of the excited gas, the intensity of its excitation, and the pressure of the surrounding atmosphere is well known to spectroscopists. The class name "normal" lines is used to designate simple lines which merely broaden and reverse as their intensity is increased, as distinguished from "composite" lines consisting of several components when fully developed. A. A. Michelson[1] and F. L. O. Wadsworth[2] from the "visibility" of lines in the interferometer calculated their width and established relations between width and atomic weight and pressure. They appear to have taken no account of the effect of intensity on the width of lines and composite lines are treated as single lines.

The echelon grating, with its high resolving power and direct reading property, exhibits the individual character of fine lines as does no other instrument. Structure, shading off at the edges, and the like are shown by the largest Rowland grating down to a line-width of about 0.05 t.-m; lines narrower than this appear merely as narrow ribbons of light without character. The echelon is just at its best at this width, and gives the details of lines as narrow as 0.005 t.-m.[3]

Previous work[4] having indicated the existence of an intimate relation between the width and structure of arc lines and their intensity, a spectrophotometric study of line width was undertaken. The plan was to measure width and intensity simultaneously, the former with an echelon provided with a micrometer eyepiece, the latter with a special spectrophotometer.

[1] *Phil. Mag.*, (5) **34**, 280–299, 1892.

[2] *Astrophysical Journal*, **6**, 30, 1897.

[3] P. G. Nutting, "Line Structure, I," *Astrophysical Journal*, **23**, 64–78, January 1906; "Line Structure, II," *ibid.*, 220–232, April 1906.

[4] Most lines are so sharp and well defined when narrower than 0.1 t.-m. that width measurements to within the width of the finest cross-hairs are easily made.

The apparatus was used as shown in the figure. Light from the source passed by one path through a dispersing apparatus and echelon, and by another path through a spectrophotometer so placed that its ocular was very near the echelon ocular. A 1 mm horizontal slit placed near the arc served to screen off all light except that from the central portion of the luminous flame, and the same portion supplying the light entering the echelon. A Nernst lamp at normal voltage, checked at intervals with a normal Hefner, served as reference standard. The light from the Nernst lamp entered the 0.2 mm spectrophotometer slit between the two right-angled prisms through which the light from the arc entered. An ocular slit cut down the central continuous spectrum to the width of the spectrum lines seen above and below it. The echelon ocular micrometer was fitted with diagonal quartz-fiber cross-hairs.

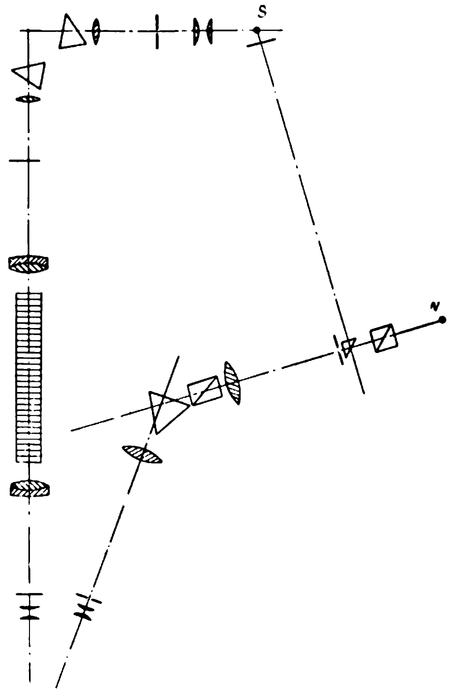


FIG. 1

The lines investigated were chiefly the brighter visible arc lines with a few Plücker tube lines. Of these attention was directed toward the normal lines—lines which show no permanent structure, but simply broaden and reverse with increase of intensity. Leaving out of account spectra filled with excessively numerous lines, fully one-third of the spectrum lines possess permanent structure, many are triple, some double, and some quadruple. Once composite, always composite, appears to be generally true; that is, a line showing structure under any conditions will appear composite under all conditions. A line which is (say) triple in a Plücker tube will be

found triple in the arc spectrum. Components differ in width and relative intensity under different conditions of production just as the lead spark spectrum varies with capacity and inductance used, but a composite line always retains its individuality as does the lead spectrum.

When a line is really composite, the fainter components disappear *in situ* as the intensity is decreased; if it is a normal single line reversed, the wings merge in a single line with decrease in intensity. The spectrum of manganese beautifully illustrates the distinctions involved. In the green, in the midst of a multitude of normal lines, are a dozen double and triple lines. These really composite lines remain as steady as though painted, while their neighbors broaden and reverse with every flash of the arc.

Copper λ 5782 and λ 5700, and calcium λ 5858, are the most striking examples of permanent double structure, while zinc, cadmium, and the alkali metals have many striking triple and multiple lines. Of the normal lines good examples may be seen in the spectra of barium, palladium, chromium and titanium.

A brief list of spectra is given below, with an indication of the character of the prominent lines:

Lithium: all five composite.

Sodium: D lines composite.

Potassium: all composite.

Rubidium: all (four red, seven green) composite?

Magnesium: all normal.

Calcium: half composite, half normal.

Strontium: all normal.

Barium: all normal.

Titanium, cerium, thorium, vanadium, chromium, molybdenum, tungsten and uranium: all normal.

Manganese: twelve green composite, all others normal.

Iron, cobalt, nickel, palladium, platinum, rhodium and osmium: all normal.

Copper: two yellow, two green composite, λ 5106 normal.

Silver: all composite.

Gold: all normal.

Iridium: blue normal.

Zinc, cadmium, mercury, thallium, tin, lead and bismuth: all the lines observed are composite, namely: zinc λλ 6263, 4810, 4722, 4680; cadmium, λλ 6438, 5086, 4800, 4676; mercury, λλ 5790, 5769, 5461, 4358, 4748, 4078, 4047; thallium, λ 5350; tin, λλ 5631, 4525; lead, λλ 6002, 4058; bismuth, λ 4722.

Roughly speaking, then, excepting calcium and manganese, the lines of all the spectra in which lines are excessively numerous are normal, while the lines of the simple spectra—except gold, indium, and magnesium—are composite. This paper is a discussion of the behavior of the normal lines. The composite lines are to be treated in a later paper.

The width of a normal line appears to depend, aside from the pressure, upon the intensity alone. When, for example, copper is fed into an arc with carbon electrodes, the line λ 5106 is of the same width at a given intensity as when copper electrodes are used. It is the same whether metallic copper or a copper salt is fed into a carbon arc. If, as a salt burns away, the current is increased to bring the intensity of a line up to its original value, its width also is restored. Nor is this ratio of width to intensity appreciably affected by the addition of an alien salt or metal to an arc. A line has less diffuse wings when produced in an oxyhydrogen flame than when produced in an arc, but it appears to possess the same mean width at a given intensity in the two cases.

The green mercury line has three satellites on the side of the primary toward the red. These three lines are the narrowest bright lines known to the writer, and the middle one of these was for this reason chosen to verify the correction for slit-width to be applied to the measured width of a line. The figures quoted below and the points on the accompanying graph are from the first set of readings taken:

Slit-Width	Width of Line	Width of Slit	Slit-Width	Width of Line	Width of Slit
5.4	121 μ	114 μ	4.8	53	44
5.3	110	94	4.7	39	34
5.2	98	84	4.6	26	24
5.1	91	74	4.5	15	14
5.0	79	64			
			4.4	faint	4
4.9	67	54	4.36	closed	0

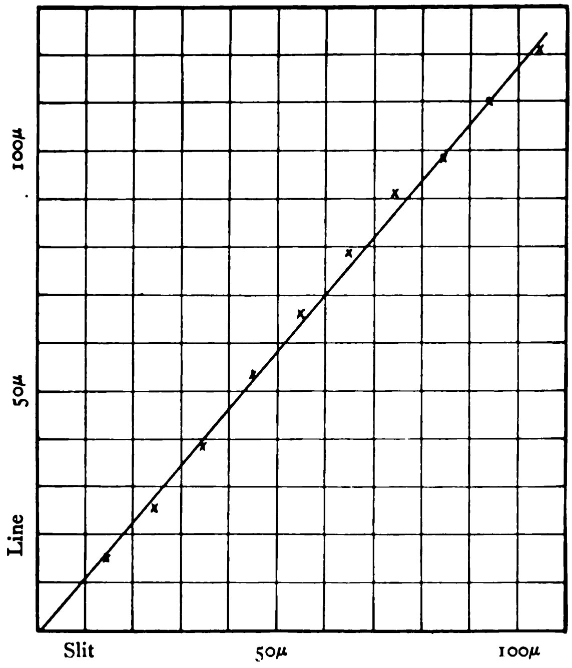


FIG. 2.—Width of Line: Width of Slit

Line-width and slit-width are seen to be proportional to each other far within the possible errors of measurement. The resolving power of the echelon is seen to be considerably (at least five times) greater than the calculated value of 0.01 tenth-meter. The actual width of the line itself is from the plot certainly not greater than 1 micron, or almost 0.001 t.-m. in terms of wave-length. In other words, the light of this satellite of the green mercury line is homogeneous to within 1 part in at least 5,000,000. The main primary of this line is fully twenty times as broad as this satellite, while the whole line, measured from extreme satellites, is three hundred times as broad, or about 0.3 t.-m.

The results of some observations on the relation of width to intensity of the yellow helium line λ 5876 are given below. They are the first and last of a series of observations. The helium was contained in short, stout Plücker tubes excited by means of a 5000-volt transformer. The intensity was varied by varying the current

in the primary. The two sets of observations quoted were made more than a week apart (two different Plücker tubes. The echelon slit was open 14 microns in one case and 24 microns in the other, the widths of the line being corrected by means of the plot reproduced in Fig. 2 above.

Tube 1			Tube 2		
Intensity	Slit-Width 24 μ		Intensity	Slit-Width 14 μ	
	Linear	Wave-Length		Linear	Wave-Length
1.0	42 μ	0.046 t.-m.	0.85	40 μ	0.044 t.-m.
1.9	47	0.052	1.44	46	0.050
3.2	57	0.062	2.6	56	0.062
4.7	66	0.072	3.9	64	0.071
6.7	76	0.084	5.0	69	0.076
9.5	93	0.102	6.0	72	0.079
12.9	109	0.120	7.1	75	0.083
19.0	115	0.126	9.0	77	0.085
			12.2	90	0.099
			20.2	115	0.126

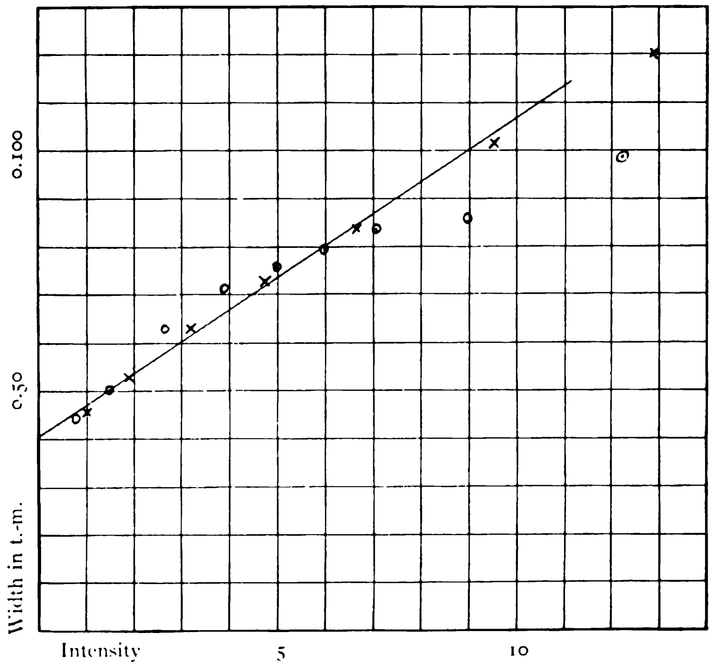


Fig. 3. Width: Intensity, *He*, λ5876.

The width of the line is seen to vary considerably with its intensity, more than doubling within the range available. The relation between width and intensity is approximately linear for low and moderate intensities. At great intensities the width is, if anything, less than this proportionality would indicate. At the intensity called 10, the capillary of the tube was just commencing to burn and show sodium lines, and possibly this might account for the falling-off from the linear relation at high intensities, but arc lines show a similar falling off. Plotting against a quadratic or logarithmic abscissa does not help matters.

Extrapolating back to zero of intensity, there is good evidence that the line then possesses a finite width of perhaps 0.040 t.-m. Visual observation tends to confirm this, for at intensities much lower than the lowest at which the width can be measured with any accuracy, the line still appears as a ribbon of light rather than as a fine line. The appearance of the line suggests that it is an unresolved composite. With exceptionally favorable conditions, I have seen a faint separation into two components. If the line had two components, each of zero width, this limiting width of 0.040 t.-m. would represent the distance between their centers.

A few normal arc lines are sufficiently isolated in their spectra and variable in intensity to permit fairly accurate observations of the width-intensity relation to be made. Arcs of from 2 to 8 amperes were used. As the arc fluctuated, the maxima and minima were found to be reasonably constant in intensity and to permit of measurement. The best of the results are given below in the form of linear equations between width W and intensity I. They are taken from the plotted curves, all of which were of the same general shape as that reproduced in Fig. 3. W is expressed in tenth-meters, and I in terms of an arbitrary unit such that the intensity of the bright green line *Fe* λ 5328 from an iron arc of 5 amperes in open air is about 0.5.

Ba	λ 6342	$W=0.034+0.0030\ I$
Ni	λ 5477	$W=0.032+0.0035\ I$
Fe	λ 5328	$W=0.025+0.0045\ I$
Cu	λ 5106	$W=0.033+0.0030\ I$
Pd	λ 5296	$W=0.032+0.0040\ I$

For comparison may be added:

He λ 5876 . . . . $W = 0.040 + 0.007\ I$
Hg λ 5461 satellite . . . . $W = 0.001$

The results for the arc lines show a striking uniformity, both in limiting width and in intensity coefficient. Other lines gave data so similar, and the accidental errors were so large, that it was found to be only duplicating work to take measurements on them, and only minimum width and intensity at reversal were recorded. With the arrangement used—echelon slit *perpendicular* to the spectrum lines formed by the preliminary dispersing apparatus—many lines were simultaneously in the field of view, so that, if any of the many hundred lines observed had possessed exceptional qualities, they would have been at once noticeable.

The minimum width of all normal arc lines produced at atmospheric pressure lies between 0.02 t.-m. and 0.05 t.-m. This width is of the same order as the width to be expected from the Doppler effect and the kinetic gas theory. For, if δ is the spectral width of a line produced by a source moving with a mean speed of u cm/sec., then if V is the velocity of light, $\delta/\lambda = 2u/V$. For air at room temperature and pressure u is usually taken as about 4.5×10^5 cm/sec., increasing with the square root of both temperature and pressure. Hence the width of a green line should be at least $\delta = 5000 \times 2 \times 4.5 \times 10^4 / 3 \times 10^{10} = 0.015$ t.-m. If the arc gases are at a temperature of 3000°, then this width should by the kinetic theory be multiplied by the factor $(3000/300)^{\frac{1}{2}} = 3.16$, hence $\delta = 0.015 \times 3.16 = 0.047$ t.-m. Similarly for a red line at λ 7000, $\delta = 0.034$ t.-m.

Reversing the argument, the widths observed indicated that only the thermal motions of the radiating particles are concerned in the broadening. The conduction of a current by the gas in the arc does not add greatly to the translatory motions which the particles of a gas would have in virtue of their high temperature alone. In other words, line-of-sight motions are unaffected by the current through an arc.

At low pressure an inclosed arc with electrodes of iron, copper, brass, and nickel were used. With nickel some observations were

obtained with a clear window without disturbing the photometric adjustments used in the previous work. For *Ni* λ 5477,

$$W = 0.006 + 0.007\, I\,,$$

with a large probable error on account of the great fluctuations and altered character of the arc flame. Other lines were observed to have roughly the same minimum width. From kinetic theory the width at 1 cm pressure should be $(1/76)^{\frac{1}{2}} = 0.11$ of the width of atmospheric pressure. The latter for λ 5477 at 3000° is 0.043 t.-m.; hence at 1 cm pressure should be 0.0047 t.-m., considerably less than the width observed.

INTENSITY AT REVERSAL

The intensity of the line when reversal just occurred was recorded in the case of about eighty normal lines in different spectral regions and belonging to a wide range of elements. The striking uniformity of these intensities led to their being plotted against wave-lengths as abscissæ. The plot (Fig. 4) showed a well-defined galaxy through which a median line could be drawn with some certainty. Owing to the enormous fluctuations in the arc at the reversing intensity, the

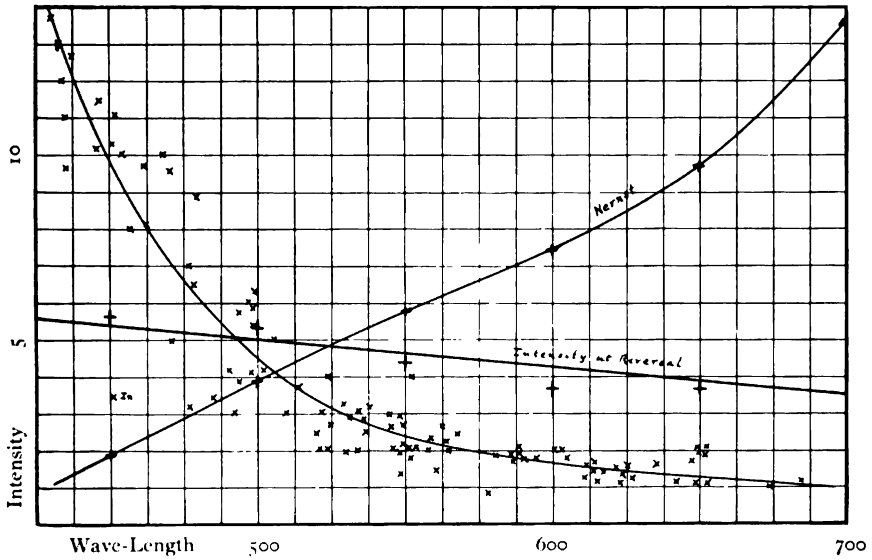


FIG. 4 —Intensity at Reversal

possible error in measurement is large, 20 to 60 per cent. But in view of the fact that the intensity of a line fluctuates by a factor of 10 to 100,000, measurements to within a factor of two are not unsatisfactory.

The points plotted in Fig. 4 relate to the Nernst lamp used as reference standard. Hence multiplication by the spectral energy of the Nernst lamp gives the relative energy of the various lines at reversal. The energy-curve for the Nernst lamp was obtained by inserting a Rubens thermopile before the ocular of the spectrophotometer.

Numerical data are given below. The thermopile was used with a Weston moving-coil galvanometer, and deflections are in millimeters. The 8-candle-power Nernst lamp, when operating at that luminosity, was found to have a drop of 106 volts across the filament, and was kept at this voltage to within 0.1 volt by hand regulation of the rheostat used as ballast for the lamp. At wavelength 650 the emission at 107 volts was 12 per cent. greater than at 106; at 105 volts, 15 per cent. less. The values of $dn/d\lambda$ used to reduce the prismatic to the normal energy-spectrum were specially determined for the prism used.

Wave-Length	400	450	500	550	600	650	700
Deflection of galvanometer......	(3)	11	33	69	123	199	347
$dn/d\lambda$ (cm^{-1})	4000	2650	1780	1261	895	730	590
Normal Nernst spectrum	(120)	291	588	870	1101	1452	2048
Intensity of reversal (relative to Nernst)......................	(20.)	9.8	4.5	2.4	1.7	1.3	1.0
Intensity of reversal (corrected)..	(240)	285	264	209	187	188	205

The actual energy intensity thus obtained is sensibly a constant, not only for all lines in the same spectral region, but for all lines in all regions. In other words, all normal lines reverse at the same intensity, provided of course that they are produced under similar conditions, say in an arc in open air. This constancy, if it be fully confirmed, will be of profound theoretical significance.

The constancy of this normal intensity is little, if any, affected by

1. The method of production within the arc, whether, e. g., salt or metal is introduced into a carbon arc, or electrodes of the pure or alloyed metal are used.

2. The element used. Iron, cobalt, and nickel points lie slightly below the median line, gold above, but these differences are well within the possible error in measurement.

3. Spectral series may affect the constancy, but only within the present experimental error. Most of the normal lines belong to non-series spectra. The five prominent gold lines, in particular, give a smooth curve.

Wave-Length	6278	5838	5230	5064	4793
Intensity Nernst	0.12	0.18	0.41	0.52	0.89
Actual intensity	103	124	197	214	267

4. Twinning[1] is not to be distinguished from reversal as regards the intensity at which it occurs. In so far as this fact is admissible evidence of the similarity of the two phenomena, it appears to indicate that all such reversal of arc lines is rather a real doubling in its early stages. At any rate, phenomena due to absorption and phenomena due to a doubling of the period of the radiator both occur, and probably together.

5. At low pressure, lines double or reverse at a considerably lower intensity. The components of the doublet being each much narrower than that at atmospheric pressure, they appear to separate when their centers are a smaller distance apart.

6. The components of composite lines appear to be capable of doubling, reversal, or multiplying like single lines, although this point will require considerable further study. There is so much overlapping that effects are difficult to observe except when components are well separated. Possible exceptions are the extremely narrow satellites like those of the green mercury line. These satellites broaden perceptibly when very intense, but show no trace of doubling or reversal.

The development of a typical normal line is then somewhat as depicted in Fig. 5, where frequency (or wave-length) is plotted against intensity. At high pressures (or temperatures), say in an open-air arc, the development of the line is shown by the dotted lines. The intercept *ab* on the fre-

[1] See "Line Structure, II," *l. c.*

quency axis represents the minimum width or impurity of the spectrum line at a very low intensity. With increase of intensity, the line broadens and finally separates into two as the intensity corresponding to the point c is reached. With further increase in intensity the two components continually broaden and separate. At low pressure the minimum width is less and the line twins at a lower intensity.

There occur to me three possible explanations of this behavior of normal lines:

1. It may be an extreme type of reversal by absorption.

2. It may be an effect of the gas motion of the radiator on its elasticity, and hence on its frequency as discussed in "Line Structure, II."

3. A selective Doppler effect might give a doubling at certain intensities. Stark[1] finds that the canal rays give *displaced* rather than *broadened* spectrum lines, and hence concludes that only the swiftest particles emit light. Applied to arc spectra, this deduction would lead one to expect doubling.

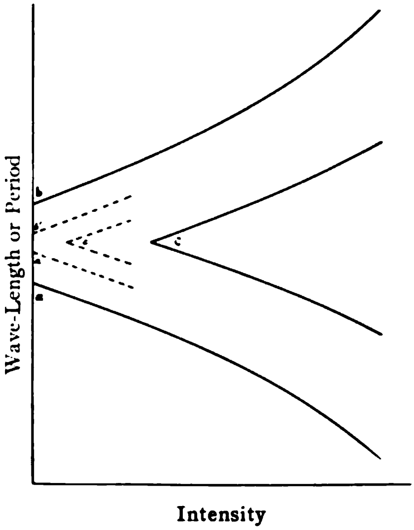


FIG. 5.—Wave-Length-Intensity.

There appears to be no conclusive evidence as yet for or against any of these three theories. It seems improbable that the doubling is an early stage of reversal proper on account of the extreme sharpness and brightness of its components. Dr. Humphreys unhesitatingly pronounced against reversal on viewing some of the twin iron lines through the echelon.

Assuming the Doppler effect, variability of speed and orientation are the two causes that would determine the character and behavior of single lines. Assuming the Maxwell-Boltzmann distribution for these and radiation independent of translatory motions of the radi

[1] *Physikalische Zeitschrift*, 7, 251–256, April 15, 1906.

ators, a spectrum line would have as definite an energy-spectrum as a black body; observation, however, shows that lines possess an individuality indicating two or more parameters in the energy-period function.

If the character of the radiation is modified by the effect of rotation on the elasticity of the radiator; speed and orientation would be operative in broadening components as well as the original single line; components of a double line would be as broad or broader than the minimum width at small intensity (cf. Fig. 5). Distance between centers of components would be proportional to the speed of rotation, which is assumed proportional to the speed of translation. Orientation of translation would be without effect on the doubling of lines. These effects are in accord with observation.

Stark's theory that radiation occurs at the higher speeds of the radiator applied to the arc would eliminate part of the effect of speed of translation, leaving orientation fully operative—in direct contrast to the preceding. Minimum width would be zero. At all pressures width would be much more than proportional to intensity; doubling would occur only if axial radiation were relatively small, and even then would be diffuse. Still, this theory is attractive on account of its simplicity.

SUMMARY

1. Spectrum lines fall into two quite distinct classes, "normal" and "composite," only the latter showing permanent structure.

2. The spectral width of normal lines is approximately a linear function of their intensity.

3. The minimum width of normal lines at atmospheric pressure is approximately 0.04 t.-m., and is the same or nearly the same for all lines.

4. At low pressures the minimum width of normal arc lines was found to be about 0.005 t.-m. in the case of a few lines measured, with indications that all other normal lines were of the same minimum width at that pressure.

5. All normal arc lines double or reverse at the same (energy) intensity within a factor of two at most.

6. When doubling or reversal occurs, each component has approxi-

mately a width equal to the minimum width of the original line. In other words, normal lines twin when their width is increased to double their miminum width. After doubling, each component broadens very rapidly with increase in intensity, chiefly on its outer side.

7. Some, at least, of the components of the triple and other composite lines broaden and double with increase of intensity, like normal lines.

8. Three of the satellites of the green mercury line are mono-chromatic to within 1 part in at least 5,000,000 when produced at moderate intensity in an ordinary mercury lamp. That these satellites appear not to be subject to the Doppler effect is very remarkable.

For absolute length standards, wave-length standards and interferometry as well, the narrowest possible lines—i. e., the purest and most monochromatic sources—are desirable. Of secondary importance are the brightness and ease of production and isolation of such sources. At moderate intensity, sodium yellow is pure only to 1 part in about 700; mercury green, to 1 in 10,000; cadmium green, to 1 in 100,000; helium yellow, to 1 in 200,000, or to 1 in 15,000 if the satellite is included. Obviously such lines are ill adapted to serve as pure sources or absolute standards.

But these are all composite lines. There is an abundance of normal lines, easily produced in arcs, whose impurity at moderate intensity is less than 1 in 100,000 in open air, and 1 in 1,000,000 at low pressures. It would appear to be in the highest degree advisable if attention were confined to these normal lines when very pure sources are desired. Work with the Michelson or Fabry-Pérot interferometers is particularly liable to error on account of the ease with which impurities may be overlooked and ignored.

I had hoped to find normal lines varying widely in minimum width and intensity coefficient, in order that the narrowest and least variant might be selected as standards and monochromatic sources. That there is so little diversity means that greater experimental skill must be exercised in producing the source, but gives a wide range of available lines.

Bureau of Standards,
Washington, D. C.,
July 1906.

PLATE II

Li

Na

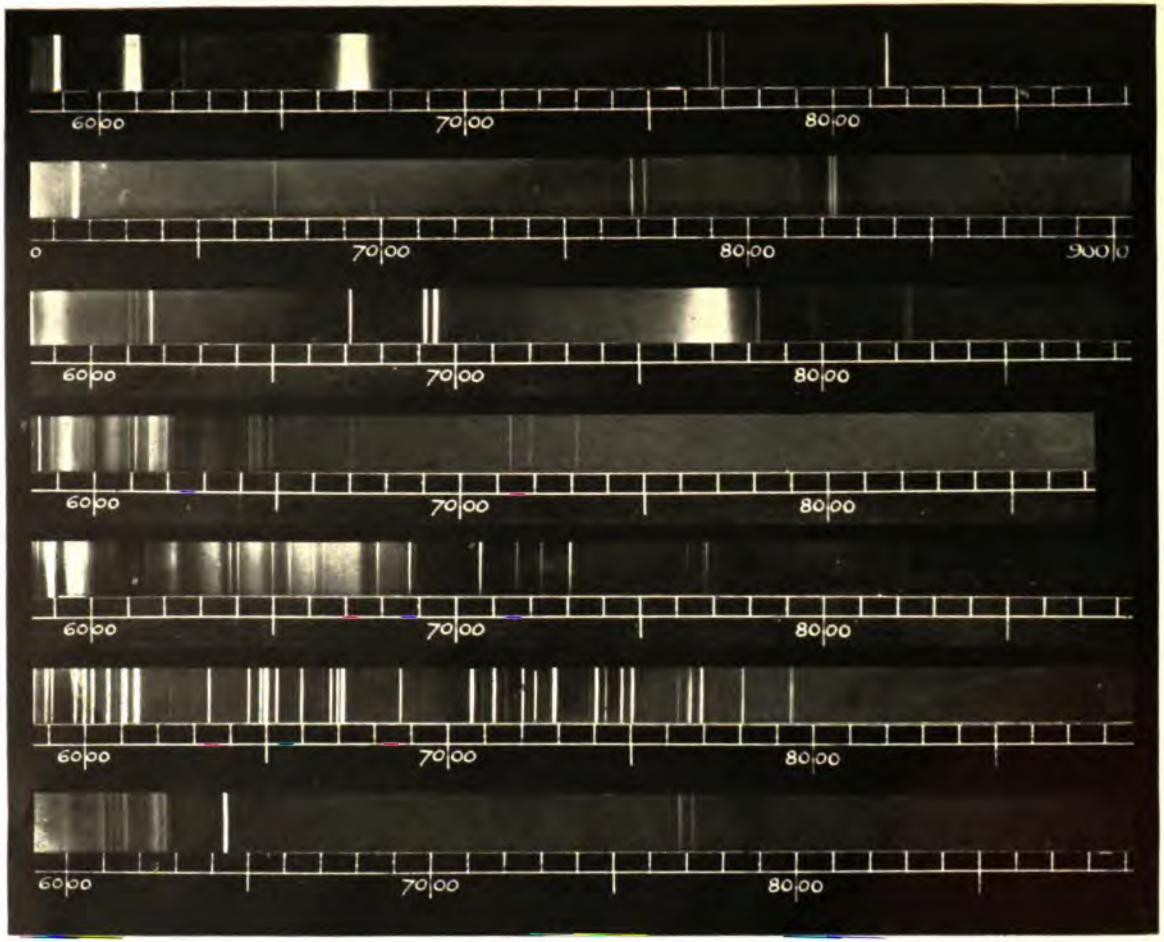
K

Ca

Sr

Ba

Cd



Photographs of Arc Spectra

PHOTOGRAPHS OF CERTAIN ARC SPECTRA FROM λ 5800 TO λ 8500

By LORD BLYTHSWOOD AND WALTER A. SCOBLE

In order to make a practical test of the diffraction gratings cut on the Blythswood dividing engine, photographs of a number of arc spectra were taken, using a concave grating of ten feet radius. At the same time it was desired to work so far as was conveniently possible into the ultra-violet and infra-red. As the total length photographed was about forty inches, two exposures were made, dyed films being used for the red end. Experiments were made to determine the relative values of various dyes for this purpose, alizarine blue *S* being found the best and pinacyanol next. When an attempt was made to dye the films with alizarine blue for the spectrum photographs, using Mr. Higgs' formula, they were found to be very capricious, only about one film in four being satisfactory. The same result was noticed by General Waterhouse, who discovered the dye. It was therefore decided to use pinacyanol. Further experiments have since indicated a new method with alizarine blue which appears to be quite certain, as a large number of exposures have been made without a single failure. The pinacyanol films have photographed to λ 7500 and alizarine blue films to λ 8750. The photographs given here were taken to extend the range beyond that of the original pinacyanol films.

Since the lines in the added region are comparatively few in number, a concave grating of 37.6 inches radius was employed to produce the spectrum. The slit was placed at the center of curvature of the grating, and a film-holder, cut with a radius half that of the grating, was fixed in a suitable position on the circle having the line from slit to grating as diameter. The arc was usually produced by feeding the element examined, or one of its salts, into a carbon arc, the carbons being held in an ordinary hand-regulated lamp with a side adjustment. The carbons were of the usual commercial kind, which were found to compare favorably with those specially prepared. The

[1] *Photographic Journal*, May 1906, "The Red Sensitiveness of Dyed Films."

arc was on a 100-volt circuit with a resistance in series, no higher voltage being available. An image of the arc was formed on the slit. To screen out the second-order violet and ultra-violet a glass cell containing a solution of potassium bichromate was placed in front of the slit. This arrangement had the advantage that the definition is unimpaired, as the screen was not between the slit and film. The strength of the solution was adjusted by using a wide slit and an arc strong in violet light. The grating was viewed directly and the strength of the solution increased until all the violet light was cut out. A test exposure was made to confirm the efficiency of the screening and the final photographs justified it.

The films used were Wellington orthochromatic, dyed as follows: To 100 cc alcohol add 2 cc strong ammonia and dissolve 0.2 gram alizarine blue *S*. Add 100 cc distilled water and filter. The bath was used immediately after it was prepared. The films were washed for two minutes in a one per cent. solution of ammonia and dyed for two minutes, then washed for an equal time in running water. The films only keep for about a day after dyeing, consequently they were exposed wet, clamped to the film-holder. To obtain good results the staining bath should be green, regularly obtained as above. The alcohol apparently causes the film to take the dye.

The films were developed with hydroquinone applied by means of a brush, thus allowing a considerable amount of control. The usual exposure was about one hour. A fact which caused the development to be rather difficult was that the density of the developed image could not be judged by the surface. When the surface is faint it frequently happens that the true image, as seen by the faint transmitted light obtainable in the dark room, is very dense. Sometimes when nothing is visible on the surface, or even by transmitted light, after fixing a considerable image is seen. This effect is not serious if the exposure has been sufficient, but is liable to cause dense parts to be overdeveloped. A deep green light was employed in the dark room.

A scale was drawn and prints from it placed against the photographs. Under these conditions the scale serves to indicate the positions of the lines approximately, but errors are sure to occur. In cases in which lines of known wave-length appear in the infra-red,

PLATE III

Cr

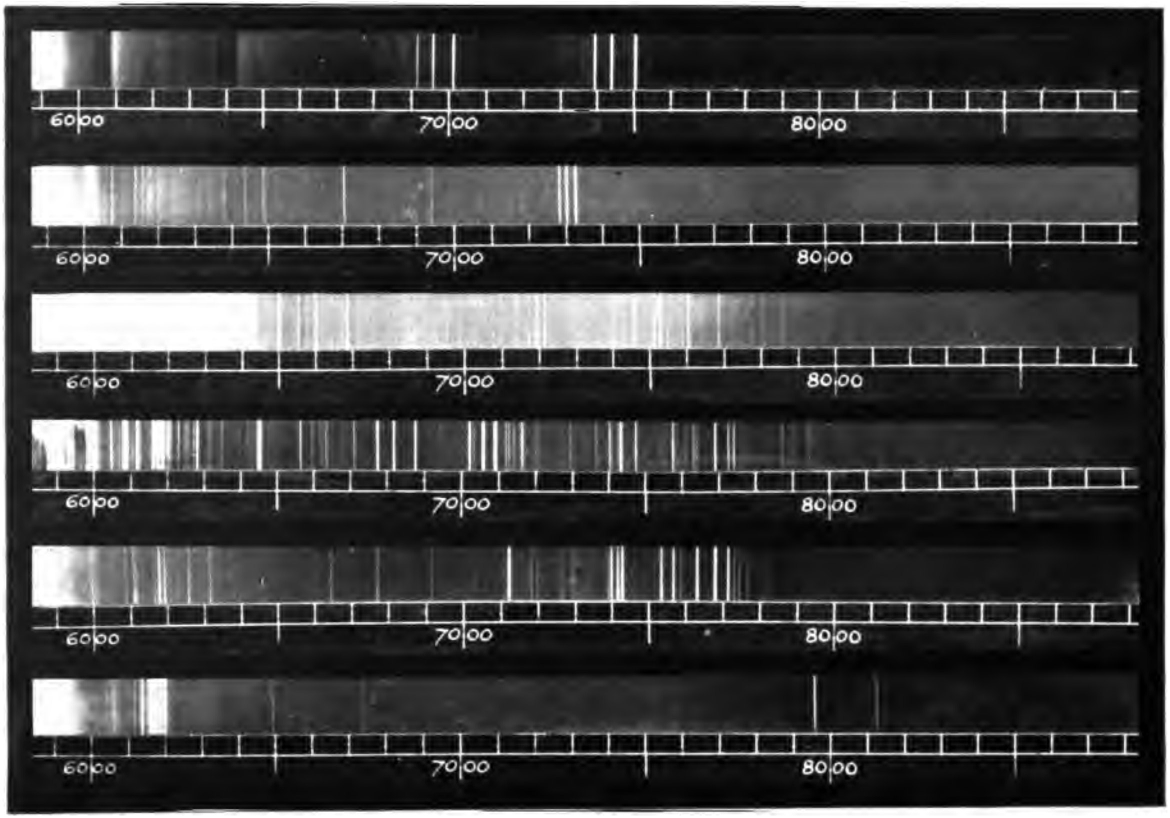
Mn

Fe

Co

Ni

Cu



Photographs of Arc Spectra

corrections can be applied. The following treats of the spectra separately:

Lithium.—Material: lithium chloride in carbon arc; impurities: sodium and potassium .

Sodium.—Material: sodium carbonate in carbon arc; impurities: lithium and potassium. Second-order potassium lines appear faintly at λ 8089 and 8095. There is a double line at λ 9000.

Potassium.—Material: potassium carbonate in carbon arc; impurities: sodium, lithium, calcium, and silicon.

Calcium.—Material: calcium chloride in carbon arc; impurities: sodium and lithium.

Strontium.—Material: strontium chloride in carbon arc; impurity: sodium.

Barium.—Material: barium chloride in carbon arc; impurity: sodium.

Cadmium.—Material: cadmium bromide in carbon arc; impurities: potassium and sodium.

Chromium.—Material: chromium metal in carbon arc.

Manganese.—Material: manganese chloride in carbon arc; impurities: sodium and lithium.

Iron.—Material: arc between metal rods. Faint lines appear at λ 8030, 8085, 8120, 8265, 8385, 8440, on the scale.

Cobalt.—Material: cobalt chloride in carbon arc; impurity: sodium.

Nickel.—Material: nickel chloride in carbon arc; impurity: sodium.

Copper.—Material: cupric chloride in carbon arc; impurity: sodium.

BLYTHSWOOD LABORATORY,
Renfrew, N. B.

Minor Contributions and Notes

THE MIDNIGHT ILLUMINATION ABOVE THE NORTHERN HORIZON NEAR THE TIME OF THE SUMMER SOLSTICE

This summer I have made observations of the illumination under the pole at midnight, near the summer solstice. This phenomenon was seen by Professor Newcomb from high altitudes in the mountains of Switzerland in the summer of 1905.[1] There were but few nights on which it could be well seen here this summer. It was satisfactorily observed, however, in particular, on the night of June 22. Careful observations show that the light is far more extensive than my casual recollections of it would imply[2]; and, unless the generally accepted ideas of the extent of our atmosphere, or the effect of the Sun in illuminating it at great altitudes, are entirely wrong, the illumination must be a manifestation of the zodiacal light, as suggested by Professor Newcomb.

1906, June 12. At midnight there was a distinct glow under the pole which reached 8° above the horizon and beyond. This glow was very diffused and extended for many degrees to the west, and to the Milky Way in the east. Near the horizon elsewhere the sky was dark.

Following are the observations made on the night of June 22 of this year.

$11^h\, 0^m$. The twilight glow is noticeable to the left of the north point.

$11^h 30^m$. It is easily seen as high as one-third the distance to *Polaris*, and is as conspicuous as the Milky Way between *Cassiopeia* and *α Cygni*. The limit of its visibility to the west would be on a vertical through *β Ursae Majoris*. The eastern limit is lost in the Milky Way. The light is conspicuous. It is a soft pale glow. In the past half-hour its motion to the right has been noticeable. It can easily be traced half-way to *Polaris*, and by averted vision diffuses even farther than that. The light is very soft and diffused with no definite limits. The sky elsewhere, which is unusually clear after several days of cloud and rain, is dark.

$12^h\, 0^m$. The glow is so decidedly visible that one would not fail to notice it if looking toward the northern sky.

$13^h\, 0^m$. The brightness has now passed to the right of the north

[1] *Astrophysical Journal*, **22**, 209, 1905.

[2] *Ibid.*, **23**, 168, 1906.

point. The glow diffuses nearly to *Polaris*. It is easily seen half-way to *Polaris*, and gradually fades farther than this. To one-third the distance to *Polaris* it is very noticeable.

$13^h 40^m$. It is now away to the right of the north point; *Capella* is in a strong dawn, which extends 10° or 20° above and to the left of that star. The western limit would have the same azimuth as α *Ursae Majoris*. I have watched this illumination at frequent intervals tonight, and have seen it pass under the pole and move to the east. The sky is fine except near the horizon.

$14^h 0^m$. The sky is now getting bright everywhere in the region of *Capella*, and the glow is widening and rising—true dawn is approaching. On this night both Mr. Barrett and Mr. Sullivan, who were working in the 40-inch dome, confirmed the observations.

June 28. The illumination passing under the pole, was visible all night.

July 17, $11^h 30^m$. The illumination under the pole well seen to one-third the distance to the pole. At 12, haze all over the north prevented observations of the illumination.

It would seem, therefore, if this is the zodiacal light, that it extends at least 65° north and south of the Sun (assuming the southern extent to be the same as the northern), which is a considerably greater extent than that given it by Professor Newcomb.

E. E. Barnard

Yerkes Observatory,
August 2, 1906

LATITUDE AND LONGITUDE OF THE SOLAR OBSERVATORY[1]

Through the courtesy of Superintendent O. H. Tittmann, of the United States Coast and Geodetic Survey, the astronomical latitude and longitude of Mount Wilson were determined in December 1905 and January 1906 by Assistants Edwin Smith and John E. McGrath, of the Survey. I am indebted to Superintendent Tittmann for the following information regarding the observations.

The latitude observations were made on three nights—December 24 27, and 29, 1905. Forty-six observations were taken on sixteen pairs. The observations were made with meridian telescope No. 3, having a focal length of 80 cm and a clear aperture of 7 cm. The value of one 2-millimeter division of its latitude level is 1″.186, and the value of one turn of the eyepiece micrometer, as determined from the latitude observations at this

[1] *Contributions from the Solar Observatory*, No. 9.

station, is 65ʺ034. The probable error of a single observation, as derived from the residuals of the separate observations on each pair from the mean result for that pair, and, therefore, not including the errors of the star places, was found to be ±0ʺ36. The mean result from each pair is given in Table I. The star numbers, as given in this table, refer to the *Greenwich Ten-Year Catalogue* of 1880, with the exception of the two numbers which are inclosed in parentheses; these refer to the *Greenwich Ten-Year Catalogue* of 1890. The declinations of nineteen of the stars observed depend upon computations of mean declinations, based upon many catalogues, made at the survey office on the Boss system in former years in connection with other latitude observations. For the remaining thirteen stars, for which sufficiently accurate declinations were not available at the Survey Office, Professor Lewis Boss kindly furnished the mean declinations from the manuscript of his *Preliminary General Catalogue*, now being prepared under a grant from the Carnegie Institution.

TABLE I

RECAPITULATION BY PAIRS OF RESULTS FOR LATITUDE (MOUNT WILSON, CALIFORNIA, 1905)

Pairs. No.	Star Nos. Gr. 10-Year 1880		Latitude	No. Obs.	W	V
1	179	191	34° 12′ 55ʺ10	3	15	−0ʺ03
2	234	(557)	55.01	3	11	+0.06
3	260	269	55.72	3	18	−0.65
4	282	321	54.83	3	16	+0.24
5	327	340	54.97	3	15	+0.10
6	362	389	54.57	3	9	+0.50
7	401	414	55.23	3	18	−0.16
8	(935)	431	55.22	2	11	−0.15
9	465	475	55.09	3	16	−0.02
10	490	500	55.62	3	16	−0.55
11	520	551	54.98	3	17	+0.09
12	557	563	54.48	3	7	+0.59
13	604	612	54.90	2	11	+0.17
14	621	662	54.57	3	13	+0.50
15	679	700	54.64	3	13	+0.43
16	719	735	55.34	3	16	−0.27

The computation of the latitude was made in accordance with Appendix 7 of the *Coast and Geodetic Survey Report* for 1898, and the weights were assigned to separate pairs by the method indicated at the bottom of page 362 of this Appendix. The weighted mean value for the latitude of the station of observation is 34° 12′ 55ʺ07±0ʺ06. The reduction to sea-level to take account of the curvature of the vertical is 0ʺ28, making the reduced latitude of the point of observation 34° 12′ 54ʺ79.

The observations to determine the difference of longitude of Los Angeles and Mount Wilson were made on six nights: December 31, 1905; January 2, 3, 4, 7, 8, 1906, by Assistant John E. McGrath at Los Angeles with transit No. 19, and Assistant Edwin Smith at Mount Wilson with transit No. 18. A description of these instruments is given on page 268 of Appendix 7 of the *Report* for 1898, with an illustration opposite page 320. Each instrument has been fitted with a transit micrometer of the type described in Appendix 8 of the *Report* for 1904, "A Test of the Transit Micrometer."

The difference of longitude Los Angeles—Mount Wilson is the first one wholly within the United States determined by using transit micrometers. Earlier in the season the transit micrometers were used in observations to determine three differences of longitude fixing three points in Alaska. The observations on each night at Los Angeles and Mount Wilson consist of two time sets, each involving ordinarily twelve stars; one time set being taken before the exchange of signals between stations by telegraphy, and the other afterward. The observed differences of longitude on each night are shown in Table II, together with the residuals, the mean and its probable error.

TABLE II

DETERMINATION OF DIFFERENCE OF LONGITUDE OF TRANSIT INSTRUMENTS AT LOS ANGELES AND MOUNT WILSON, CALIFORNIA

Date	Difference of Longitude	Residual
1905, Dec. 31	$0^m\ 47^s.580$	$-0^s.009$
1906, Jan. 2	.601	$-$ .030
3	.561	+ .010
4	.608	$-$ .037
7	.532	+ .039
8	.544	+ .027
	Mean = $+0^m\ 47^s.571 \pm 0^s.008$	

The longitude of the transit of 1892 at Los Angeles is $7^h\ 53^m\ 01^s.561$, fixed by the adjustment of the longitude net of the United States, see Appendix 2 of the *Report* for 1897. The transit used by Assistant John E. McGrath was $0^s.140$ west of the transit of 1892, and its longitude was, therefore, $7^h\ 53^m\ 01^s.701$. Applying to this the measured difference of longitude, as given in Table II—namely, Mount Wilson transit east of Los Angeles transit $47^s.571$—there is obtained as the longitude of the transit at Mount Wilson $7^h\ 52^m\ 14^s.130 = 118°\ 3'\ 31''.95$.

The transmission time, as determined on different nights, varied from 0ˢ.003 to 0ˢ.010, with a mean of 0ˢ.007.

The local triangulation at Mount Wilson, done by Assistant Edwin Smith, shows that the Mount Wilson triangulation station is 4″.93 north and 13″.59 west of the Mount Wilson longitude station, and that the Snow Telescope Pier is 4″.74 north and 2″.94 west of the Mount Wilson longitude station. Hence the astronomical position transferred to each of these points is as follows:

MOUNT WILSON TRIANGULATION STATION

Latitude	34°	12′	59″.72
Longitude	118	3	45.54

SNOW TELESCOPE PIER

Latitude	34°	12′	59″.53
Longitude	118	3	34.89

By comparing the astronomical latitude and longitude, as given in this report, with the geodetic position of the Mount Wilson triangulation station on the United States Standard Datum, as printed on page 541 of Appendix 9 of the *Report* for 1904, "Triangulation in California," it is found that the astronomical latitude is 26″.50 less than the geodetic, and the astronomical longitude is 5″.63 greater than the geodetic. The deflection in the prime vertical at the Mount Wilson longitude station is therefore moderate in magnitude, but the deflection in the meridian is very large; one of the largest, if not the largest, yet observed in the United States.

I take pleasure in extending the hearty thanks of the Solar Observatory to Superintendent Tittmann, and to Messrs. Smith and McGrath, for their kindness in undertaking this work, which was carried out with the precision and dispatch characteristic of the Coast and Geodetic Survey

GEORGE E. HALE.

MOUNT WILSON, CALIFORNIA,
June 1906.

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXIV OCTOBER 1906 NUMBER 3

THE ORIGIN OF COLOR. I

By E. C. C. BALY and A. W. STEWART[1]

In the previous paper it was shown how the presence of two carbonyl groups in juxtaposition gives rise to the appearance of an absorption band in the spectrum very much nearer to the red than that which is produced by the process known as enol-keto tautomerism,[2] or a reversible equilibrium such as

$$\begin{array}{ccc} -\underset{\underset{O}{\|}}{C}-CH_2- & \rightleftarrows & -\underset{\underset{OH}{|}}{C}=CH- \end{array}$$

In the case of pyruvic ester $CH_3CO\ COOEt$, in which it was first discovered, the absorption band is situated at a frequency of 3100, while the band due to enol-keto tautomerism is always very near to 3800. Now, in pyruvic ester there is only one true carbonyl group, for it is well known that the $-CO-$ group of a carboxyl radicle is not endowed with all the properties usually appertaining to this group. In order to investigate this process more fully, we have examined the absorption spectra of a series of compounds which contain two true carbonyl groups in juxtaposition, and we then found that the new absorption band is still nearer to the red than in the case of pyruvic ester. Camphorquinone $C_8H_{14}\left\langle\begin{array}{l} C=O \\ | \\ C=O \end{array}\right.$ was dealt with in the pre-

[1] Carnegie Research Fellow.

[2] Baly and Desch, *Astrophysical Journal*, 23, 110, 1906.

vious paper where its absorption curve was given; a very persistent absorption band is exhibited with its head at a frequency of 2100. The simplest compound of this type is of course diacetyl

$$\begin{array}{l} CH_3 \\ | \\ C=O \\ | \\ C=O \\ | \\ CH_3 \end{array}$$

and the absorption curve of this substance is shown in Fig. 1, where the new absorption band appears at a frequency of 2400. Since these two compounds, camphorquinone and diacetyl, show absorption bands in the visible blue ($\lambda = 4760$ and 4170 t.-m. respectively), they are naturally colored yellow, and clearly therefore the process which produces these absorption bands is the origin of color in the case of these compounds.

In the previous paper we showed that the origin of the new absorption band is to be found in some form of oscillation between the residual affinities of the oxygen atoms of the carbonyl groups, and for this oscillation we proposed the name *isorropesis*. Before dealing with the theoretical aspect, we may say that we have extended our observations to include many other compounds containing two carbonyl groups in juxtaposition, and in every case we find the new absorption band present. For example, the absorption curves of acenaphthenequinone $C_{10}H_8\left\langle\begin{array}{l} C=O \\ | \\ C=O \end{array}\right.$ and phenanthraquinone $C_{12}H_8\left\langle\begin{array}{l} C=O \\ | \\ C=O \end{array}\right.$ are shown in Fig. 2; isatin, whose spectrum has already been recorded by Hartley and Dobbie,[1] is another case in point and shows the same absorption band with head at a frequency of 2400. The formula of isatin $C_8H_4\left\langle\begin{array}{c} CO \\ NH \end{array}\right\rangle CO$ at once explains the appearance of the band, as there are present two carbonyl groups in juxtaposition.

[1] *Chem. Soc. Trans.*, **75**, 640, 1899.

Another very interesting case of an α-diketone is that of benzil, $C_6H_5.CO.CO.C_6H_5$, the absorption-curve of which is shown in Fig. 3. It is well known that the oscillating double linking of the

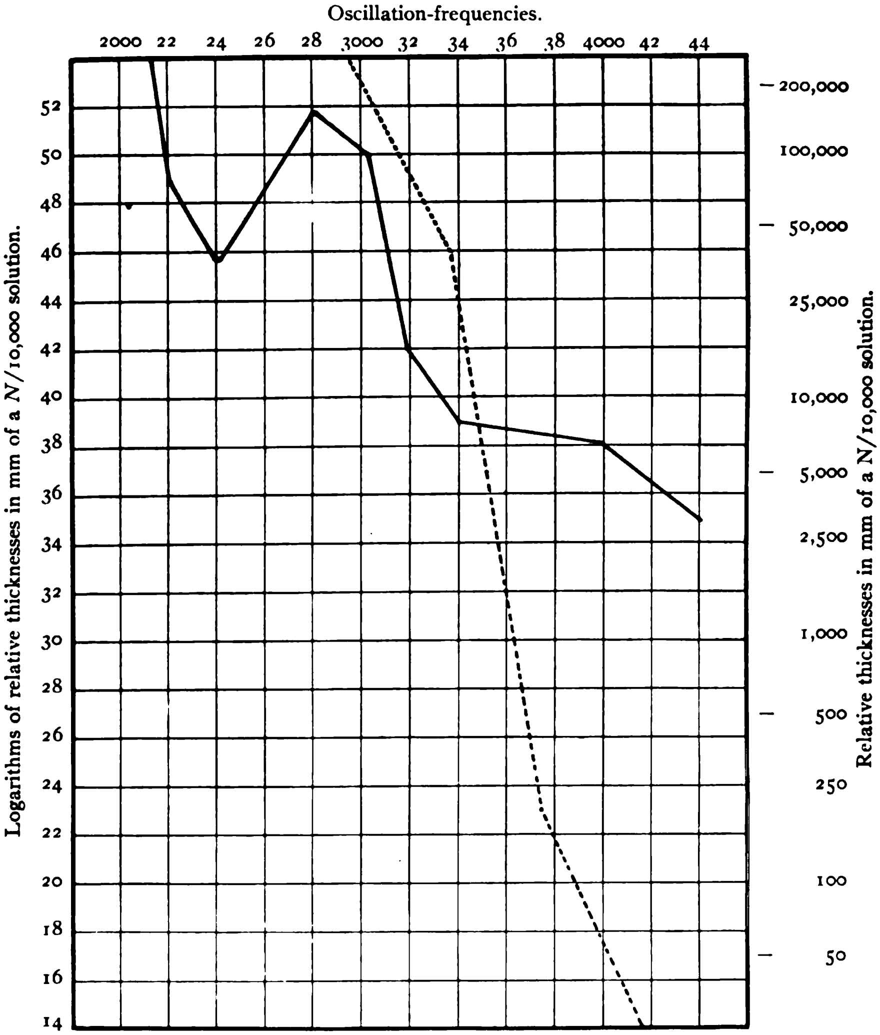


FIG. 1.—Diacetyl (full curve): Diacetyl dioxime (dotted curve).

benzene ring, or the benzenoid tautomerism, produces absorption bands which have about the same frequency as the absorption band due to enol-keto tautomerism. In the case of benzene itself there

are seven narrow absorption bands[1] with heads at frequencies of 3725, 3765, 3830, 3915, 4025, 4110, and 4200. These absorption

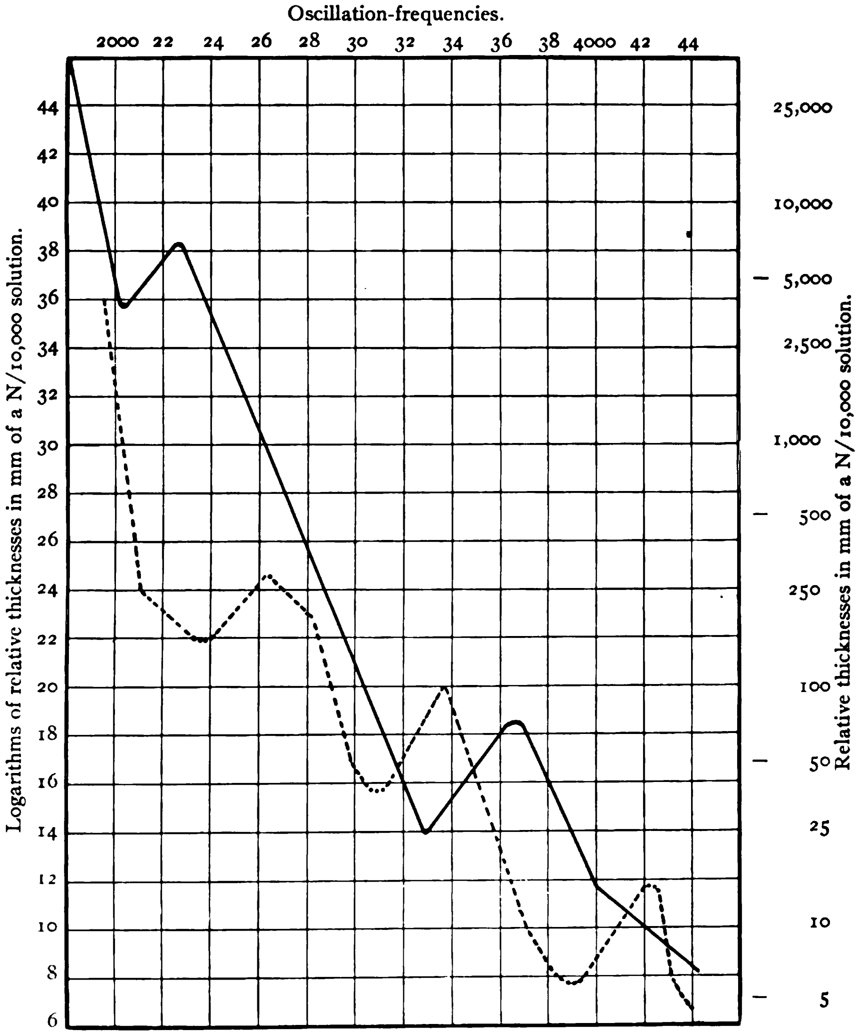


FIG. 2.—Acenaphthenequinone (full curve).
Phenanthraquinone (dotted curve).

bands are considerably modified by substituting different groups for the hydrogen atoms, especially if the substituent groups possess

[1] Baly and Collie, *Ibid.*, 87, 1332, 1905.

residual affinity. Thus acetophenone, $C_6H_5.CO.CH_3$, in which one hydrogen of benzene has been replaced by the acetyl group —$CO.CH_3$, shows an absorption very different from that of benzene.

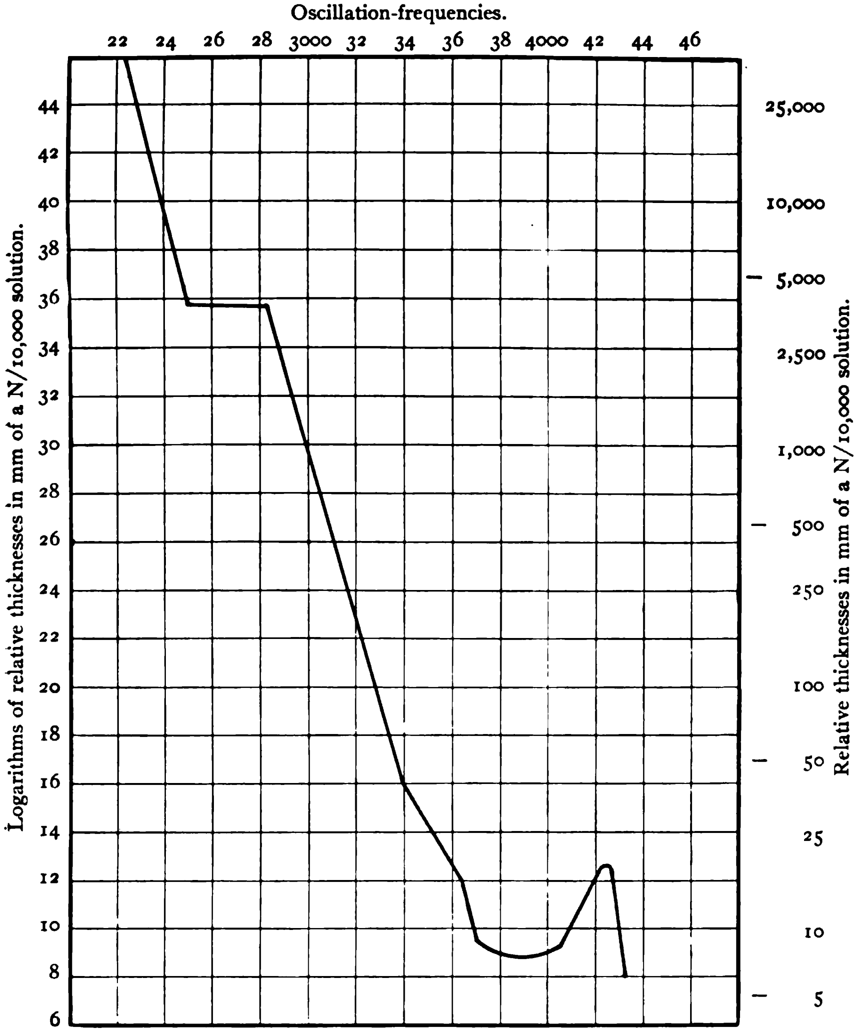


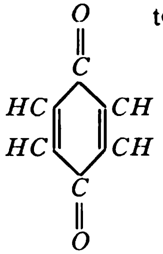
FIG. 3.—Benzil.

All the characteristic absorption due to the benzene ring has disappeared, owing no doubt to the fact that the residual affinity of the benzene ring has been fixed by the attraction between the car-

bonyl group and the atoms of the ring. This accounts for the fact that the carbonyl group of acetophenone is unusually inactive toward sodium bisulphite, etc., because the group does not readily pass into the nascent state necessary to the formation of additive compounds. It might be expected, therefore, that in benzil the residual affinities of the two carbonyl groups would each be occupied with and fixed by the adjacent benzene nucleus, and that therefore no isorropesis would occur. In reference, however, to the absorption-curve of this substance (Fig. 3) it will be seen that in the region of least concentration there is an absorption band with head at a frequency of 3900. The presence of this band argues that the benzenoid tautomerism is undoubtedly present to a small extent. For this reason we may conclude that the residual affinities of the carbonyl groups are not entirely fixed, and that a small amount of isorropesis between them is possible. It is evident that this conclusion is justified from an inspection of the upper portion of the absorption-curve of benzil, where a shallow band with head at a frequency of 2650 appears. The existence of this band shows that isorropesis is taking place, and its shallowness proves that it is present to a small extent only. It may be noticed that the yellow color of benzil is not very pronounced, and readily disappears in dilution. The measurements of the additive capacity of the benzil carbonyl groups made by Petrenko-Kritschenko agree very closely with the above observation.

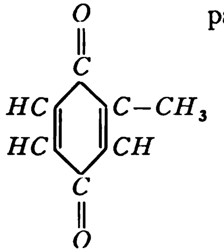
There is thus little doubt that the color of the compounds, diacetyl, camphorquinone, acenaphthenequinone, phenanthraquinone, isatin, and benzil is due to the two carbonyl groups in juxtaposition, since this configuration gives rise to a new type of oscillation or isorropesis between the residual affinities upon the two adjacent oxygen atoms. The most striking application of this principle is in the case of the true benzenoid quinones, for in these compounds, which are all strongly colored, we have a type of compound resembling an α-diketone, and in these compounds, too, the new absorption band is exhibited showing the undoubted existence of the process of isorropesis between the quinonoid oxygen atoms. Quinone itself was dealt with in the preceding paper and its absorption-curve there reproduced. It may, however, be again emphasized that all the chemical

evidence supports the view that the two para positions in the benzene ring are very close together, and that we should therefore expect the two $=CO$ groups of quinone

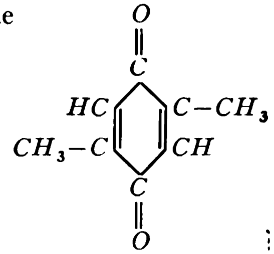


to have the same properties as those of an α-diketone. We have also examined the absorption spectra of

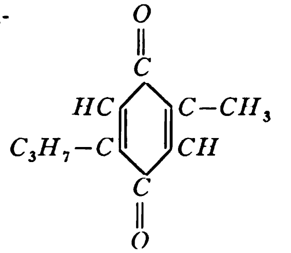
toluquinone



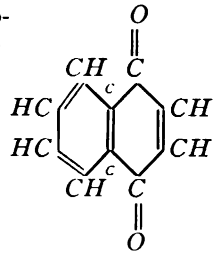
para xyloquinone



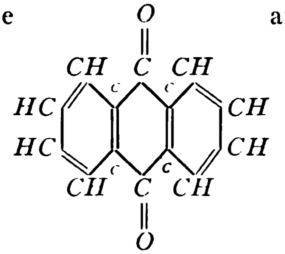
thymoquinone



α naphthoquinone

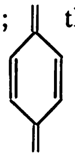


and anthraquinone



and find that the same absorption band is present in each case. The absorption-curves of the two first compounds are reproduced in Figs. 4 and 5; the remaining compounds show very similar curves. There is no

doubt therefore that the process of isorropesis exists in the quinones and is the origin of color of these compounds.

Now, Armstrong has developed a theory of color in which he attributes this property to the quinonoid linking;  the importance of these results in relation to his theory is manifest. They would seem to supply the key to his generalizations and at the same time to explain the colors of many substances which are difficult of interpretation by Armstrong's theory alone. Armstrong's theory gives no explanation of why color is produced by the quinonoid linking; there is no esoteric value in any of the linkings of the formula as light-absorbing centers. The results we have given, however, show that when the quinonoid configuration exists, isorropesis is set up between the residual affinities of the groups in the para position, with the result that an absorption band is developed in the visible region of the spectrum, producing a yellow or orange color.

In considering the whole question of color, there is no doubt that the new principle may be extended to include every case; that is to say, that isorropesis may occur between any adjacent atoms possessing residual affinity. It must be remembered, however, that in order for the new oscillation to take place, it is absolutely necessary for some exciting or disturbing influence to be present. For example, in the group $\underset{\underset{O}{\|}}{-C}\underset{\underset{O}{\|}}{-C}-$ of the α-diketones, each oxygen atom possesses a definite amount of residual affinity, and it is evident that no oscillation can arise between these atoms unless one or both residual affinities are disturbed. Now, in diacetyl, $CH_3-CO-CO-CH_3$, this influence is furnished by the hydrogen atoms of the methyl groups. There is an attraction between the hydrogen and oxygen atoms, with the result that the residual affinities on the latter tend to be altered. We have direct evidence of this potential enol-keto tautomerism in the absorption-curve of diacetyl (Fig. 1), for the curve shows a sudden extension at the ordinate 38. This extension undoubtedly means the incipient formation of an absorption

band which occupies the position of the band due to the tautomerism of a labile hydrogen atom.[1] Clearly therefore the residual affinities of the two oxygen atoms are being slightly disturbed, and it is owing

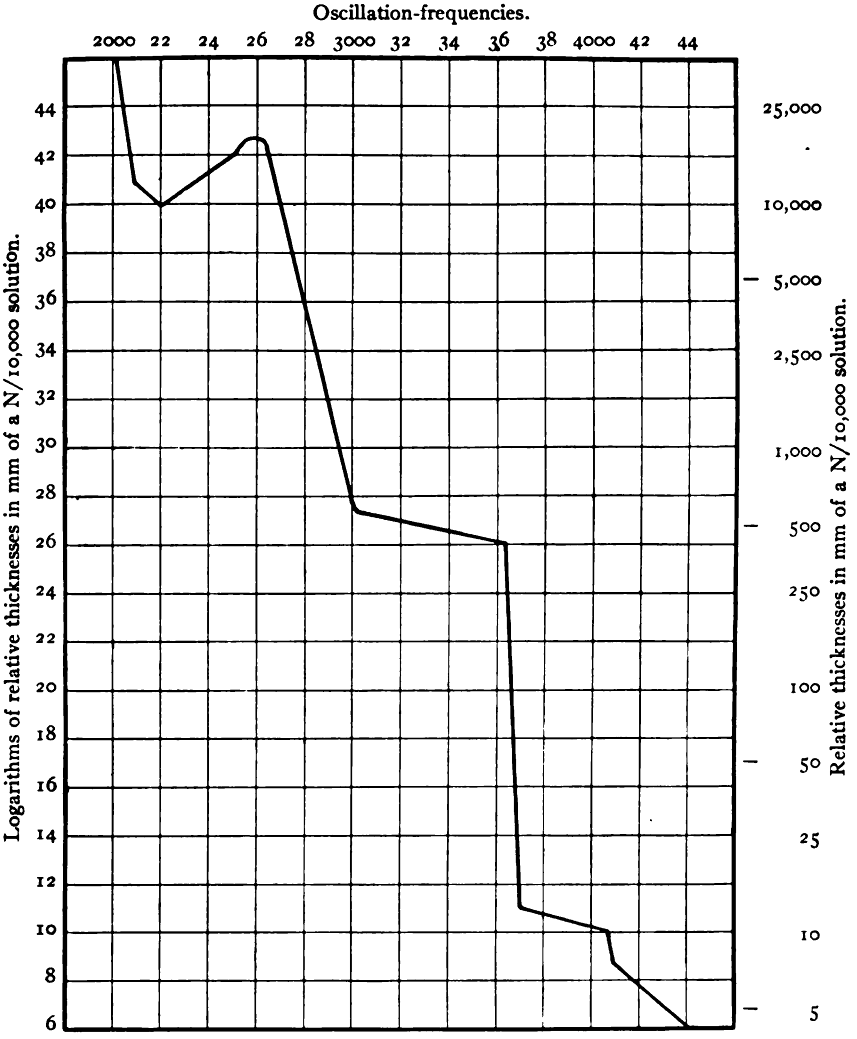


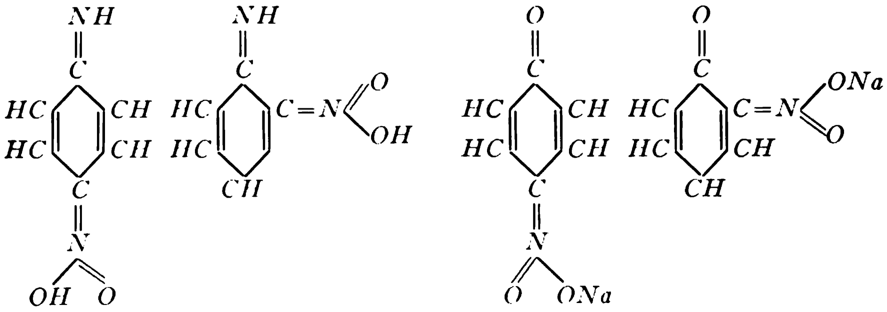
FIG. 4.—Toluquinone.

to this disturbance that the new oscillation or isorropesis takes place. We may now understand why diacetyl-dioxime is colorless

[1] Baly and Desch, *loc. cit.*

$CH_3-\underset{\underset{NOH}{\|}}{C}-\underset{\underset{NOH}{\|}}{C}-CH_3$, for in this compound we have apparently the condition for color, and yet only general absorption is exhibited. The residual affinity of the nitrogen atoms exerts no attraction on the hydrogen atoms of the methyl groups, and therefore is not disturbed in any way. No isorropesis therefore is set up and the compound is colorless. The absorption-curve of diacetyl-dioxime is shown in Fig. 1. It follows from this that all assumptions that two compounds must have essential differences in constitution if one is colored and the other white are untrustworthy.

It is evident that, if our generalizations upon color are correct—namely, that isorropesis occurs between the residual affinities of unsaturated atoms in juxtaposition, there is a large field for investigation among compounds of the quinonoid type in which the oxygen atoms of quinone are replaced by other unsaturated atoms. It should be noticed that in the quinones the necessary disturbing influence is provided by the tautomerism of the benzene ring, and that we are not entirely dependent upon the near presence of hydrogen atoms.[1] We have investigated the nitroanilines and the nitrophenols, in which unsaturated nitrogen atoms are present.[2] The color of the former and of the latter in alkaline solution has been accounted for by Armstrong on the assumption that they exist in the quinonoid form thus



para- ortho-
nitroaniline

para- ortho-
nitrophenol (sodium salt)

[1] Stewart and Baly, *Chem. Soc. Trans.*, **89**, 618, 1906.

[2] Baly, Edwards and Stewart, *Ibid.*, **89**, 514, 1906.

Considerable difficulty was met with by Armstrong in the case of the meta compounds, because it is impossible to imagine the static

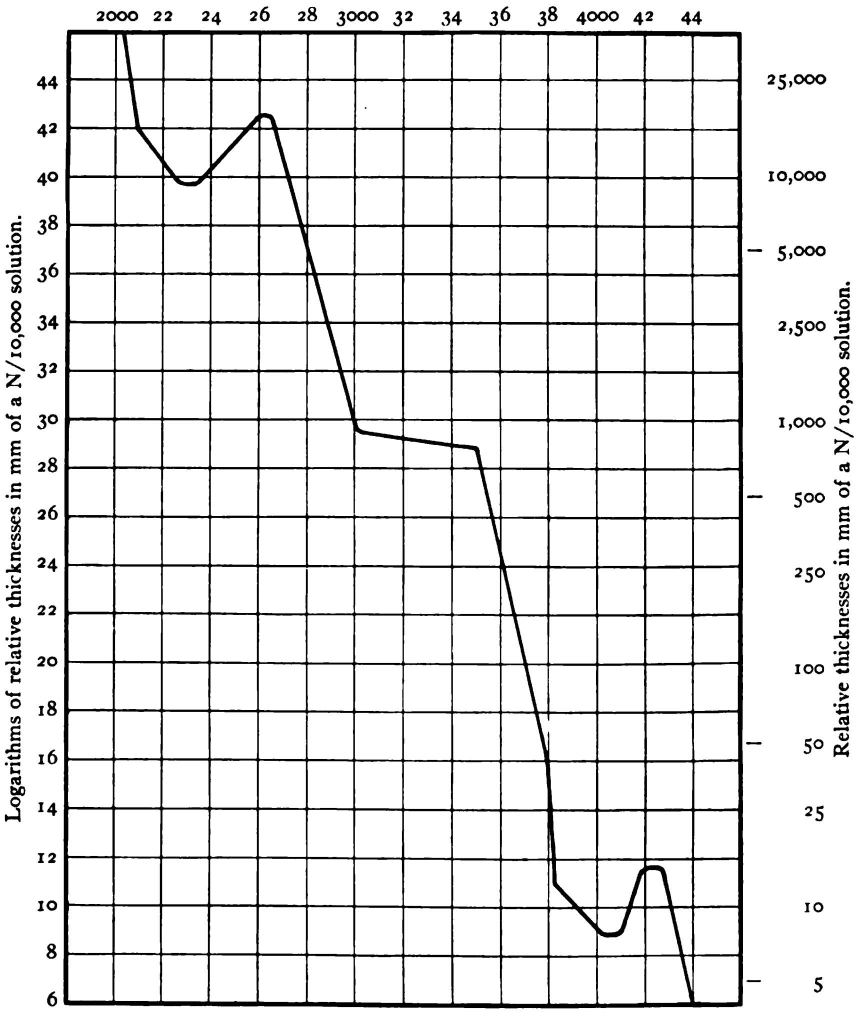
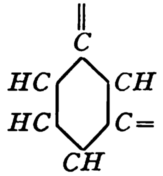
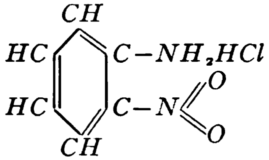


FIG. 5.—Para-xyloquinone.

existence of a linking of this type, as can readily be seen from the formula, there being no satisfactory way of linking the four remaining carbon atoms of the ring,



In Fig. 6 are reproduced the curves of meta- and paranitroaniline, and they show the presence of the absorption band due to isorropesis; this band has, however, much less persistence in the case. of the meta compound. The absorption of the ortho compound is practically identical with that of the para derivative. Now, these substances all give colorless solutions in the presence of strong hydrochloric acid; the absorption of these solutions is the same in each case and is exemplified by the curve in Fig. 7. There is no doubt from this that in the presence of hydrochloric acid the compounds possess the structure of the true hydrochloride, e. g.:



and that in neutral solution they possess the quinonoid form as given above. Isorropesis therefore occurs in these compounds between two unsaturated nitrogen atoms. Similarly, it has been proved that isorropesis occurs between the unsaturated nitrogen and oxygen atoms in the quinonoid forms of the nitrophenols. In these latter compounds, as in the case of the nitroanilines, the isorropesis is very much less in the meta than in the ortho and para isomers, showing that the metaquinonoid form undoubtedly exists, but only to a small extent. It is evident, therefore, that some restraining influence is at work against the formation of the meta-quinone. It was stated above that the static existence of a meta-quinone is impossible, but we have in these compounds undoubted evidence of the meta-quinone existing in proportionately small amounts; we may conclude, therefore, that this form has not a static but only a transitory existence. This may be explained as follows.

The space-formula proposed by Collie[1] has the advantage of representing the benzene molecule as a system of atoms in a state

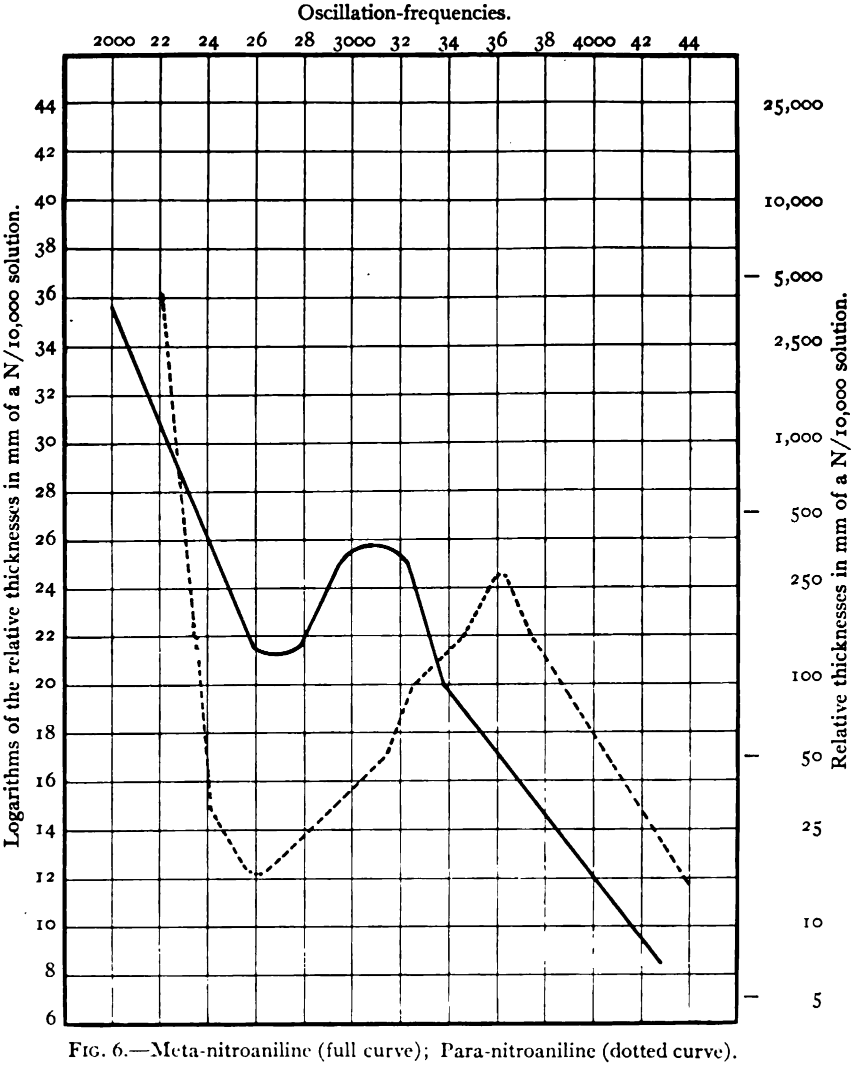


FIG. 6.—Meta-nitroaniline (full curve); Para-nitroaniline (dotted curve).

of continual vibration, and by this means it was possible to express all the various formulae which had then been put forward as phases

[1] *Chem. Soc. Trans.*, 71, 1013, 1897.

of one formula. We consider that this idea of a system in motion is extremely important, but at the same time it is evident that vibra-

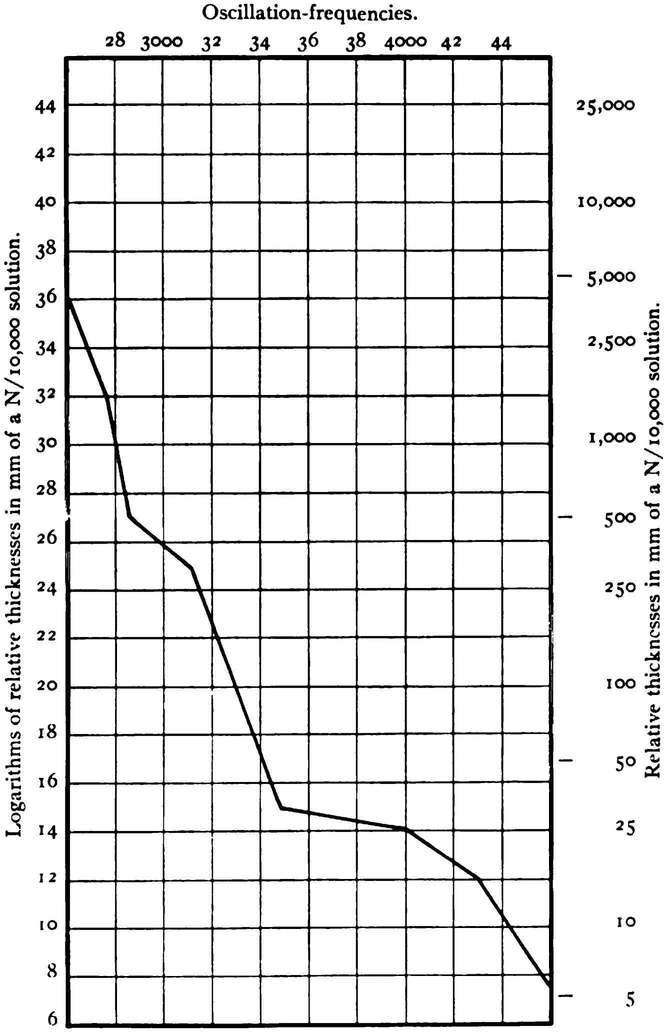
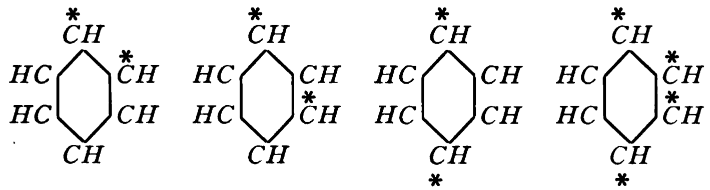


FIG. 7.—Ortho-nitroaniline in hydrochloric acid.

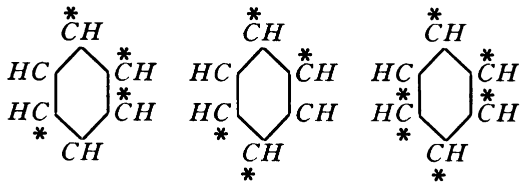
tions of the atoms not expressly described in Collie's original paper must be introduced in order to bring the theory into line with the spectroscopic and chemical evidence now at our disposal. It has been shown[1] that benzene gives seven very similar and closely situ-

[1] Baly and Collie, *loc. cit.*

ated absorption bands, and it was pointed out that the formation of these can be accounted for by assuming that each band is due to a separate make-and-break of linking between the carbon atoms of the ring. There are seven such makes and breaks possible, as can be seen from the following figures, the asterisks being attached to those atoms which are changing their linking:



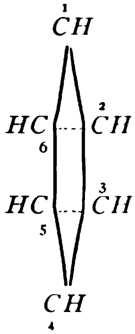
(1) (2) (3) (4)



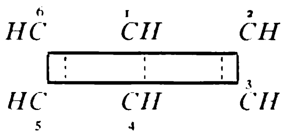
(5) (6) (7)

It will be seen that in case (2) a single meta-linking is being formed or broken; this throws some light on the possibility of the existence of meta-quinones.

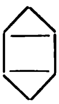
Now, in order to bring the seven phases into existence, it is necessary to assume the displacement of the carbon atoms of the ring, and we can do this in the simplest way possible—that is to say, by the ordinary vibration as is accepted by any elastic ring. Thus we may say that the benzene ring is pulsating between the two displaced forms *a* and *b*



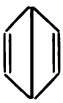
a



b



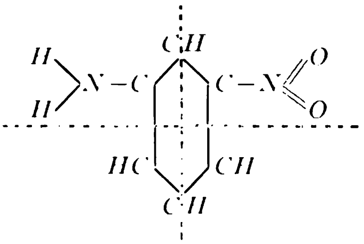
a′

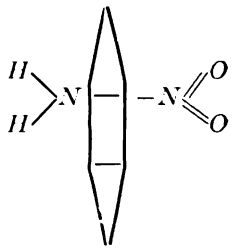


b′

Each carbon atom possesses residual affinity, and consequently in the condition represented in *a*, when the atoms 2 and 6 and the atoms 3 and 5 are brought close together, these residual affinities will produce linkings as shown by the dotted lines. The atoms 1 and 4, however, are far removed from one another and from the other atoms, and are therefore unsaturated. On the other hand, when the ring has passed into the other phase *b*, then the three atoms 2, 1, and 6 come very close to the three atoms 3, 4, and 5 respectively, and linking may be considered to be formed between these pairs of atoms. The linkings existing in phases *a* and *b* are shown for greater convenience on the ordinary hexagons in *a'* and *b'*. As the ring is pulsating between the forms *a* and *b*, many of the seven phases of linking-change described above will be obtained. For example, let us consider the ring to have reached the form *b;* as it starts opening, the first break will occur between the atoms 1 and 4, giving phase No. 3. This will be followed by the breaking of the two ortho-linkings 2:3 and 5:6, giving phases Nos. 3 and 6. When the ring passes through the half-way stage—that is, the circular form—then we shall have the centric formula, with the result that phase No. 7 is produced. We can in this way account for phases 1, 2, 3, 6, and 7; Nos. 4 and 5 can readily be understood if the motions described above are slightly interfered with by collisions between adjacent molecules. In the above it was assumed that the displacement takes place so that atoms 1 and 4 are at the ends of the ellipse in the form *a*, but in general the displacement will take place along any of the three possible axes.

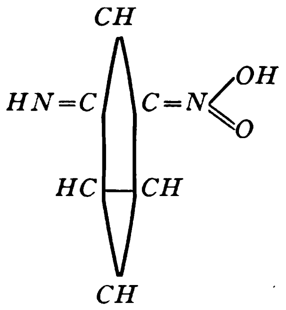
This scheme of displacement of the benzene ring renders it perfectly possible for meta-quinones to have a transitory existence. Let us take meta-nitroaniline,





a

and let the displacement take place along the dotted axes, when we shall obtain phase *a*. When in the form *a*, then the metaquinone can exist thus:



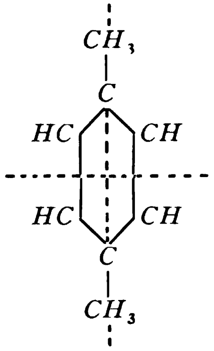
It must be remembered that this meta-quinone can exist only when the displacement occurs in the way shown. It is not therefore necessary to conceive the static existence of a metaquinone, but it is clearly possible for such a linking to exist during part of the motions of the ring.

It has been stated above that meta-nitroaniline and that meta-nitrophenol in alkaline solution exist only partly in the quinonoid form. Inasmuch as a special form of vibration is necessary in order that the meta-quinone may exist, we may say that in this fact is to be found the undoubted restraining influence against the formation of the meta-quinone referred to above.

This pulsation of the benzene ring explains very satisfactorily many of the characteristic physical and chemical properties of benzene and its derivatives. The explanation of the chemical properties need not be detailed here,[1] but one most striking result observed in the absorption spectra of disubstituted benzene derivatives[2] is readily accounted for. In these compounds the para isomer is always more symmetrical than the ortho and meta isomers; that is to say, the internal motions of the benzene ring are less disturbed by the para- than by either the ortho- or meta-substitution. This fact is clearly explained by the theory of the pulsating ring, because it is evident that in a compound such as para-xylene the vibration will take place very readily along the dotted axes shown in the figure

[1] *Cf.* Baly, Edwards, and Stewart, *loc. cit.*

[2] Baly and Ewbank, *Chem. Soc. Trans.*, **87**, 1355, 1905.



para-xylene

In the ortho-and meta-compounds the unsymmetrical loading of the ring will to a great extent militate against the vibration of the ring, with the result that the ring is distorted and the several absorption bands become confused.

Again, this theory of pulsation readily explains the well-known fact that the two para positions are very close together.

A strong point in favor of this hypothesis is its simplicity. The motion described is the simplest possible, and is the form of vibration adopted by any elastic ring—as, for example, a bell when struck.

These results leave no doubt that when two carbonyl groups are adjacent to one another in a molecule, a new free period of vibration is established; and, further, that when both the groups are true carbonyl as distinct from carboxyl carbonyl groups, the frequency of the new free period is situated in the visible region so that the substance is colored. In general, our results go to prove that the new free period or isorropesis is caused by the existence of the linking:

$$R_1-\underset{\underset{X_1}{\|}}{C}-\underset{\underset{X_2}{\|}}{C}-R_2$$

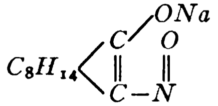
with the proviso that the residual affinity as expressed by the $-\underset{\underset{X}{\|}}{C}-$ group is disturbed by the influence of the groups R_1 and R_2. We have attempted to express the process of isorropesis chemically by stating that the new free period is connected with the equilibrium expressed by

$$\begin{array}{c} R_1-C-C-R_2 \\ \quad\| \quad \| \\ \quad X_1 \; X_2 \end{array} \rightleftarrows \begin{array}{c} R_1-C=C-R_2 \\ \quad | \qquad | \\ \quad X_1-X_2 \end{array}$$

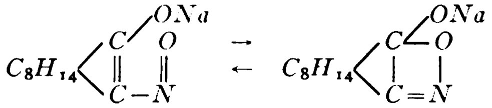
Quite recently[1] we have obtained some experimental evidence in favor of the static existence of the ring form *b*. It is well known that the substances known as the *iso*-nitroso compounds are yellow in alkaline solution. This color, as we have shown, in conjunction with Miss Marsden, is due to the isorropesis occurring with the form

$$\begin{array}{l} R_1 \\ | \\ C=O \\ | \\ C=NONa \\ | \\ R_2 \end{array}$$

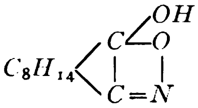
the necessary disturbing influence being provided by the hydrogen atoms upon the radicle R_1 as regards the $-CO-$ group and by the fact that the sodium atom is labile, in the case of the $\overset{|}{\underset{|}{C}}{=}N-$ grouping. Now, in the case of *iso*-nitroso-camphor, the stable form of this compound is yellow in alkaline solution owing to the isorropesis occurring with the form



This may be expressed chemically by the equilibrium



There is direct chemical and spectroscopic evidence that stable *iso*-nitrosocamphor in neutral solution has the form



[1] Baly, Marsden, and Stewart, *Chem. Soc. Trans.*, **89**, 1906.

so that our original method of chemically expressing isorropesis has found experimental verification in the case of *iso*-nitroso-camphor.

At the present time we have no physical explanation to offer of the new free period; there is, however, another way of looking at the phenomenon which perhaps may throw more light upon the process. Taking the simplest case of diacetyl

$$CH_3-CO-CO-CH_3$$

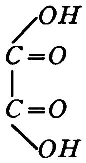
there are two CH_3-CO- groups in juxtaposition, and each one of these, by virtue of the potential enol-keto tautomerism they possess, causes or tends to cause the appearance of an absorption band in the ultra-violet. Inasmuch as these two groups mutually influence one another, it is possible that the two free periods in the ultra-violet may so far interfere or combine together to give a new free period in the visible region. On these grounds, therefore, we should look upon the acetyl group or any other group showing enol-keto tautomerism and the benzine nucleus showing benzenoid tautomerism as being potential color systems. The juxtaposition of two of these systems in certain definite ways gives rise to isorropesis or the combination of the two systems to give a new free period. In the compounds described above the new free period is situated in the visible region, so that the substances have visible color. It must be remembered that the conditions may occur in which the isorropic free period is not in the visible region; in this case the substance would not be colored. Such a condition occurs in both pyruvic ester and in *iso*-nitrosoacetic acid in alkaline solution.

$$\begin{array}{l} CH_3 \\ | \\ C=O \\ | \\ C=O \\ | \\ OC_2H_5 \end{array} \qquad\qquad \begin{array}{l} H \\ | \\ C=NONa \\ | \\ C=O \\ \diagdown ONa \end{array}$$

Pyruvic ester — *Iso*-nitrosoacetic acid (dissolved in sodium hydroxide)

In the case of both these compounds the frequency of the isorropic free period is about 3100, which is not in the visible region, and the substances are colorless. It appears that the presence of the hydroxyl oxygen next to the carbonyl group produces this effect, and experi-

ments are at present in progress with a view of explaining this influence. It is interesting to note that the frequency of 3100 obtained with the above two compounds is exactly half-way between the frequency of the isorropesis of diacetyl (2400) and of the enol-keto tautomerism absorption (3800), and further that no isorropesis occurs in oxalic acid where both the carbonyl groups form part of a carboxyl group:



Oxalic acid

It is very noteworthy that the wave-length of the free period of vibration established by isorropesis is about the same as that emitted by the simpler fluorescent organic substances ($\lambda = 4800 - \lambda = 4000$). It may be that there is an intimate connection between fluorescence and isorropesis, and that the former is only a manifestation of the latter. There is nothing inherently improbable in this idea. In both cases, visible color and fluorescence, the free period is established by the isorropesis; in the former case the free period is established by the isorropesis and excited by the incident light and we have absorption, while in the latter case the free period is established and excited by the isorropesis, and we have emission. An important fact bearing on the connection between isorropesis and fluorescence has been recorded by Nichols and Merritt;[1] these authors have observed that, when the fluorescence of fluorescein and certain other substances is excited by a beam of ultra-violet light, a distinct absorption occurs of light of the same wave-length as that emitted by the substance when fluorescent.

CONCLUSIONS

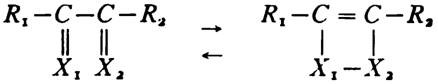
1. When two true ketonic groups $\left(\begin{array}{c}-C-\\ \| \\ O\end{array}\right)$ are in juxtaposition in the molecule, an oscillation (isorropesis) occurs between the residual affinities of the oxygen atoms, which results in the establishment

[1] *Phys. Rev.*, 18, 447, 1904.

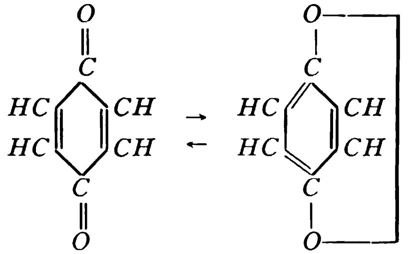
of a free period of vibration in the visible region of the spectrum. These substances are therefore colored.

2. This isorropesis occurs also between the residual affinities of the oxygen atoms in the benzenoid quinones, of the nitrogen atoms of the quinonoid form of the nitroanilines, and of the nitrogen and oxygen atoms of the quinonoid form of the nitrophenols. It also occurs between the residual affinities of the oxygen and nitrogen atoms of the *iso*-nitroso compounds.

3. The process of isorropesis may be expressed chemically by the equilibrium expressed by



or



4. It is necessary to assume the transitory existence of a meta-quinonoid linking to account for the phenomena observed with meta-nitroaniline and meta-nitrophenol.

5. Many of the physical and chemical properties of benzene are explained by considering that the benzene ring is elastic and undergoes the same vibrations as are suffered by any elastic ring.

6. The metá-quinonoid linking is possible during one phase of the displacement of the benzene ring.

7. In order to start the isorropesis, it is necessary that some influence be present to disturb the residual affinities upon the atoms concerned.

8. This influence is provided in compounds of the type of diacetyl by the neighboring hydrogen atoms which are attracted by the oxygen atoms; in the benzoquinones it is provided both by the hydrogen atoms and also by the benzenoid tautomerism.

9. Subject to the proviso referred to in 7, there is no doubt that this principle may be extended, and that all the phenomena of visible color are due to the oscillation between residual affinities on atoms or groups of atoms in juxtaposition.

10. Any assumption that two compounds must be fundamentally different in constitution if one is colored and the other white is quite untrustworthy.

11. It is possible that color and fluorescence are evidences of the same phenomenon—isorropesis. In the former case the isorropesis provides the mechanism, and the incident light actuates it; in the latter case the isorropesis both provides and actuates the mechanism.

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THE GEOMETRICAL THEORY OF OPTICAL IMAGERY

By JAMES P. C. SOUTHALL

INTRODUCTION

In the admirable preface of Professor S. P. Thompson's translation of Lummer's *Contributions to Photographic Optics* (Macmillan, 1900) he calls attention to the conspicuous superiority of the Germans in all matters relating to the theory and construction of optical instruments, and asserts that, in spite of the existence of some more or less excellent English works on optics—particularly Lord Rayleigh's article in the *Encyclopædia Britannica* and Heath's *Geometrical Optics*—there is nothing in the English language at all comparable, for example, with Dr. S. Czapski's *Theorie der optischen Instrumente nach Abbe* (the first edition of which was published at Breslau in 1893) or the volume on optics written by Dr. Otto Lummer for the ninth edition of Müller-Pouillet's *Lehrbuch der Physik* (Braunschweig, Friedrich Vieweg & Sohn, 1897). Professor Thompson attributes "the badness of almost all recent British textbooks of optics" to the fact that "the examination curse lies over them all;" but, however true this may be, it is only a partial explanation. The secret of the "badness" lies deeper than that. The simple truth of the matter is that neither the English nor any other people have cultivated this domain of mathematical and applied science in a degree at all approaching the methods and activity of the Germans, and in recent years at least all the remarkable developments both in the theory and manufacture of optical instruments are traceable to Germany. To anyone at all familiar with the mathematical investigations of Abbe and Seidel, and the scientific methods of construction employed by such firms as those of Carl Zeiss and Schott & Genossen in Jena, such treatises as Parkinson's *Optics* and Heath's *Geometrical Optics* appear to be more than half a century behind the times.

Of Dr. Czapski's book (of which a new and revised edition has been recently published in Leipzig) it is not too much to say that it has long been recognized as epoch-making; in it was set forth for

the first time a complete and masterly exposition of the remarkable theories of Professor Abbe, of Jena.

One of the particular merits of Abbe's methods consists in the fact that he discerned clearly (what other investigators,[1] including even Moebius and Gauss, appear to have perceived more or less dimly) that the physical agency or mechanism which was employed in the actual formation of an optical image was in no wise involved in the geometrical theory of optical imagery; so that, without any special assumptions whatever as to the construction and mechanism of the optical apparatus, and also without reference to the physical laws of reflection and refraction (which in some form or other always entered into the theories of the earlier investigators), he deduced all the laws concerning the relative positions and dimensions of the object and image, and showed for example, that the so-called "cardinal points," "focal lengths," etc., of the optical system were purely geometrical ideas and essentially independent of the *modus operandi* whereby these conceptions might be said to have a physical existence.

The function of an optical instrument is to produce an image of an external object. Each point of the object is the base (or vertex) of a bundle of rays, of which, in general, only a part is utilized in the formation of the image. These object-rays which are affected by the instrument are called the "incident" rays. Within the apparatus these rays undergo certain refractions (or reflections) at the plane or curved boundary surfaces of suitably disposed optical media; and, thus modified, they "emerge" into the last medium and form somewhere an image of the object, which may be "real" or "virtual," etc.; the nature of the image in the several respects of position, dimensions, and orientation depending primarily on the peculiarity and design of the instrument itself. Proceeding from any point P of the object, a bundle of incident rays "enters" the optical instrument, and, emerging therefrom, a portion of these

[1] In this connection see, however, a noteworthy paper, entitled "On the general Laws of Optical Instruments," contributed by J. Clerk Maxwell, in 1858, to the *Quarterly Journal of Pure and Applied Mathematics;* wherein the author expressly discards all assumptions as to the physical mode of producing an optical image, and constructs a purely geometrical theory. In the new edition of his book Czapski calls attention to this paper of Maxwell's, of which, however, Abbe was ignorant.

rays at least, if not all of them, will intersect ("really" or "virtually") in the corresponding (or "conjugate") point P' of the image. In the case of an *ideal* (or geometrically perfect) image, *all* of the emergent rays corresponding to the bundle of incident rays P will intersect in the image-point P'; so that a bundle of homocentric object-rays will be "imaged" (*abgebildet*) by a homocentric bundle of image-rays.

The fundamental and essential characteristic of optical imagery is, therefore, a point-to-point correspondence, by means of rectilinear rays, between object and image. From this one assumption —at once the most natural and obvious—Abbe deduces all the general laws of optical images. The advantage of this is that in investigating an actual image produced by an optical instrument it will be possible to separate what in the laws of this image depends on the general fundamental laws of optical imagery and what is due to the particular mode of producing the image. Moreover, although today a certain optical instrument may be a mechanical impossibility, we can determine by this theory whether or not it is theoretically practicable, and thus the theory indicates the way of future invention and discovery.

In the modern geometry this unique point-to-point correspondence by means of rectilinear rays between image and object is called "collineation"—a term introduced by Moebius in his *Der barycentrische Calcul* (Leipzig, 1827).

Two regions of space Σ and Σ' are said to be "collinear" with each other if to every point P of Σ there corresponds one (and only one) point P' of Σ', and to every straight line of Σ which goes through P there corresponds one straight line of Σ' which goes through P'.

In the theory of optics the two regions Σ and Σ' are designated as the "object-space" and "image-space," respectively. Since the relation between the two spaces is perfectly reciprocal, there is no essential difference between them; whence is deduced at once the principle known as "The Reversibility of the Light-Path."

A direct consequence of the unique point-to-point and ray-to-ray correspondence between object-space and image-space is plane-to-plane correspondence; so that to every plane ϵ in the object-space there corresponds also a definite plane ϵ' in the image-space,

and vice versa. This plane-to-plane correspondence, which is likewise characteristic of the collinear relation of two regions of space, is used by Czapski as the basis of his mathematical investigation. Employing the method of analytic geometry, and denoting the co-ordinates of any object-point P, with respect to an arbitrary system of rectangular axes in the object-space, by x, y, z, and the co-ordinates of the conjugate image-point P', also with respect to an arbitrary system of rectangular axes in the image-space, by x', y', z', he shows that the following equations, involving fifteen independent constants, are the analytical expression of the relation of collinear correspondence between the object-space and image-space:

$$\left.\begin{aligned} x' &= \frac{a_1x+b_1y+c_1z+d_1}{ax+by+cz+d}, \qquad y' = \frac{a_2x+b_2y+c_2z+d_2}{ax+by+cz+d}, \\ z' &= \frac{a_3x+b_3y+c_3z+d_3}{ax+by+cz+d}. \end{aligned}\right\} \qquad (A)$$

From these general image equations, involving, as we have said, no assumptions as to the apparatus itself, and no restrictions as to the position, form, or dimensions of the object, nor as to the angular aperture of the imaging bundles of rays, all the laws of optical imagery can be deduced.

The geometrical character of the imagery defined by these equations has been extensively investigated in treatises on modern geometry; but, so far as the present writer is aware, there is no book which, *using the methods of projective geometry, treats the theory of collinear correspondence in its special application to the laws of optical imagery.*

Inasmuch as these methods are not only interesting of themselves, but appear in certain respects to possess decided advantages, the writer has proposed to employ them in the following outline of Professor Abbe's theory.

PART I

COLLINEATION OF TWO PLANE FIELDS

ART. 1. PROJECTIVE RELATION OF TWO COLLINEAR PLANE FIELDS

For the purposes of this article (which does not pretend to completeness) it will be entirely sufficient and much simpler to restrict

our investigations to two conjugate planes of the object-space and image-space. The advantage of this is obvious, especially as both the methods which we shall use and the results which we shall obtain can be easily extended to include the whole of the two collinear regions of space.

A "plane field" is the name applied to the totality of points and lines contained in a plane.

Definition.—Two plane fields ϵ and ϵ' are said to be "collinear" if to every point P of ϵ there corresponds one point P' of ϵ,' and to every straight line p of ϵ which passes through P there corresponds a straight line p' of ϵ' which passes through P'.

Two collinear plane fields ϵ and ϵ' are also called "projective," because to each harmonic range of four points of ϵ there corresponds a harmonic range of four points of ϵ'.

Thus, if P, Q, R, S are a range of four harmonic points of the plane field ϵ', and P', Q', R', S' the four corresponding points of the collinear plane field ϵ', in the first place, since the points P, Q, R, S, are all situated on a straight line u, the points P', Q', R', S' must all likewise lie on a straight line u' which is conjugate to u. Let $ABCD$ be any quadrangle of the plane field ϵ, such that the two opposite sides, AB and CD, intersect in P, and the other two opposite sides, AD and BC, intersect in Q, while the fifth and sixth sides, BD and AC, go through the points R and S, respectively. To this quadrangle of ϵ there will correspond a certain quadrangle A', B', C', D' of ϵ', such that the two opposite sides $A'B'$ and $C'D'$, intersect in P', the other two opposite sides, $A'D'$ and $B'C'$, intersect in Q', and the fifth and sixth sides, $B'D'$ and $A'C'$, go through the points R' and S', respectively. Hence, the points P', Q', R', S' are also a harmonic range of points; and this is the condition that the two plane fields ϵ and ϵ' shall be *projective*.

ART. 2. THE SO-CALLED "FLUCHT" POINTS OF CONJUGATE RAYS

Let u and u' denote two conjugate rays of the collinear plane fields ϵ and ϵ'. Since the point-ranges u and u' are "projective" (as has just been shown), it follows that *the anharmonic ratio $(ABCD)$ of any four points A, B, C, D of u is equal to the anharmonic ratio $(A'B'C'D')$ of the four corresponding points A', B', C', D' of u'.*

That is,

$$\frac{AC}{BC} : \frac{AD}{BD} = \frac{A'C'}{B'C'} : \frac{A'D'}{B'D'} .$$

If the points A, B, C are supposed to be fixed while the point D travels along u, the anharmonic ratio $(ABCD)$ will vary in value; and if the point D moves away to an infinite distance until it coincides with the infinitely distant point E of u, we shall have:

$$(ABCE) = (A'B'C'E') = \frac{AC}{BC} ,$$

where E' denotes the point on u' which corresponds with the infinitely distant point of u. Since A,B,C are actual (or finite) points of u, no two of which are supposed to be coincident, the value of AC/BC is finite; and, hence, *the point E' is a determinate and, in general, a finite point of u'.*

Similarly, if F' is the infinitely distant point of u', *the point F, which corresponds to the infinitely distant point F' of u', is likewise a determinate, and, in general, a finite point of u.*

In general, therefore, the points F and E', corresponding to the infinitely distant points F' and E of u' and u, respectively, are finite (or actual) points having perfectly determinate positions on u and u', respectively. The German writers call F and E' the *flucht* points of the two projective ranges of points u and u'.

NOTE.—We are careful to say that the so-called "flucht" points are, "in general," "finite" (or actual) points; for in *one special case*, viz., when

$$(A'B'C'E') = \frac{AC}{BC} = \frac{A'C'}{B'C'} ,$$

the point E' will coincide with the infinitely distant point F' of u'; and *in this case the infinitely distant points E and F' of the projective point-ranges u and u' will also be conjugate points.*

ART. 3. THE "FOCAL LINES" OF CONJUGATE PLANES

In the plane field ϵ let us consider a quadrangle $ABCD$ (Fig. 1) such that the two pairs of opposite sides form two pairs of parallel straight lines. The two parallel sides, AB and CD, intersect in the infinitely distant point P; and, similarly, the other two parallel sides, AD and BC, intersect in the infinitely distant point Q. If therefore R and S designate the infinitely distant points of the remain-

ing sides, *BD* and *AC*, respectively, the four points *P*, *Q*, *R*, *S* are a harmonic range of points of the infinitely distant straight line *e* of the plane ϵ.

In the collinear plane field ϵ' the ray *A'B'*, conjugate to *AB*, will contain the point *P'* which corresponds to the infinitely distant point *P* of the ray *AB;* so that the point *P'* is the so-called "flucht" point of the ray *A'B'*. Obviously, the point *P'* is also the "flucht" point of the ray *C'D'*, conjugate to the ray *CD*. Precisely in the same way, the point *Q'*, conjugate to the infinitely distant point *Q* of the parallel rays *AD* and *BC*, is the common "flucht" point of each of the rays *A'D'* and *B'C'*, conjugate to the rays *AD* and

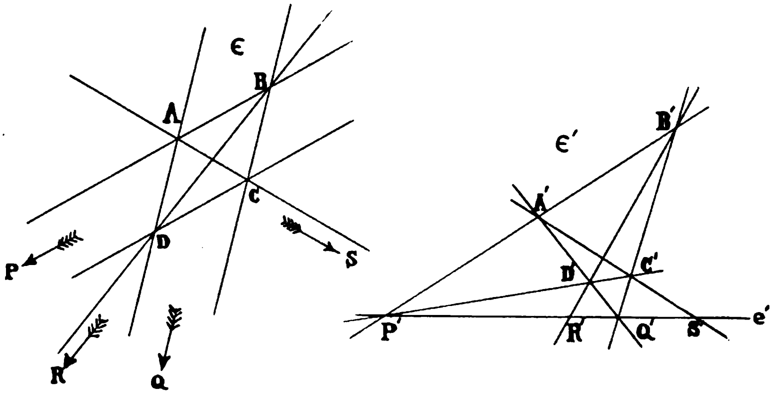

FIG. 1

BC, respectively. Let *R'* and *S'* designate the "flucht"-points of the rays *B'D'* (conjugate to *BD*) and *A'C'* (conjugate to *AC*), respectively.

Since (Art. 1) the points *P'*, *Q'*, *R'*, *S'* are a harmonic range of points, they all lie on a certain definite straight line *e'* of the plane field ϵ'; and this line *e'*, which is conjugate to the infinitely distant straight line *e* of the plane field ϵ, is *the locus of the "flucht" points of all the rays of the plane field* ϵ'.

Similarly, *there is a certain straight line f of the plane field* ϵ, *conjugate to the infinitely distant straight line f' of* ϵ', *which is the locus of the "flucht" points of all the rays of* ϵ.

These two straight lines *f* and *e'* are called in the German treatises on geometry the "*flucht*" *lines* (or *Gegenaxen*) of the two pro-

jective plane fields. We shall designate them hereafter (from the standpoint of optics) as the *focal lines* of the two conjugate planes ϵ and ϵ'. Hence,

If two plane fields are collinear, then, in general (that is, except in one particular case, to be presently remarked), *to the infinitely distant* (*or ideal*) *line of one field there corresponds a finite* (*or actual*) *line of the other field, the so-called "focal line" of that field.*

To a pencil of parallel rays in one plane field there corresponds therefore a pencil of rays in the other field which all intersect in a point situated on the focal line of that field; or, as we might say, the focal line of one plane field is the locus of the bases of pencils of rays which are conjugate to pencils of parallel rays in the other plane field.

The points P', Q', R', S' of ϵ', corresponding to the infinitely distant points P, Q, R, S, of ϵ, are, in general (as we have said), finite (or actual) points, and determine an actual straight line e'; except in the one particular case when the quadrangle $A'B'C'D'$ has each pair of its opposite sides parallel. In this special case the points P', Q', R', S' will be ranged along the infinitely distant straight line f' of the plane field ϵ', and the focal line e' will therefore coincide with the infinitely distant line f'.

This special case in which *the two focal lines f and e' are also the infinitely distant lines e and f' of the object-plane and the conjugate image-plane, respectively,* is the so-called case of *telescopic imagery* in optics (called "affinity" in modern geometry); of which we shall treat separately in another place. (The name "telescopic imagery" is derived from the fact that the focal points of the optical instrument known as a telescope are both at infinity.)

ART. 4. THE FOCAL POINTS AND THE PRINCIPAL AXES OF THE OBJECT-PLANE (ϵ) AND THE IMAGE-PLANE (ϵ')

As a general thing, therefore, we may say that to a pencil of parallel rays of one plane field there will correspond a pencil of non-parallel rays of the other plane field, the base (or vertex) of which is a finite (or actual) point of the focal line of that plane field. Or, expressing the same fact differently ,we can say:

To a parallel (or translatory) displacement of an object-ray there

will (in general) correspond both a translation and a rotation of the conjugate image-ray.

There are two cases, however, in which this is not true:

1. *When the ray is parallel to the focal line.* To the infinitely distant point of the focal line (f) of the object-plane (ϵ), which is the point of intersection of f and the infinitely distant straight line of the plane field ϵ, there corresponds the infinitely distant point of the focal line (e') of the image-plane (ϵ'), which is the point of intersection of e' and the infinitely distant line of the plane field ϵ'. Hence:

To rays of the object-plane which are parallel to the focal line (f) correspond rays of the image-plane which are parallel to the focal line (e'); and vice versa.

In general, this is the only case of a pencil of parallel rays of one plane field corresponding to a pencil of parallel rays of the other plane field. The other case referred to above is:

2. *The case of "telescopic imagery."* In this case the infinitely distant lines of the two plane fields are conjugate to each other and are at the same time the focal lines. Here, therefore, it is obvious that parallel rays of one plane always correspond to parallel rays of the conjugate plane. However, this is a distinctly exceptional case.

A pencil of parallel object-rays will therefore determine a definite point of the focal line (e') of the image-plane; and vice versa. Thus, for example, if we select a pencil of parallel rays of one plane which are all *perpendicular* to the focal line of that plane, we can say:

To a pencil of parallel $\left\{\begin{matrix}\text{object-rays}\\ \text{image-rays}\end{matrix}\right\}$ which are perpendicular to the focal line $\left\{\begin{matrix}f\\ e'\end{matrix}\right\}$ of their plane, there corresponds a pencil of non-parallel $\left\{\begin{matrix}\text{image-rays}\\ \text{object-rays}\end{matrix}\right\}$ which intersect in a certain point $\left\{\begin{matrix}E'\\ F\end{matrix}\right\}$ of the focal line $\left\{\begin{matrix}e'\\ f\end{matrix}\right\}$ of the $\left\{\begin{matrix}\text{image-plane}\\ \text{object-plane}\end{matrix}\right\}$.

This unique pair of points, F and E', thus determined, are the so called *focal points* of the object-plane and image-plane, respectively.

The two straight lines drawn through F and E' perpendicular to the focal lines f and e' are called the ***principal axes*** of the object-plane and of the image-plane, respectively. These lines we shall designate by the symbols x and x', and we shall refer to them as the x-axis and the x'-axis.

Since the ray x' passes through the focal point E' (which corresponds to the infinitely distant point of x) and also through the infinitely distant point of x' (which corresponds to the focal point F, likewise situated on x), it follows that x and x' are a pair of conjugate rays; and *the only pair of conjugate rays which are at right angles to the focal lines.*

METRIC RELATIONS OF TWO COLLINEAR PLANE FIELDS

Art. 5. Relation Between Conjugate Abscissae

Let u and u' be two conjugate rays of the object-plane and the image-plane, and let R and S' designate the points where these rays cross the focal lines f and e', respectively. Moreover, let S and R' denote the infinitely distant points of u and u' conjugate to S' and R, respectively. Now, if P, Q and P', Q' are any other pair of conjugate points of u and u', we shall have:

$$(PQRS)=(P'Q'R'S')\,,$$

or

$$\frac{PR}{QR}:\frac{PS}{QS}=\frac{P'R'}{Q'R'}:\frac{P'S'}{Q'S'};$$

or, finally,

$$\frac{PR}{QR}=\frac{Q'S'}{P'S'};$$

which may be written:

$$RP\cdot S'P'=RQ\cdot S'Q'=a\ constant.$$

Stated in words, this *characteristic metric relation of optical imagery* may be expressed as follows:

The product of the abscissae[1] *of two conjugate points, P and P',*

[1] The term "abscissa" is employed here, and generally also throughout this paper (unless otherwise specified), to describe the position of a point on a ray with respect to the "flucht" point of the ray as origin. Thus, for example, the abscissa of the point P of the ray u is RP, which means the segment of the ray included between the points R and P, and *reckoned from R to P;* that is, reckoned always in the sense in which the letters are written.

In this place we also take occasion to say expressly (although it is doubtless not

of the object-plane and image-plane, with respect to the so-called "flucht" points, R and S', of two conjugate rays u and u' which go through P and P', respectively, is constant.

In particular, let us suppose that the two conjugate rays are the two principal axes themselves; then we shall have:

$$FP \cdot E'P' = a \text{ constant.}$$

If we denote the value of this constant for the principal axes (x and x') by a, and if we put

$$FP = x, \quad E'P' = x',$$

we obtain:

$$xx' = a. \tag{1}$$

We shall call this the "*abscissa equation.*"

Art. 6. The Lateral Magnification

Let m, m' (Fig. 2) denote two conjugate straight lines of the object-plane and image-plane which are perpendicular to the prin-

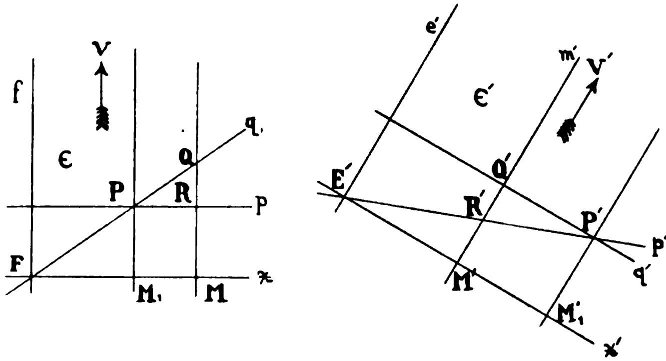

Fig. 2

cipal axes x and x', respectively, and therefore parallel to the focal lines (see Art. 4). The infinitely distant points, V, V', of the projective ranges of points m, m' are corresponding points. Let M, M' designate the points of intersection of the lines m, m' with the prin-

necessary) that if A, B, C, D, ... J, K are any number of points ranged along a straight line in any order whatever, we have always the following relations:

$$AB + BA = 0; \quad \text{or } AB = -BA;$$
$$AB + BC + CA = 0;$$
$$AB + BC + CD + \ldots + JK = AK; \text{ etc.}$$

cipal axes x, x', respectively. And let Q, Q' and R, R' be any other two pairs of corresponding points of m and m'; then

$$(QRMV) = (Q'R'M'V');$$

or

$$\frac{QM}{RM} = \frac{Q'M'}{R'M'};$$

which may be written:

$$\frac{M'Q'}{MQ} = \frac{M'R'}{MR} = \beta,$$

where β denotes the value of this constant ratio of corresponding line-segments of the two given conjugate lines m and m' which are parallel to the focal lines f and e', respectively.

The two conjugate lines m and m' are said to be "projectively similar." This ratio β for each line m of the object-plane perpendicular to the x-axis is called the *lateral magnification* for that line, and has for it a constant value. If, therefore, we put

$$y = MQ, \quad y' = M'Q',$$

we can write:

$$\frac{y'}{y} = \beta;$$

which states that the value of β is independent of the magnitudes of y and y' themselves.

It will be convenient now to select the two focal points F and E' as the origins of two rectangular systems of co-ordinates of the object-plane and image-plane, respectively. The principal axes of the two planes will be the axes of x and x', while the two focal lines f and e' will be the other two lines of reference (Fig. 2). Let m and m' be two corresponding straight lines at right angles to the axes of x and x', respectively, and meeting these axes in the points M and M'. And let β denote the value of the lateral magnification for m, m'.

Through any point P of the object-plane draw two rays; viz., the ray p parallel to the x-axis and the ray q which passes through the focal point F. Let R and Q designate the two points on m where the rays p and q, respectively, meet m. On m' lay off $M'R' = \beta \cdot MR$ and $M'Q' = \beta \cdot MQ$; then the intersection of the ray q' drawn through Q' parallel to the x'-axis and the ray p' joining E'

and R' determines the image-point P' corresponding to the object-point P.

If the object-point P is supposed to move along a line perpendicular to the x-axis which cuts this axis in the point M_1, we see at once that the image-point P' must also traverse a line perpendicular to the x'-axis in such fashion that the lateral magnification $\frac{M'_1P'}{M_1P}$ shall be constant.

Again, if the object-point P is supposed to move along the ray q which passes through the focal point F, the image-point P' will travel along the conjugate ray q' which is parallel to the x'-axis; so that as the ordinate M_1P assumes all values from $-\infty$ to $+\infty$, its image M'_1P' remains constant as to both magnitude and sign.

Thus we see that *the lateral magnification β has different values for each pair of conjugate lines which are parallel to the focal lines.* That is, *the magnification β is a function of the abscissa x.*

Art. 7. The Image Equations

We proceed, therefore, to ascertain in what way the lateral magnification β depends on x. Let x, y denote the co-ordinates of an object-point Q and x', y' the co-ordinates of the corresponding image-point Q'; and let $\frac{y'}{y}=\beta$ be the magnification-ratio for the point Q. If θ denotes the angle made with the x-axis by the ray q which joins the focal point F and the object-point Q, then

$$y=x\tan\theta\,,$$

and

$$\beta=\frac{y'}{x\tan\theta}\,.$$

If x_1, y_1 and x'_1, y'_1 denote the co-ordinates of the pair of conjugate points P and P' which are situated on the conjugate rays q and q', and if β_1 denotes the value of the magnification-ratio for the point P, then, since

$$y'_1=M'_1P'=M'Q'=y'\,,$$

we have:

$$\beta_1=\frac{y'_1}{x_1\tan\theta}=\frac{y'}{x_1\tan\theta}\,.$$

Therefore,

$$\frac{\beta}{\beta_1}=\frac{x_1}{x}.$$

That is,

$$\beta x = \text{a constant} = b \text{ (say)}.$$

Hence, *the lateral magnification β is inversely proportional to the abscissa x.*

Referring, therefore, to (1) of Art. 5, we are able now to express the co-ordinates x', y' of any point of the image-plane in terms of the co-ordinates x, y of the corresponding point of the object-plane as follows:

$$x'=\frac{a}{x}, \quad y'=\frac{by}{x}. \tag{2}$$

These two equations are the general image-equations for two conjugate planes in their simplest forms.

NOTE.—If, instead of confining the investigation, as we have done here, to two collinear plane fields, we had extended it to two collinear *regions of space*, we should have obtained by precisely analogous processes the following results—which are here given in brief:

1. If we have two collinear regions of space Σ and Σ', which we shall call in the language of optics the "object-space" and the "image-space," respectively, then, except in the case of "telescopic imagery" (or "affinity"), to the infinitely distant plane of one region of space there will correspond an actual (or finite) plane, the so-called "flucht" plane or *focal plane*, of the other region of space.

2. In general, to a sheaf of parallel planes of one of the two regions of space there will correspond a sheaf of non-parallel planes of the other region of space. However, there is one exception to this statement: *The two sheaves of parallel planes to which the focal planes themselves belong are conjugate sheaves of planes.*

Two conjugate planes which are parallel to the focal planes are in "affinity" with each other; that is, their infinitely distant straight lines are conjugate to each other. And, hence, any range of points u in the object-space which is parallel to the focal plane of that space will be "imaged" in a projectively similar range of points u' of the image-space which is likewise parallel to the focal plane of that space.

3. To a bundle of parallel rays in the object-space will correspond a bundle of non-parallel rays in the image-space the vertex of which lies in the focal plane of that space; and vice versa. If the bundle of parallel rays in one space meets the focal plane of that space at right angles, the vertex of the corresponding bundle of rays in the other space will determine a certain point in the focal plane of that space—viz., the *focal point* of the object-space or image-space, as the case may be.

The object-space and the image-space have therefore only one pair of conjugate straight lines each of which is perpendicular to the focal plane of the space to which it belongs; these two conjugate lines at right angles to the focal planes are called the ***principal axes*** of the object-space and image-space, respectively; we shall denote them as the axis of x and the axis of x'.

4. To the sheaf of planes in the object-space which has for its base the x-axis corresponds the sheaf of planes in the image-space which has for its base the x'-axis which is conjugate to the x-axis. Of these so-called "meridian" planes there is one pair at right angles to each other in the object-space to which correspond a pair in the image-space which are also at right angles. Following Czapski, we shall denote these planes as the xy-plane and xz-plane in the object-space and the $x'y'$-plane and $x'z'$-plane in the image-space.

Thus, selecting as axes of a system of rectangular co-ordinates the three lines of intersection of the focal plane and the two meridian planes above-mentioned, in the case of both the object-space and the image-space, Czapski shows in his book that the general image-equations (A), which are the analytical expression of the relation of collinear correspondence of two regions of space (see "Introduction") will reduce to the following simple form:

$$x' = \frac{a}{x}, \quad y' = \frac{by}{x}, \quad z' = \frac{cz}{x}, \tag{B}$$

where, instead of fifteen independent constants, the number has been reduced to three (although it is hardly necessary to say that a, b, c of equations (B) do not denote the same constants as those same letters denote in the general equations (A)).

Accordingly, in the most general case of optical imagery, as defined by equations (B), there are involved at least three constants, a, b, and c. In the general case the imagery is not symmetrical around the principal axes of the object-space and image-space; that is, the two magnification-ratios y'/y and z'/z have different values corresponding to the same value of x. In most actual cases, however, the principal axes are axes of symmetry; and as we are concerned primarily with the practical applications of these laws to the theory of optical instruments, we shall assume that this is the case. Thus we shall put $b=c$; in which case the character of the imagery is defined by the two constants a and b, and of the three equations (B) we need deduce only the first two, since the third equation follows at once from the condition of symmetry.

In the case of symmetry with respect to the principal axes of x and x', it is sufficient, therefore, to ascertain the laws of collinear correspondence of *two conjugate meridian planes* of the object-space and image-space. This is one of the reasons why we have confined the investigation in this paper to the case of two collinear plane fields.

In regard to the constants a and b of equations (2) of Art. 7, it is perhaps not superfluous to say that, in general, the values of these constants are different for different pairs of conjugate planes of the two regions of space. However, for

each pair of conjugate planes of the two sheaves of meridian planes the constants a and b have the *same* values; and these values are also the same as the values of a and b in equations (B)—assuming that we have symmetry with respect to the principal axes. For any pair of conjugate meridian planes the principal axes of the two planes coincide with the principal axes of the two regions of space.

In the following we shall *assume that the two conjugate planes are meridian planes of the object-space and image-space;* although generally the results can be applied to any two conjugate planes, as we shall do in certain special cases.

ART. 8. THE FOCAL LENGTHS f AND e'

It was remarked above (Art. 6), and is evident at once from the image-equations, that the lateral magnification $\frac{y'}{y}=\beta$ may have any value from $-\infty$ to $+\infty$, depending on the value of the abscissa x. Accordingly, there must be a pair of conjugate points, H and H', situated on the axes of x and x' of the object-plane and image-plane, respectively, for which $\beta=+1$. In other words, *there is one pair of conjugate straight lines parallel to the focal lines of the object-plane and image-plane, respectively, which are "projectively congruent."*

If the two conjugate planes are meridian planes (as we shall now assume), the axes of x and x' are the two principal axes of the object-space and image-space; and the points H and H' are the so-called "principal points" of the optical system—as Gauss designated them in his *Dioptrische Untersuchungen*, published in 1840.

The abscissae of the principal points H and H' with respect to the focal points F and E' are called the *focal lengths* of the object-space and the image-space, respectively. If we denote the two focal lengths by f and e', then

$$f=FH\,,\quad e'=E'H'\,.$$

If in the image-equations (2) of Art. 7 we put $y=y'$, $x=f$ and $x'=e'$, we find:

$$a=fe'\,,\quad b=f\,;$$

so that now we may write the image-equations (2) in terms of the two new constants f and e' as follows:

$$\left.\begin{aligned} &xx'=fe'\,,\\ &\frac{y'}{y}=\frac{f}{x}=\frac{x'}{e'}=\beta\,. \end{aligned}\right\} \tag{3}$$

For general purposes these are the most useful forms of the image-equations.

Czapski defines the focal lengths without reference to the principal points; but our object here was merely to show the final form of the image-equations. It would take too much space to discuss these equations in this place; but the reader who is interested will find in the chapter on the "Geometrical Theory of Optical Images" of Dr. Czapski's book a complete discussion, not only of these equations, but of many other matters connected therewith, which we are compelled to omit, as they are not included within the scope of this article.[1]

ART. 9. TELESCOPIC IMAGERY

The image-equations which we have deduced above are not applicable in the special case of telescopic imagery, because in this case the focal lines are the infinitely distant lines of the object-plane and the image-plane.

In the language of geometry, two collinear plane fields ϵ and ϵ' are said to be in "affinity" with each other if their infinitely distant straight lines are corresponding straight lines.

Since, therefore, to each infinitely distant point of one field corresponds an infinitely distant point of the other field, it is obvious that a pencil of parallel image-rays will correspond to a pencil of parallel object-rays. Moreover, to any range of points in one field there will correspond a "projectively similar" range of points of the other field. The peculiarity of such ranges of points is that corresponding segments of them are in a constant ratio to each other.

Let x, x' and y, y' be any two pairs of non-parallel corresponding rays, and let m and n denote the values of the ratios of corresponding segments of each of these two pairs of conjugate rays, respectively. Let PQ and $P'Q'$ be two conjugate line-segments drawn parallel to x and x', respectively. If then we draw through the points P and Q lines parallel to y, the corresponding lines in the other plane-

[1] The reader is also referred to E. Wandersleb's article in the first volume of *Die Theorie der optischen Instrumente*, edited by M. von Rohr (Berlin, 1904). This valuable and exhaustive work, of which only the first volume has appeared, is published under the auspices of Dr. Czapski himself, who contributes the preface. Each chapter is the work of one or more of the staff of eminent optical engineers associated with the firm of Carl Zeiss, in Jena.

field will be a pair of lines drawn through the points P' and Q' parallel to y', and we see at a glance that

$$\frac{P'Q'}{PQ} = m .$$

Hence, all lines parallel to x have the same magnification-ratio. Thus, we can say:

In the case of telescopic imagery, all parallel rays of the object-plane have the same value for the magnification-ratio.

If we select any two conjugate points, O and O', as origins of the axes of co-ordinates of the object-plane and the image-plane, there will always be in the two projective pencils of rays through O and O' one pair of rectangular rays in the object-plane which correspond to a pair of rectangular rays in the image-plane. Hence, in the case of telescopic imagery, the image-equations referred to rectangular axes may be written as follows:

$$x' = mx , \quad y' = ny . \tag{4}$$

ART. 10. IDEAL IMAGE NOT ATTAINABLE

The geometrical theory of optical imagery is seen, therefore, to be identical with the theory of collineation in modern geometry. As to the mechanical means of producing an optical image we have not inquired at all in the foregoing; although, properly speaking, the geometrical theory is at the foundation of the theory of all optical instruments, and a knowledge of the theory has proved of the utmost advantage in the design and construction of optical apparatus.

In general, there may be said to be only one actual optical system which perfectly satisfies the condition of collinear correspondence—viz., the *plane mirror;* which, inasmuch as it produces only a virtual image without magnification, hardly deserves to be ranked as an optical instrument at all. The *pin-hole camera* is no exception to the statement just made, because only when the aperture is a mathematical point would there be strict point-to-point correspondence of object and image—even then assuming that there were no exceptions to the law of the rectilinear propagation of light, such as we encounter in physical optics.

Practically speaking, nearly all optical instruments consist of combinations of certain isotropic optical media separated from each

other by spherical boundary-surfaces, the centers of which are ranged along a straight line, called the "optical axis" of the system. This optical axis coincides with the principal axes of the object-space and image-space, and is an axis of symmetry. An optical system of this kind is called a "centered" system of spherical surfaces. (In this statement it seems superfluous to add that plane boundary-surfaces are likewise included, inasmuch as they may be regarded as spherical surfaces with infinite radius.) By means of an optical apparatus of this description it is practically quite impossible to produce, with the employment of wide-angle bundles of rays, a theoretically perfect image of an extended object; that is, a homocentric bundle of rays after passing through the instrument is, in general, no longer homocentric. Instead of the ideal case of collinear correspondence of object-space and image-space, the theory of optical instruments is complicated by numerous practical and irreconcilable difficulties due chiefly to the so called "*aberrations*"—a number of which are aberrations of sphericity, while others are "chromatic" aberrations. Nevertheless, in spite of these apparently insurmountable obstacles in the way of the attainment of even an approximately perfect image, the wonderful performance of modern optical apparatus is a most extraordinary and convincing proof, not only of the genius and skill of the designers of these instruments, but of the achievements that are possible under science as the leader. However, it would be foreign to the purpose of this paper to do more than merely refer to the intricate problems of applied optics.

The point in which the optical axis of a centered system meets one of the spherical surfaces is called the "vertex" of that surface. If we render opaque all the parts of these spherical "interfaces" (or boundary-surfaces) except the very small zones of which these vertices are the summits, the only rays of light that can be utilized by the optical instrument are the narrow bundles of rays which meet the spherical surfaces at very nearly normal incidence. These rays which are all very near to the optical axis are called ***paraxial* rays**. *In the case of the refraction (or reflection) of **paraxial rays** by any optical system consisting of a series of centered spherical surfaces, there is strict collinear correspondence between object and image*

for "monochromatic" rays of light. All of the formulæ, therefore, which we have obtained are applicable in the case of paraxial rays. These formulæ were obtained for paraxial rays by the earlier investigators in optics, chief among whom may be mentioned Moebius, Gauss, Bessel, and Listing, who made great advances in the theory of optics, so that they may be regarded as the founders of modern optics. The first writers on optics (such as Kaestner, Euler, Kluegel, Herschel, and Littrow) did not employ general formulæ and calculated the paths of the rays from surface to surface without simplifying the problem by introducing the ideas of the so-called "cardinal points" of the optical system.

PART II

THE ELEMENTARY PROBLEMS OF GEOMETRICAL OPTICS

Most textbooks of geometrical optics begin with the problems of reflection and refraction of paraxial rays at a spherical surface, and proceed thence to the problem of infinitely thin lenses. As already stated, we have always in the case of paraxial rays complete collinear correspondence between object-space and image-space. Moreover, in each of the problems above mentioned we have a simple case known in geometry as "central collineation." Inasmuch as these problems afford pretty applications of the general theory outlined in the preceding pages, and are at the same time not without interest for their own sake, a consideration of them here should not be deemed out of place. Perhaps, too, to teachers of physics, especially, as the author ventures to hope, this mode of viewing these problems may be in some respects novel and suggestive.

ART. 11. CENTRAL COLLINEATION OF TWO PLANE FIELDS

If two collinear plane fields are so situated relative to each other that they have *in common a self-corresponding range of points*, we have the case of *central collineation* of two plane fields. The straight line common to the two fields, which corresponds with itself point by point, is called the "*axis of collineation.*" Obviously, any pair of corresponding rays of the two collinear plane fields will meet in this line. If the two plane fields are not in the same plane, the axis of collineation will be the line of intersection of their planes.

In the problems in optics, however, the two plane fields are usually in the same plane of space. We shall assume here that this is the case, and also that this plane is a meridian plane; that is, a plane which contains the optical axis of the system.

It is easy to show that *in the case of central collineation of two plane fields the straight lines which join each pair of conjugate points all intersect in one point.*

This point which is situated on every straight line which joins a pair of conjugate points is called the *center of collineation.*

By way of illustration of this kind of collineation, we may consider the following problem:

Given the axis of collineation (y) and the center of collineation (C) and the positions of two conjugate points P and P': it is required to construct the image-point Q' of a given object-point Q (Fig. 3).

Through the object-points P and Q draw the straight line k meeting the axis of collineation (y) in the self-corresponding point K; the corresponding straight line k' which connects K with P' must also pass through the image-point Q'. And since Q' must likewise be situated on the self-corresponding ray which passes through Q and the center C, the image-point Q' will be uniquely determined by the intersection of the straight lines KP' and QC.

Similarly, the image-ray l corresponding to any object-ray l' may be constructed as follows: Let L designate the point where l meets the axis of collineation (y), and let G be the point of intersection of l and any ray k which passes through the given object-point P. The image of G is the point of intersection of the straight lines joining C with G and K with P'. Hence, the image-ray l' is the straight line joining G' and L.

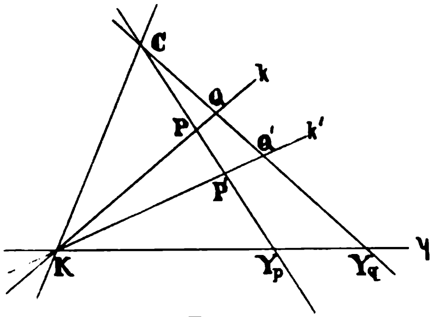

FIG. 3

The self-corresponding ray at right angles to the axis of collineation (y) is the optical axis of the system. This ray we shall refer to as the ray x of the object-space and the ray x' of the image-space. The point where it crosses the axis of collineation will be desig-

nated by the letter H, and this point will be selected, in the special case of central collineation, as the most convenient point for the origin of a system of rectangular co-ordinates the axes of which are the optical axis and the axis of collineation.

A good illustration of central collineation is afforded by the *refraction of paraxial rays at a spherical surface* (Fig. 4). The center of the spherical surface is evidently the center of collineation (C), because the rays which pass through this point (really or virtually) meet the refracting surface normally and proceed in unchanged directions. Moreover, since we are concerned only with

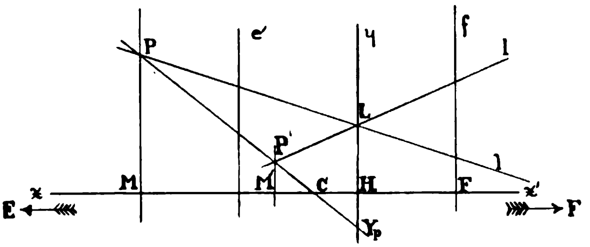

FIG. 4.—On the assumption that light is propagated from left to right, this diagram shows imagery by refraction of paraxial rays from air to glass at concave spherical surface.

paraxial rays which meet the refracting surface at points infinitely near to the vertex (H), it is plain that the straight line in the meridian plane which is *tangent to the spherical surface at its vertex* is the axis of collineation (y).

ART. 12. THE FOCAL POINTS

It is obvious that the *focal lines*, f and e', of the object-plane and image-plane are both parallel to the axis of collineation (y); because each focal line meets its corresponding straight line on the axis of collineation and at the same time in an infinitely distant point. The *focal points* are the points, designated by F and E', where the optical axis meets the focal lines. Now, if E and F' are the infinitely distant points of x and x'—that is, if E and F' are the points conjugate to the focal points E' and F, respectively—we can write the following equation immediately:

$$(CFHE) = (CF'HE');$$

that is,

$$FH = CE' . \tag{5}$$

Hence, also:

$$E'H = CF . \tag{6}$$

The important relation expressed by these equations is characteristic of central collineation; it may be stated in words as follows:

In the case of central collineaton, the two focal points are always so situated that the step from one of them to the point H is identical with the step from C to the other focal point.

(The reader will find this a very useful thing to remember in connection with problems of refraction of paraxial rays at a spherical surface.)

If we denote the abscissae of the center of collineation (C) and the two focal points (F and E') by the symbols r, s_F and s'_E; that is, if we put

$$HC = r , \quad HF = s_F , \quad HE' = s'_E ,$$

then since

$$FH = CE' = CH + HE' ,$$

we obtain:

$$s_F + s'_E = r . \tag{7}$$

That is, *the sum of the abscissae of the focal points is equal to the abscissa of the center of collineation.*

The abscissae s_F and s'_E are not the focal lengths; if f and e' denote the focal lengths of the optical system, then $f = FH$, $e' = E'H$, so that

$$s_F = -f , \quad s'_E = -e' .$$

If therefore we are given the center of collineation (C) and the axis of collineation (y), together with (say) the ratio of the two focal lengths of the optical system, we can construct the two focal lines and determine completely the nature of the imagery.

Art. 13. The "Invariant" in the case of Central Collineation

Since all the rays of the pencil C are self-corresponding, each of these rays is the base of two projective ranges of points, a range of object-points and a range of corresponding image-points. Moreover, each of these self-corresponding rays has two self-corresponding points, of which one is the center of collineation itself, and the other is the point where the ray crosses the axis of collineation.

Similarly, every point on the axis of collineation is the common vertex of two projective pencils of rays, a pencil of object-rays and a pencil of corresponding image-rays. Each pair of such pencils of rays contains two self-corresponding rays, of which the axis of collineation itself is one, and the ray joining the common vertex of the two pencils with the center of collineation is the other.

If P, P' (Fig. 3) are two conjugate points, and if the line joining P and P' crosses the axis of collineation in the point Y_p, the anharmonic ratio of the two conjugate points P, P' with the two self-corresponding points C, Y_p of the two projective ranges of points which have the common base PP' has a fixed value c which is independent of the positions of the conjugate points P and P'; that is,

$$(CY_pPP')=c.$$

This is an elementary proposition of projective geometry. If Q, Q' are any other pair of conjugate points, and if Y_q designates the point where the ray QQ' crosses the axis of collineation, and, finally, if S is the point on the axis of collineation where the two corresponding rays PQ and $P'Q'$ intersect, it is obvious at once that the range of points C, Y_p, P, P' is in perspective with the range C, Y_q, Q, Q', since they are sections of the pencil of rays S. And, hence, the anharmonic ratios of each of these ranges of four points are equal. Accordingly, we have the following remarkable relations:

$$\left.\begin{aligned} c&=(CY_pPP')=(CY_qQQ')=\text{etc.},\\ &=(CHFF')=\frac{CF}{HF},\\ &=(CHEE')=\frac{HE'}{CE'}; \end{aligned}\right\} \tag{8}$$

where, as heretofore, F and E' are the two focal points and F' and E the corresponding infinitely distant points (see Fig. 4).

The most striking characteristic of central collineation, therefore, consists in the fact, above discovered, that it has an *invariant*, which is the constant anharmonic ratio of any two corresponding points with the two self-corresponding points of the ray connecting the corresponding points. The value of this invariant, as above stated, is:

$$c=\frac{CF}{HF}=\frac{HE'}{CE'}.$$

Using the same symbols as in Art. 12 to denote the abscissae, with respect to H, of the axial points C, F, and E', we obtain:

$$\frac{s_F - r}{s_F} = \frac{s'_E}{s'_E - r} = c\,;$$

which gives the relation obtained in Art. 12, viz., $s_F + s'_E = r$; accordingly we have:

$$\frac{s'_E}{s_F} = -c\,. \tag{9}$$

Thus we ascertain that *the value ot the invariant* (c) *is equal to the ratio of the abscissae of the focal points, with the sign changed.* (We may likewise write $e'/f = -c$, since the focal lengths have the same ratio as the abscissae of the focal points).

In particular, if M, M' designate the positions of any two conjugate axial points (that is, conjugate points situated on the optical axis), then

$$(CHMM') = c\,,$$

or

$$\frac{CM}{HM} : \frac{CM'}{HM'} = c\,.$$

We shall denote the abscissae of M, M', with respect to H, by the symbols s, s', respectively; that is,

$$HM = s\,; \quad HM' = s'\,, \quad \text{also, } HC = r\,.$$

Substituting these symbols in the above, we obtain the following relation between the abscissae of any pair of conjugate points *for any central collineation:*

$$\frac{c}{s'} - \frac{1}{s} = \frac{c-1}{r}\,. \tag{10}$$

The applications of this formula will be shown in the following section.

ART. 14. THE CASES WHICH WE HAVE IN OPTICS

If the *sign of the invariant* (c) *is positive*, the conjugate points M, M' are not "separated" (in the geometrical sense) by the axis of collineation (y) and the center of collineation (C). That is, for $c > 0$, the points M and M' are either both situated between C and H, or neither of them is between C and H. This case occurs always whenever light is *refracted* from one medium into another; so that

in optics a positive value of c denotes refraction; whereas, on the contrary, whenever the light-rays are *reflected* at a mirror, the imagery is of the kind that corresponds to a *negative* value of c ($c<0$); in which case one of the points M or M' will lie between C and H, but not the other point.

I. Refraction of Paraxial Rays; $c>0$

1. Suppose that r *is not equal to zero;* that is, suppose that the center of collineation (C) does not lie on the axis of collineation (y). In this case formula (10) is the formula for the *refraction of paraxial rays at a spherical surface* (Fig. 4). The invariant (c) in this case proves to be identical with the "relative index of refraction" (n) from the "first medium" to the "second medium;" while the other constant (r) denotes here the radius of the spherical surface. Accordingly, the points H and C designate the positions of the "vertex" and center of the spherical surface, respectively. If we suppose in our diagrams that the light is represented as being propagated from left to right, then, according as the center C is to the right or left of the vertex H, the spherical refracting surface will be "convex" or "concave." If, therefore, we choose the direction of propagation of light along any ray as the *positive direction of the ray*, a convex surface will be indicated by a positive value of r. Writing n in place of c in the general formula (10), we obtain the special formula for the refraction of paraxial rays at a spherical surface in the usual form given in the textbooks, viz.:

$$\frac{n}{s'}-\frac{1}{s}=\frac{n-1}{r};$$

There are several special cases under this head which may be noticed in passing:

a) Suppose that $c=+1$ (the value of r, as above specified, being different from zero). Now the value of the refractive index is unity ($n=c=+1$), and the equation above gives $s=s'$. Hence, object-space and image-space coincide point by point; in fact, the two spaces are identical. When $n=+1$, there is no optical difference between the first medium and the second.

b) *The case when* $r=\infty$.—This merely means that the center C is at an infinite distance away in the direction of a line at right

angles to the axis of collineation; so that now the refracting surface is a *plane* surface. The formula in this case becomes:

$$s' = ns\,,$$

which is, therefore, the abscissa-relation in the case of the *refraction of paraxial rays at a plane surface.* Since the center of collineation (C) is at an infinite distance in a direction perpendicular to the axis of collineation (y), all straight lines which join pairs of conjugate points are parallel to the abscissa-axis (Fig. 5). The two infinitely distant straight lines of the two collinear plane fields must both pass through the point C, and therefore they must be self-corresponding rays. Consequently, this is a case of *telescopic imagery*, for which the lateral magnification $y'/y = +1$ (see Art. 9).

2. *The invariant* $c = +1$ *and* $r = 0$.—If $r = 0$, the center of collineation (C) is situated on the axis of collineation (y). Putting $r = 0$ in equation (7), we find:

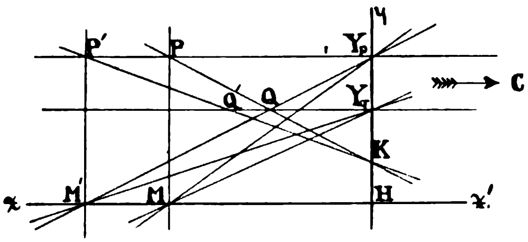

FIG. 5.—Refraction of paraxial rays at plane surface.

$$s_F + s'_E = 0\,;$$

so that this type of imagery is characterized by the fact that the two *focal points are equidistant from the axis of collineation* (y), *and on opposite sides thereof.* This will be immediately identified as the imagery produced by the *refraction of paraxial rays through an infinitely thin lens.* In the diagram (Fig. 6) the lens itself is represented by the axis of collineation (y), and the point H which coincides with the center of collineation is the "optical center" of the lens. The anharmonic ratio $(CHMM') = +1$ merely states that the points C and H are coincident—that is, $r = 0$, as we know already. Formula (10) is therefore of no value here, but we can obtain the so-called "lens formula" by writing:

$$(HMFE) = (HM'F'E')\,,$$

where M, M' denote any pair of conjugate points on the optical axis and E and F' denote the infinitely distant points corresponding to the focal points E' and F, respectively. Hence, we have:

$$\frac{HF}{MF}=\frac{M'E'}{HE'}$$

and, since here $HF+HE'=0$, that is,

$$HE'=FH,$$

we obtain:

$$\frac{HF}{MF}=\frac{M'E'}{FH};$$

which gives:

$$FM \cdot E'M'=-FH^2;$$

so that, if we put $FM=x$, $E'M'=x'$, and $FH=f$, we can write:

$$x \cdot x'=-f^2;$$

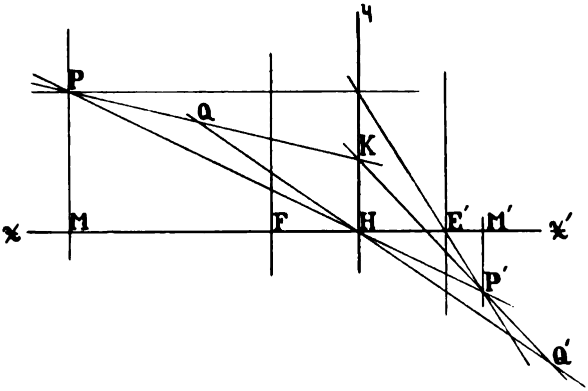

Fig. 6.—Refraction of paraxial rays by infinitely thin lens.

which is a useful form of the *lens formula.* Or, putting $HM=s$, $HM'=s'$, so that we have:

$$x=s+f, \quad x'=s'-f,$$

and substituting these values of x and x' in the above equation, we shall derive the lens formula in its usual form:

$$\frac{1}{s'}-\frac{1}{s}=\frac{1}{f}.$$

(If we take a point L on the optical axis such that $HL=2HM=2s$, this equation shows that we have $(LHFM')=-1$; that is, the points L and H are harmonically separated by F and M'; which suggests an easy geometrical way of constructing the image-point M' corresponding to the object-point M.)

The same formula is applicable also to the refraction of paraxial rays through any number of thin lenses in contact with each other; in this case f denotes the focal length of the combination of lenses.

II. Reflection of Paraxial Rays; $c<0$

The only negative value of the invariant (c) that has any practical meaning in optics is the value $c=-1$. For this value we have:

$$(CHMM')=-1\ ;$$

so that each pair of conjugate points is harmonically separated by the center (C) and the axis of collineation (y). The general formula (10) becomes for this case:

$$\frac{1}{s}+\frac{1}{s'}=\frac{2}{r},$$

which is seen to be the formula for the *reflection of paraxial rays at a spherical mirror.* Putting $c=-1$ in (9), we find $s_F=s'_E$; hence, in the case of a spherical mirror the focal points F and E' are coincident. The fact that the points C, H, M, M' are a harmonic range of points affords an easy method of constructing the image-point M' of a given object-point M.

If $r=\infty$, we have $s=-s'$ (case of *reflection at a plane mirror*).

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PRELIMINARY PAPER ON THE CAUSE OF THE CHARACTERISTIC PHENOMENA OF SUN-SPOT SPECTRA[1]

By GEORGE E. HALE, WALTER S. ADAMS, AND HENRY G. GALE

In considering the characteristic features of the spectra of sun-spots, three points especially attract attention:

1. The fact that certain lines in the spectrum of a given element are strengthened, while others are weakened; the remainder of the lines being unaffected.

2. The inclusion of all the strengthened lines within the visible spectrum, none of them occurring in the ultra-violet, and their predominance in the red, yellow, and green.

3. The relatively great intensity of the continuous background[2] of the spot spectrum in the less refrangible region.

From our general knowledge of spectra corresponding to various temperatures we are aware:

1. That in passing from a high temperature to a lower temperature, certain lines are relatively strengthened, some are unaffected, and others are diminished in intensity.

2. That such a reduction of temperature is accompanied by an increase in the relative intensity of the less refrangible lines, and a shift of the maximum of a continuous spectrum toward the red.

The general correspondence of these two groups of facts led us to seek for an explanation of the spectra of sun-spots, on the hypothe-

[1] *Contributions from the Solar Observatory*, No. 11.

[2] Careful visual observations of the spot spectrum have been made with the Littrow spectroscope of 18 feet (5.5 m) focal length in the third and fourth orders of a 4-inch (10 cm) grating, having 14,438 lines to the inch (5,672 to the cm). Although an immense number of fine lines can be seen in the spot spectrum, they nevertheless seem to lie on a continuous dark background, which we have not been able to resolve into lines. This background, however, is interrupted at certain points by lines or breaks, which seem to be nearly as bright as the spectrum of the adjacent photosphere. They do not appear to us like genuine bright lines, and we are unable to offer an adequate explanation of them, unless the dark background is resolvable. Up to the present time we have observed none of the bright reversals (narrow bright line on wider dark line) of spot lines described by Mitchell, although the high resolving power of our spectroscope should render such reversals easily visible.

sis that the metallic vapors within the spots have a temperature lower than that of the photosphere.

Fortunately, the best of material was available for the investigation. Photographs of spot spectra, made with the Snow telescope and the Littrow spectrograph of 18 feet (5.5 m) focal length, show an immense number of affected lines.[1] These plates, which cover the region from D to $H\beta$, have been supplemented by photographs of the entire spectrum of recent large spots, extending from A in the red to the ultra-violet, made by Mr. Ellerman. Plates extending into the ultra-violet, which had previously been used for a study of the H and K lines, were also available.

The laboratory work began with a study of iron and other metals in a synchronous rotating arc, designed by Professor Crew, and constructed for us under his supervision. This ingenious instrument permits the spectrum of the alternating arc to be photographed at any desired phase. In an account of his experiments with the arc, published last year in the *Astrophysical Journal*, Professor Crew showed that the changes in the relative intensities of lines photographed at phase angles varying from 90° to 0° correspond to the changes observed in passing from high to low temperatures. The arc gives excellent results, but the brightness decreases rapidly with the phase, involving an undesirable increase of exposure time. It therefore occurred to Mr. Gale to try the effect of varying the current strength in an ordinary 110-volt direct current arc, the difference of potential between the poles being kept approximately constant. Spectra photographed with currents of 30 amperes and 2 amperes respectively show changes of intensity similar to those observed with the synchronous arc, with the advantage that the spectrum given by the arc with low current requires a much shorter exposure than that of the low-phase synchronous arc, which is necessarily intermittent. We have thus photographed, with currents of 30 amperes and 2 amperes, the spectra of iron, titanium, vanadium, chromium, manganese, calcium, and other metals characteristic of sun-spots. As the work progressed a correspondence was observed between lines that are "enhanced" in the spark and those that are weakened in sun-spots. For the further study of this effect we have also photo-

[1] For a partial list of the strengthened lines see *Astrophysical Journal*, **23**, 11, 1906.

graphed the spectra of the same elements in the discharge of a 600-watt transformer, giving about 6,000 volts at the secondary terminals. A condenser was used in the discharge circuit, and the potential was increased by an auxiliary air spark, in series with the observed spark, both being exposed to a strong blast of air from an electric fan. Under these conditions the "enhanced"[1] lines of the spark are well shown in the photographs. We are about to investigate them more effectively with the aid of a 5 K. W. transformer, giving 1,000, 2,000, 4,000, 8,000, 16,000, 32,000, or 64,000 volts, as desired.

The instrument with which by far the greater part of the laboratory spectra used in this investigation have been secured is a grating spectroscope in the Littrow form of 13 feet (3.96 m) focal length. The grating is by Michelson, with 700 lines to the millimeter, and gives a bright first-order spectrum on one side with excellent definition. The slit is provided with an occulting bar, by means of which two spectra that are to be compared can be photographed in the usual way. In most of this work, however, we have preferred to make the separate exposures through the same window in the bar, moving the plate between exposures. In this way we have been able to obtain a greater range of exposure time, and the arrangement has proved satisfactory, since the slight displacements between the separate spectra are of no consequence for our purpose. The window used has a diameter of about 1.2 mm. Our usual procedure in comparing the weak and the strong arcs has been to place on each side of the weak arc spectrum two spectra of the strong arc, giving different exposure times in the two cases. From the four strong arc exposures obtained in this way that one is selected which is most nearly comparable in general strength with the weak arc spectrum. When the spark spectrum has also been added, it has usually been placed immediately adjoining the weak arc, with the arrangement of the strong arc spectra as before. A few plates have been taken with a Fuess quartz spectrograph, but these have been used exclusively for qualitative purposes.

The arc employed in the work is of the self-feeding type, with carbon poles, working on a direct current (storage battery) circuit

[1] Throughout this paper the word "enhanced" is used to denote lines that are strengthened in the spark as compared with the arc.

of 110 volts. The metal has in all cases been placed in the positive pole.

EXPLANATION OF THE TABLES

The tables which follow contain the results of a study of the elements titanium, vanadium, iron, chromium, and manganese, for a region extending from the ultra-violet to λ 5800. Our investigation of both spot and laboratory spectra is far from complete, but the material here presented seems sufficient for the purpose of a preliminary communication. The tables include all the lines which are affected prominently, and which, through rise or fall in intensity in spot as compared with disk, or in spark or weak arc as compared with strong arc, are especially significant in the present investigation. Until it becomes possible, by the use of improved experimental methods, to increase the magnitude of the effects observed in the laboratory, it is necessary to omit from the discussion such slight apparent changes as are now beyond the limits of accurate observation.

In a discussion of the behavior of the lines of these elements, the investigation naturally divides itself into two parts: the relation of the weak arc to the strong arc, and of the arc to the spark. Accordingly, two sets of tables are given. The first contains the wavelengths of all the lines which are affected prominently in spots, the amounts by which they are affected, their behavior in the weak arc as compared with the strong arc, and in the spark as compared with the weak arc. For these last two determinations a scale of 0 to 5 has been used, 5 denoting the maximum change. In the column "Identification, Rowland," the absence of an entry indicates that in the Preliminary Table the line is identified with the element considered, a dash that the line is unidentified, while identifications with other lines are given in full. As comparatively few of the fainter lines given in Rowland's table are identified by him, it has been necessary to obtain such identifications for a considerable number of lines, particularly of vanadium and titanium. For the green and yellow region of the spectrum these have been made from measures of plates taken in our laboratory, further results of which will be published at a later time. Excellent tables by Hasselberg of *Ti*, *V*, *Mn*, and *Cr*, and by Rowland and Harrison of *V*, are available, and have been used for the blue and violet region of the spectrum. The

TABLE I

SUN-SPOT LINES IN SPARK AND ARC

TITANIUM

λ Rowland	Identification Rowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhancement Spark	Remarks
4060.42		1	2	1		
4112.87		1	2	0–1		
4299.80	*Fe,Ti*	2	3	n.c.		
4300.21		3	2	–2	4	
4300.73		2	3	1		
4338.08		4	3	–2	3	
4351.00		1	0	–2	2	
4387.01	*Ti?*	1	0	–1	3	
4426.20		0Nd?	2	1		
4471.41		0	2	1		
4475.03		0	1–2	n.c.		
4496.32		1	2	0–1		
4501.44	*Ti*, —	5	4	–2	4	
4512.91		3	4–5	1		
4518.20		3	4–5	1		
4522.97		2	4	1		
4533.42		4	5–6	0–1		
4534.95		4	5	1		
4548.94		2	3	0–1		
4555.66		3	4	0–1		
4572.16	*Ti*—	6	5	–2	5	
4617.45		3	4	1		
4623.28		2	3	0–1		
4639.54		2 }	4	1		
.68	*Cr*	0 }				
4639.85		2	3	1		
4640.12		1	3–4	1		
4645.37		0	1–2	n.c.		
4656.64		3	5	1–2		
4682.09		3	5	1–2		
4693.85		0	1–2	0–1		
4722.80		0	2	0–1		
4742.98		1	2	0–1		
4758.31		1	2	1		
4799.98		1	2	0–1		
4820.59		1	3	1		
4841.07		3	4	1–2		
4856.20		1	3	1		
4868.45		0	1–2	0–1		
4870.32		1	2	1		
4885.26		2	3	1		
4900.10	*Ti, La*	2	3	1		
4913.80		2	3	1		
4928.51		0	1–2	1		
4981.91		4	5	1–2		
4991.25		3	4–5	1–2		
4997.28		0	2	1		
4999.69	*Ti, La*	3	4–5	2		

TABLE I—*Continued*

λ Rowland	Identification Rowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhancement Spark	Remarks
5009.83	*Ti, Co*	oo	2-3	1-2		
5016.34		2	4	1		
5020.21		2	3-4	1-2		
5023.05		2	3	1		
5025.03		3	4-5	1		
5025.75		1	2	1		
5036.64		2	4	1-2		
5038.58		2	3	1		
5040.14		3	4	1-2		
5064.84		3	4-5	2		
5087.24		o	2	1-2		
5147.65		o	2	1-2		
5173.92		2	3	1-2		
5186.07		2	1	-1	2	
5193.14		2	3	2		
5210.56		3	4-5	2		
5219.88		o	3	1		
5238.74		oooN	2	1-2		
5266.14		o	1	1-2		
5282.58		oo	1	o-1		
5336.97	*Ti*, —	4	3	-2	3	
5426.47	—	oo	3	1-2		H.
5460.72	—	oo	3	1		H.
5471.41		ooo	2	1-2		
5474.44	*Ti?*	oo	1	1		
5477.90		oo	1-2	1		
5482.08	—	oo	1-2	o-1		H.
5490.37		o	3	1-2		
5490.90	—	o	2	n.c.		H.
91.04	—	ooo				
5504.12		o	1	1		
5512.74		2	4	2		
5514.56		2	3-4	2		
5514.75		2	3	2		
5523.47		ooo	o	1		
.55		ooo				
5565.70		oo	2	1-2		
5644.26	—	oo	2-3	2		
.37		o				
5648.80		oo	2	1		
5662.37		o	2-3	1		
5680.15	—	ooo	o	1		H.
5702.88		ooo	o-1	1-2		
5716.67		oo	2	1-2		
5720.67	*Ti*, A	o	2	1		
5739.70	—	o	1-2	o-1		H.
5740.20	—	o	1	o-1		H.
5766.55		o	1-2	1		
5774.25	*Ti*, A	o	1	1		
5786.19	*Ti, Cr*	oN	2	o-1		
5804.48		o	1-2	1		
5866.68		3	5	2		

TABLE I—*Continued*

CHROMIUM

λ Rowland	Identification Rowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhancement Spark	Remarks
4339.62		4	5	2		
4351.22		3	4	2		
4371.44		2	3–4	1–2		
4373.42		1	3	1		
4540.67		2	3	1		
4555.16	—	2	1	–1	1	L.
4558.83	*Cr?*	3	2	–1	4	
4571.85		1	1–2	n.c.		
4580.23		3	4–5	1		
4588.38	—	3	2	–1	3	H., L.
4600.93		3	5	2		
4616.30		4	6	2		
4618.97	*Fe*—	4d ?	3		1–2	L.
4626.36		5	7	1–2		
4646.35		5	6	2		
4651.46		4	6	1		
4652.34		5	7	2		
4756.30		2	3–4	1–2		
4789.53		2	3	1–2		
4824.32	*Fe*	3	2	–1	4	L.
4829.55		2	3	1		
4836.42	—	0	00		1	L.
4848.44	—	2	1		3	H., L.
4862.03		0	2	1		
4887.19	*Ni, Cr*	2	2–3	1		
4936.51		1	2	0–1		
4942.66		2	3	1		
5073.11		1	2	1		
5123.64		000	0	1		
5144.85	*Cr, C*	00	1	0–1		
5166.45	*Cr–Fe*	3	4	1		
5227.04	*Fe–Cr*	3	3–4	1		
5230.38	*Co, Cr*	00	0–1	0–1		
5237.49	*Cr?*	1	0–1		3	
5239.14		00	1	n.c.		
5243.53		00	0–1	0–1		
5247.74		2	3–4	2		
5264.33		4	10	2		
.42	*Ca*	3				
5275.93		1	1–2	1		
5296.87		3	5	1–2		
5298.46		4	6	2		
5300.93		2	3	1–2		
5335.05	*Co*	1	0		2	
5345.99		5	6–7	2		
5348.51		4	5	1–2		
5410.00		4	6–7	3		
5442.63		00	0–1	n.c.		
5620.72	*Fe*	0	00		2	

TABLE I—*Continued*

λ Rowland	Identification Rowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhancement Spark	Remarks
5694.96		0	1	1		
5702.54		0	0–1	1		
5712.99		0	1	1		
5783.29		2	2–3	0–1		
5784.08		3	4	1		
5785.19		2	3	0–1		
5785.95		1	2–3	n.c.		
5788.14		4	5	1		
			VANADIUM			
3910.98	*Fe–V*	4	5	1		
4090.73		1	2	1		
4099.94		2	3	1		
4105.32		2	3	1		
4115.33		3	4	1		
4116.63		1 }	2–3	1–2		
.71	*V, Fe?*	0 }				
4128.25	*Ce–V,–*	6d	7	1		
4232.76		00	1	0–1		
4330.19		0N	2	1–2		
4332.99		0	2	2		
4341.17		0	2	2		
4379.40		4	5	2		
4390.15		2	3	2		
4392.24	*V?*	1N	2	n.c.		
4395.41	*V, Zr*	2	3	1–2		
4400.74		1	2	1		
4406.81	*V–*	2	4	2		
4416.64		0	2	1		
4421.73		0	2	1–2		
4428.71	*V–Cr*	1d?	2	1		
4436.31		0	2	1–2		
4438.01		0	2	2		
4441.88	*V–*	3Nd?	5	2		
4444.57	*V–Ti*	00	2	1–2		
4452.17		0N	2	0–1		
4459.92		1	3	2		
4545.51	*Cr–V*	0	1	1–2		
4577.36		0	2	2		
4580.59		1	2	2		
4586.55		1	2–3	2		
4594.30		2N	4	2–3		
4635.35		00N	1	1		
4831.83		00	2	2		
4832.62		00	2	2		
4851.69	*Ca, V*	1	3	2–3		
4864.92		0	3	3		
4875.67		1	3	3		
4881.74		1N	3	3		

TABLE I—*Continued*

λ Rowland	Identification Rowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhancement Spark	Remarks
5176.95		000	0	n.c.		
5625.10	—	000	0–1	1–2		H.
5626.25	—	000N	1–2	n.c.		H.
5627.86		00	3	2		
5632.67	—	000Nd ?	0	0–1		H.
5646.32	—	000N	1–2	1–2		H.
5657.67	—	000	1–2	1		H.
5668.59		000	2	1		
5671.07		0	3	1–2		
5698.75		1	4	2		
5703.80		1	3	2		
5707.20		0 }	3	1–2		
.26	*Fe*	1 }				
5727.27	*Ti–V*	2N	4–5	2		
5727.87	—	00	3	1–2		H.
5731.44	—	00	3–4	1–2		H
5734.26	A ?	000	1	n.c.		H.
5737.29	—	0	3	1–2		H.
5743.64	—	00	3	2		H.
			IRON			
4258.48		2	3–4			Widening probably due to 58.64 = Hasselberg's *Ti* 58.68
4508.46	*Fe ?,–*	4	3		2	
4522.80	—	3	2	–1	2	L., Re.
4556.06	—	3	2		1	L., Re.
4584.02	*Fe–*	4	3	–1	3	
4603.13		6	7–8	1		
4630.31		4	5	n.c.		
4733.78		4	6	1		
4924.11		5	4	–1	3	
5018.63		4	3	–1	3	
5022.41		3	2	–1		
5110.57		5d	6–7	1		
5169.07		3 }	5–6	–2	4	
.22		4 }				
5198.89		3	4	0–1		
5218.37		1	0		2	Enhancement in spark may vary
5225.70		2	3–4			Widening probably due to 25.88 = Rowland's *V* 25.92
5250.82		3	4	0–1		
5269.72		8d ?	10	2		
5276.17	*Fe ?*	3 }	4	n.c.	0–1	
.24	*–Cr ?*	2 }				
5316.70		4	2–3	–1	2	
5328.24		8d ?	9	1		
5370.17		6	5	–1		
5371.66	*Cr ?*	4 }	10	2		
.73		3 }				
5373.90	*Fe, Cr*	2	3	0–1		

TABLE I—*Continued*

λ Rowland	Identification Rowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhancement Spark	Remarks
5381.22		2	1			*Ti* line 5381.20 is enhanced 3
5397.34		7d ?	8–9	1–2		
5404.36		5	4	–1		
5405.99		6	7	1–2		
5429.91		6d ?	7–8	2	0–1	
5447.13		6d ?	8–9	2	0–1	
5455.67	*Fe ?*	2	7	1		
.83		4				
5497.74		5	7	1		
5501.68		5	6	0–1		
5507.00		5	6–7	1		
5586.99		7	8	0–1		
5712.10		3	6			Hasselberg gives *Ti* at 5712.07
MANGANESE						
4762.57		5	6	1		
4823.70		5	6–7	1–2		
5255.49		0d ?	0–1	n.c.		
5377.80		2N	1	–1	0–1	
5394.84		1	4–5	2		
.91		1				
5407.59		0	2	1		
.69		0				
5420.51		0N	3–4	1–2		
.61		0N				
5432.75		1Nd ?	3–4	2		
5470.80		0	3	1		
.88		0				
5506.10		1	1–2	n.c.		
5516.95		0	2	1		
17.03		0				
5537.93		00	2	1		
38.02		00				

abbreviation "H", under "Remarks," in the last column of our tables, indicates that Hasselberg gives a line at this point which we consider to be identical with the solar line in question. Similarly "L" and "Re" refer to the tables of enhanced lines published by Lockyer and Baxandall, and Reese, respectively. The spot intensities of the lines for the region extending from the violet to λ 5000 have been determined from photographs of the spectrum of the large spot of the latter part of June. The remainder are obtained from a combination of results from these plates with those from plates of numerous small spots taken previously.

TABLE II

"ENHANCED" LINES IN ARC AND SPARK

TITANIUM

λ Rowland	Identification Rowland	Solar Intensity	Amount Enhancement Spark	Amount Decrease Weak Arc	Remarks
3722.73	*Ti–Fe*	6	2	1	
3757.82	*Cr–Ti*	4	3	2	
3759.45		12d ?	5	2	
3761.46		7	5	1–2	
3900.68	*Ti–Fe*	5	5	1	
3913.61	*Ti–*	5d ?	5	1	
4012.54	*Ti, Ce*	4	2	0–1	
4028.50	*Ti–Ce*	4	3	1	
4053.98	*Cr–Fe–Ti*	3	3	n.c.	
4161.68	—	4	1–2	0–1	H.
4163.82	*Cr–Ti,–*	4	5	2	
4172.07	*Ti, Fe*	2	5	1	
4290.38		2	3	1	
4294.20		2	2	2	
4300.21		3	4	2	
4302.08		2	1–2	1	
4313.03		3	3	2	
4314.96		1 }	2	1	
15.14		3 }			
4316.96	*Ti?*	1	1	1	
4338.08		4	3	2	
4341.53	*Ti?*	2	1	1	
4344.45	*Ti–*	2	2	2	
4351.00		1	2	2	
4367.84		2	4	2	
4387.01	*Ti?*	1	3	1	
4395.20		3	4	1–2	
4399.94	*Ti, Cr*	3	2	2	
4411.24	*Cr–*	1	4	0–1	
4443.98		5	4	2–3	
4468.66	*Ti–*	5	4	2	
4488.49	—	1	3	1	H.
4501.44	*Ti, –*	5	4	2	
4534.14	*Ti–Co*	6	3	2	
4549.81	*Ti–Co*	6d ?	5	2	
4563.94		4	4	2	
4572.16	*Ti–*	6	5	2	
4590.13	—	3	2		H.
4805.28	—	3	2	0–1	H.
4911.37	—	1	4		H.
5072.48		0	1		
5129.34	*Ti?*	3	3	1–2	
5154.24	*Ti–Co*	2	1	1	
5186.07		2	2	1	
5188.86		2	5	1	
5226.71	*Ti–*	2	4	2	
5336.97	*Ti,–*	4	3	2	
5381.22	*Fe*	2	1–2	1–2	H.
5418.98	*Ti?*	1	1–2	1	

TABLE II—*Continued*

CHROMIUM

λ Rowland	Identification Rowland	Solar Intensity	Amount Enhancement Spark	Amount Decrease Weak Arc	Remarks
3603.83	—	2	3	1	H.
3678.04	—	2	4	n.c.	H.
3715.32	—	2	3–4	1	
3865.67	*Fe–C*	7	3		H.
3979.66	*Nd–Co*	4	1–2	1	L.
4012.63		0	0–1	1	
4145.91	—	1N	3		L.
4179.41	*Cr, Co*	0	1	n.c.	
4225.02	—	2N	1–2		L.
4242.54	—	2	3		L.
4284.38	—	2Nd?	1	1	L.
4555.16	—	2	1	1	L.
4558.83	*Cr?*	3	4	1	
4588.38	—	3	3	1	H.
4592.23	*Cr–*	1	0–1		
4634.25	—	2	1	1	H.
4824.32	*Fe*	3	4	1	L.
4848.44	—	2	3		L.
4876.59	—	1	2		L.
5237.49	*Cr?*	1	3		
5335.05	*Co*	1	2		
5620.72	*Fe*	0	2		

VANADIUM

λ Rowland	Identification Rowland	Solar Intensity	Amount Enhancement Spark	Amount Decrease Weak Arc	Remarks
3899.28	—	2	4	2	H.
3903.40	*V–Ce?*	2	4	2	
3916.55	—	3	3	2	H.
3952.10	*Mn, V*	2	4	2	
3968.24			2	1	Hasselberg's λ
3973.80	*Nd, V, Fe*	1	3	2	
3997.26	*Cr?*	1	2	1	H.
4003.08	—	2	2	1	H.
4005.86	*V*	3	5	2	
4023.53	*V, Co*	3	4	2	
4035.77			4	2	Hasselberg's λ
4036.92	*V,–*	1	1	1	
4202.51	—	0Nd?	2	1	H.
4205.24	—	1	2–3	1	H.
4225.38	—	0	2	1	H.

IRON

λ Rowland	Identification Rowland	Solar Intensity	Amount Enhancement Spark	Amount Decrease Weak Arc	Remarks
4173.62	—	3	0–1	0–1	L.
4179.02	—	3	0–1	1	L.
4219.52		4	1	0–1	Re.
4233.33	*Mn*	4	1–2		L., Re.
4303.34	—	2	1		L., Re.

TABLE II—*Continued*

λ Rowland	Identification Rowland	Solar Intensity	Amount Enhancement Spark	Amount Decrease Weak Arc	Remarks
4385.55	—	2	1		L., Re.
4433.39		3	0–1	1	
4466.73		5	1	0–1	
4508.46	*Fe?*,–	4	2		
4515.51	—	3	2	1	L., Re.
4520.40	*Fe?*,–	3	1	0–1	
4522.80	—	3	2	1	L., Re.
4541.48	—	0	1		L., Re.
4549.64		2	3	1–2	
4556.06	—	3	1		L., Re.
4576.51	—	2	0–1		L., Re.
4584.02	*Fe*—	4	3	1	
4789.85		3	0–1	n.c.	
4924.11		5	3	1	
5018.63		4	3	1	
5169.07		3 }	4	2	
.22		4 }			
5276.17	*Fe?*	3	0–1	n.c.	
5316.79		4	2	1	
5429.91		6d?	0–1	–2	
5447.13		6d?	0–1	–2	

The second series of tables consists of a comparison of the intensities of the lines of these elements which are considerably enhanced in the spark, with their intensities in the weak arc. Most of these lines which occur in the less refrangible part of the spectrum are diminished in spots, and such lines will appear in both sets of tables. As is well known, however, the great majority of strongly enhanced spark lines occur in the violet and ultra-violet, where the spot lines seem to have the same intensity as the Fraunhofer lines. Accordingly, independent lists of these lines have been added, since the evidence afforded by them as to the relation of spark to weak arc is of extreme importance. Almost all of these lines are unidentified in Rowland's table—a result which is of course due to his use of the arc lines for purposes of identification. Consequently we have been obliged to identify many of them, and in the course of this work have used the tables of Hasselberg and the lists of enhanced lines by Lockyer and Baxandall, and by Reese, to great advantage. References to these lists are made in the same way as in the preceding tables.

We have attempted throughout these tables to include only those

cases of blends in which the element under which they are classified in each instance seems to be the predominant factor in producing the effect.

DISCUSSION OF THE TABLES

Before entering upon a discussion of the evidence afforded by these tables, it will be well to indicate to what extent they are to be regarded as comprehensive, and to examine the character of the lines not appearing in them. At the outset the important statement should be made that we have found very few lines which are certainly strengthened or weakened in the low-current arc that are not either spot lines or lines enhanced in the spark. There are, of course, in spot, in weak arc, and in spark many instances near the limits of accuracy of estimates of intensity, but for unquestionable cases the statement above seems to be true. To what extent the converse, that all spot lines are affected in the weak arc, the spark, or both, fails to hold, the tables indicate, and this side of the subject we shall discuss later in this paper.

The lines which are affected in spots and which do not appear in these lists fall into two classes. The first consists of lines so faint that they fail to appear upon our laboratory photographs, or are too weak to furnish quantitative results. The second class consists of lines for which the means of our determinations of change of intensity fall below what we consider the limits of accuracy. In the second class, accordingly, there should be no lines which are prominently affected in our photographs of spot spectra, and we believe this to be the case. The majority of the lines which are too weak upon our photographs to admit of satisfactory discussion belong to iron and chromium. Since any marked cases of strengthening in the low-current arc as compared with the strong arc would be visible for most of these lines, it is clear that such indirect evidence as they afford is in agreement with the fact that the fainter spot lines belonging to these elements are inconspicuous. The evidence furnished by lines enhanced in the spark, for which the absence of an entry in the column "Amount Decrease Weak Arc" indicates that the line is not visible in the weak arc, is considerably more important. Since most of these lines can be seen in the strong arc, it is clear that they are weakened in the low-current arc and that only quantitative data are lacking.

In order to simplify the analysis of these tables we shall discuss separately the cases of lines strengthened in spots, and those weakened in spots. An inspection of the results for lines of the first sort gives the following summary:

TABLE III

Element	No. of Lines Strengthened in Spots	No. of Lines Strengthened in Weak Arc	Unchanged Weak Arc
Ti	88	83	5
Cr	46	42	4
Fe	19	18	1
V.	56	52	4
Mn . . *t*	11	9	2
	220	204	16

The relatively small number of *Fe* lines in this summary is due to three causes: first, numerous *Fe* lines are weakened in spots; second, a large number of lines are only very slightly affected, if at all; third, many *Fe* lines are very faint in the arc spectrum, and do not appear on our plates.

The evidence afforded by this comparison of results certainly indicates very strongly the connection of spot lines with those strengthened in the weak arc. The exceptions form but seven per cent. of the total number, and it seems probable that many of these may be accounted for either by misidentification in Rowland's table or by the presence of close companions in the solar spectrum to which the change of intensity in the spot is due. In the table for iron three cases of the latter kind are indicated by notes, and no doubt others exist.

The relation of the lines weakened in spots to those in the spark and the low-current arc is one of the most interesting results of this investigation. The following brief table gives a summary for such lines:

TABLE IV

Element	No. Lines Weakened in Spot	No. These Enhanced in Spark	No. Not Changed in Spark	No. Diminished in Weak Arc	No. Not Seen in Weak Arc	No. Not Changed Weak Arc
Ti	8	8	0	8	0	0
Cr	10	10	0	6	4	0
Fe	13	10	3	9	3	1
Mn	1	1	0	1	0	0
	32	29	3	24	7	1

We have not included in the above list the interesting case of the line λ 5381.22. This line, identified by Rowland as *Fe*, is diminished in spots. No line is given here in the arc tables of Kayser and Runge, nor does it appear upon spark plates of this region. There is, however, a line of *Ti* given by Hasselberg at λ 5381.20 which is enhanced 3 in the spark, and this is probably identical with the solar line.

Further important evidence as to the relation between weak arc, strong arc, and spark is furnished by a number of lines which lie so far to the violet that they apparently are beyond the range of lines affected in sun-spots. This region is particularly rich in lines enhanced in the spark, and it has seemed desirable to consider here also the changes produced in passing from the strong arc to the low-current arc. The following summary derived from Table II includes the most important of these lines. For the sake of completeness the lines comprised in Table IV are also included.

TABLE V

Element	No. Lines Enhanced in Spark	No. These Diminished in Weak Arc	No. Not Seen in Weak Arc	No. Not Changed in Weak Arc	No. Strengthened in Weak Arc
Ti	48	44	3	1	0
Fe	25	14	7	2	2
Cr	22	10	10	3	0
V	15	15	0	0	0
Mn[1]	1	1	0	0	0
	111	84	20	6	2

As stated before, we consider the evidence afforded by enhanced lines which fail to appear in the weak arc to be confirmatory of the conclusion that these lines are, in general, diminished in passing from strong to weak arc.

The two lines of iron which appear in the above table as increased in intensity both in spark and in weak arc are the strong yellow lines λ 5429.91 and 5447.13. Both lines are considerably increased in the weak arc, and both are strengthened in sun-spots. We are obliged at present to regard these lines as exceptions. Attention

[1] Manganese has no prominently enhanced lines in the region which we have investigated. The single line given occurs in the yellow, and it is but slightly affected.

should, however, be called to the fact that both are double in the Sun and probably in the arc as well. This may perhaps account for their contradictory behavior in the spark and the weak arc. The contradiction in the case of spots would, however, still exist, since visual observations in the third order of the 18-foot spectroscope have shown that the amount of strengthening of the two components for each of the lines is about the same, so far as can be judged with the resolving power available.

A consideration of the quantitative relations of the enhanced lines of Tables I and II shows that the general agreement between the amount of enhancement in spark and of decrease of intensity in low-current arc is reasonably good, in view of the obvious difficulties in the way of making a comparison on account of the marked difference in the magnitude of the effect in the two cases. The agreement between the amount of decrease in weak arc and in spot is still better, apparently, but too much emphasis should not be laid on this fact, since the range of effect is relatively small. The behavior of silicon, which we have not investigated as yet, should be especially interesting in this respect, its lines being weakened remarkably in sun-spots.

There remains one other feature of this investigation to which attention should be called. This has to do with the negative evidence and serves as a valuable check upon the results obtained. It is evident that in cases in which the intensities of lines in separate spectra are to be compared with each other the best criterion for the correctness of exposure time is given by the lines which are not affected in the two cases. In the present instance we have selected at random from the tables of arc lines for the various elements a series of prominent lines which are not affected in sun-spots. The behavior of these lines in the weak arc and in the flame has then been examined, and the results are indicated in Table VI.

The lines in the "doubtful" column all show some evidences of weakening, but the amounts are at the limit of accuracy of determination and of slight value.

This table affords two kinds of important testimony. The first bears upon the general validity of our hypothesis, which assumes the existence of lines which are affected neither in spots nor in the

TABLE VI

Element	No. of Lines	Strengthened in Weak Arc	Unchanged	Doubtful
Ti..........	37	0	32	5
V...........	39	0	38	1
Fe..........	30	0	26	4
Cr..........	27	0	20	7
Mn..........	19	0	17	2
	152	0	133	19

weak arc. The second goes to show that our quantitative values cannot be affected in any considerable degree by errors in the relative exposure times of our spectra.

In concluding this preliminary discussion of the observational materials it is well to call attention to one or two essential difficulties in the way of agreement between laboratory and solar results, arising from the nature of sun-spot spectrum photographs. The first of these is that in the sun-spot spectrum the blending of close lines or the superposition of lines of different elements is very liable to hide the true behavior of those under consideration. This is especially true in the blue and violet regions of the spectrum, where the number of lines in the solar spectrum becomes very great. The second difficulty arises from the fact that the number of lines shown to be affected upon a sun-spot photograph depends upon a variety of conditions, in particular the character of the definition at the time, and the size of the sun-spot. Accordingly, until a very large amount of observational material shall have furnished what will actually be a definitive list of sun-spot lines (assuming that the sun-spot spectrum itself does not vary essentially), it will be better to confine comparisons of sun-spot and laboratory results to the more important lines in each case.

TEMPERATURE AS THE PROBABLE CAUSE OF THE OBSERVED PHENOMENA

From the above discussion of the lines in our tables it appears that, in general, (1) the lines which are strengthened in spots are strengthened in the 2-ampere arc; (2) the lines weakened in spots are weakened in the 2-ampere arc; (3) the lines weakened in the 2-ampere arc are "enhanced" in the spark.

Although our work must be carried much farther before final conclusions can be drawn, it nevertheless seems probable that the observed differences of intensity, both in the arc and spark and in sun-spots, may be adequately accounted for by temperature differences. Accordingly, the simplest way of accounting for the characteristic phenomena of spot spectra seems to be on the hypothesis that the metallic vapors in the spot are cooler than the corresponding vapors in the reversing layer.

However, the production of enhanced lines has been ascribed to such diverse causes that we shall defer judgment as to the arc and spark changes until our experiments can be repeated with an electric furnace. The present discussion is consequently to be regarded as a preliminary one, serving to indicate why we are inclined to ascribe the observed phenomena to the effect of varying temperature.

1. In his paper on the synchronous arc Crew states unequivocally: "Increase of phase in these experiments undoubtedly means increase of temperature."[1] The gradual weakening and finally the complete disappearance of the carbon flutings as the phase was reduced; the decreasing intensity of H and K, and the simultaneous increase of the blue calcium line; the increasing relative intensity of the *Al* pair, as compared with the neighboring H and K lines; and the resemblance of the iron spectrum at zero phase to the flame spectrum; are typical of the phenomena observed by Crew. All of these criteria, and others as important, apply as well to our weak arc plates as to those obtained by Crew with the synchronous arc.[2] From this standpoint, therefore, there is no conflict of testimony: Crew's conclusion as to the comparatively low temperature of the low-phase synchronous arc should apply also to the 2-ampere direct-current arc, at least in so far as it depends on the above criteria.

2. In their investigation on the temperature of the arc, Waidner and Burgess found the temperature of the crater to be reduced 70° when the current was reduced from 30 to 15 amperes. They give no results for smaller currents. In view of the fact that the relative intensities of the lines undergo no material change in passing from

[1] *Astrophysical Journal*, **22**, 201, 1905.

[2] Although the carbon flutings do not disappear in the 2-ampere arc, they are considerably weakened.

30 to 15 amperes, while the change between 30 and 2 amperes is very pronounced, it is probable that the temperature of the crater is considerably reduced at 2 amperes. As for the arc itself, the great reduction in brightness, demanding exposures of four minutes at 2 amperes as compared with 12 seconds at 30 amperes, is strong evidence in favor of a lower temperature, since we know that the total radiation varies as the fourth or fifth power of the temperature. It should be recognized, however, that the change in brightness may be due in considerable part to the change in the amount of luminous vapor.

3. Since the enhanced lines of the spark appear with lower intensity in the 30-ampere arc, and are still further reduced in passing to the 2-ampere arc, no explanation hitherto advanced to account for these lines appears adequate in the present case, unless it be the explanation based on change of temperature. However, the fact that for the observed lines the changes of intensity with current strength are the reverse of those described by Hartmann in certain other cases, indicates the necessity of caution. For example, Hartmann found the enhanced line *Mg* λ 4481 to be greatly strengthened in the arc, as the current was reduced from 8 to 0.4 amperes. He concluded that "the condition for the development of those molecular vibrations to which the line λ 4481 corresponds were much more favorable in the small arc in spite of its lower, or at least certainly not higher, temperature than in the larger arc.[1] In order to keep the 0.4-ampere arc burning, it was necessary to start it several hundred times during the exposure. Hartmann considers it probable that λ 4481 was produced only at the moment of make or break. Sir William and Lady Huggins also ascribe the production of this line to a sudden discharge,[2] and Crew concludes that in a constant voltage circuit small currents, which heat the electrodes less, will result in a higher voltage between the electrodes, and probably a much quicker break than larger currents. According to him, the essential condition for the appearance of spark lines in arc spectra is a high and rapidly changing E. M. F.[3] It may be mentioned here that our exposures to the weak arc were never commenced until it

[1] *Astrophysical Journal*, **17**, 273, 1903.

[2] *Ibid.*, **17**, 145, 1903. [3] *Ibid.*, **20**, 281, 1904.

was burning steadily, after the poles had been separated sufficiently to prevent all light except that from the arc proper from entering the short slit.

4. The behavior in stars of the lines affected in sun-spots appears to be consistent with the view that temperature changes alone are sufficient to account for their variation in intensity.[1] In *a Orionis*, which we have other reasons (such as the great strength of the blue calcium line and the presence of titanium oxide flutings) to regard as much cooler than the Sun, lines that are strengthened in sun-spots are still further increased in intensity.[2] In *Arcturus*, which is always assumed to have a temperature between that of *a Orionis* and the Sun, the intensities of the lines agree remarkably with those observed in sun-spots.[3] We hope in a future paper to discuss this subject more fully, especially in its bearing on stellar evolution and the classification of stellar spectra.[1]

5. A bolographic study of spot spectra, on which Mr. Palmer is engaged, indicates that the maximum of intensity is very considerably shifted toward the red, as compared with its position in the spectrum of the photosphere. This result is thus in harmony with the view that the temperature of the vapors in spots is lower than that of the photosphere.

6. The application of our results to chromospheric lines may prove to be of importance, but we are not yet in a position to enter into a full discussion of the subject. In a recent paper[4] Fowler has called attention to the weakening of a number of enhanced lines of iron, titanium, and chromium in spots, these lines having been taken from a list of "long or high-level" chromospheric lines. Our observations of the large spot of June, which was particularly favorable for determinations of this sort, give results in close agree-

[1] Formerly we were inclined to the view that the presence of spot lines in the spectra of red stars indicated the presence of spots like those on the Sun. Our recent work has led us to the opinion that the comparatively low temperature of these stars offers the simplest explanation of the observations.

[2] Hale and Adams, *Contributions from the Solar Observatory*, No. 8, *Astrophysical Journal*, **23**, 400–405, 1906.

[3] Adams, *Contributions from the Solar Observatory*, No. 12, *Astrophysical Journal*, **24**, 69–77, 1906.

[4] *Monthly Notices*, **65**, 361, 1906.

ment with his for these lines. From t
present communication, however, we s
of these lines to be explained much more s
a temperature basis than on any which invo
they originate at a higher level in the chromosph
lines of the spectrum. In regard to this question,
no occasion to differ from Evershed's conclusion[1] that
evidence of differences in the relative intensities of the lines of an element in the higher or lower regions of the flash layer, and the enhanced lines appear to predominate throughout the entire depth of the radiating stratum."[2]

SUMMARY

1. This paper describes a preliminary study of the more important sun-spot lines in the region above λ 5800, belonging to titanium, chromium, iron, vanadium, and manganese—the metals most characteristic of sun-spots.

2. Over ninety per cent. of the lines in our tables, which are strengthened in sun-spots, are found to be strengthened in passing from a 30-ampere arc to a 2-ampere arc.

3. Over ninety per cent. of the lines shown by our tables to be weakened in sun-spots are weakened or absent in the 2-ampere arc.

4. Over ninety per cent. of all the "enhanced" lines included in our tables are weakened or absent in the 2-ampere arc.

5. In a list selected at random of 152 lines which are not spot lines no cases were found of lines strengthened in the low-current arc or in the flame.

6. We are not yet ready to express a final opinion, but are inclined to the view that temperature differences are adequate to account for the above phenomena. Our reasons for this view may be summarized as follows:

[1] "Solar Eclipse of 1900, May 28," *Philosophical Transactions*, Series A, **201**, 477.

[2] Bright lines, due to overhanging flocculi or to eruptive phenomena, frequently appear in spot spectra. Indeed, H_3 and K_3 are always present. The weakening of the enhanced lines in spots, however, should in our opinion be classed with the strengthening of other lines, and attributed to the same cause.

a) The similarity of the spectroscopic phenomena of the weak arc to those of the low-phase synchronous arc, held by Crew to correspond to a low temperature.

b) The probable decrease in the temperature of the arc with decreasing current strength.

c) The behavior of the enhanced lines in the 2-ampere arc.

d) The presence of sun-spot lines in red stars.

ADDENDUM

When the above paper was written, it was not supposed that the competence of temperature differences alone to account for the observed phenomena could be tested at present. It was subsequently decided, however, to construct an electric furnace for immediate use in our Pasadena laboratory, where sufficient current is available. A Littrow spectrograph was also built for observations with the furnace. This is supplied with a combined collimating and camera objective, by Zeiss, of 6 inches (15 cm) aperture and 18 feet (5.49 m) focal length, and a Rowland plane grating, with ruled surface 5×3.75 inches (12.7×9.5 cm), having 15,000 lines to the inch (5,900 to the cm), for the use of which we are indebted to the kindness of Professor Ames.

In addition to our work with the furnace, which leaves no doubt that the arc phenomena can be accounted for as effects of temperature, we have obtained independent evidence, by observations of the outer portions of the flame of an ordinary arc, which leads to the same conclusion. In this investigation the metal under consideration, in the form of a powder, was placed in the positive pole. The vapor in the long flame which rises from such an arc is undoubtedly of lower temperature than that between the poles. Photographs of the spectrum of the flame show the bands of the oxide, and give other evidences of low temperature, such as the strengthening of the blue calcium line, which is more intense than H and K. The spectrum of the flame, compared with that of the core of the arc (between the poles), shows changes of line intensity similar to those observed with the 2-ampere arc and the synchronous arc.

In order to test fully the important question of the identity of these variations, we have made an extensive series of comparisons

The more important of these objections are as follows:

1. The comparative absence in sun-spots of strengthened and weakened lines in the blue and violet portions of the spectrum. While it is true that the differences in the relative intensities of lines in the weak arc and in the strong arc become as a rule less in the more refrangible parts of the spectrum, there are still some marked cases of strengthening in the weak arc of lines which are not affected in sun-spots. Perhaps the most important instances of this sort are the three *Cr* lines λ 4290, λ 4275, and λ 4255. All of these lines are decidedly strengthened in the weak arc and the flame, but are not affected in spots.

To the same category of lines belongs the blue line of *Ca* at λ 4227. This increases in intensity with a diminution of temperature. It does not, however, seem to be affected in spots.

In the case of enhanced lines the matter is still more striking. As is well known, the strongest enhanced lines of nearly all of the elements are found in the ultra-violet. As against this is the fact that the most refrangible line which we have yet observed upon our photographs as unquestionably affected in sun-spots is λ 3906.

Though we have at present no sufficient explanation for this condition of affairs, there are one or two considerations which seem to bear upon it. One of these is the striking similarity in this respect of a photograph of an abnormal solar spectrum obtained in 1894 at the Kenwood Observatory.[1] On this plate the character of the solar spectrum from λ 4100 to λ 3900 is totally changed. Beyond λ 3900, however, the spectrum gradually returns to the normal solar type. While the analogous behavior of sun-spot spectra may be no more than a coincidence, the similarity calls for remark. Furthermore, in view of the fact that certain elements, such as vanadium, manganese, and, in general, titanium, show very close agreement with our hypothesis, while others show important exceptions, the possibility is suggested that the various elements are differently distributed in spots, and that the vapors are not all of the same temperature.

A tentative explanation of the above difficulty might be founded on the hypothesis that the vapors producing the greater part of the characteristic changes in sun-spot lines occur at the base of the

[1] *Astrophysical Journal*, 16, 211–233, 1902.

reversing layer, where they may form a comparatively thin stratum. Consequently, in the more refrangible part of the spectrum, the absorption of the overlying vapors may be so great as to mask almost completely the selective absorption of this low layer. From this point of view the spectrum of a spot in the more refrangible region would be due mainly to the higher vapors, and would show few differences from the ordinary solar spectrum. The very great falling off in the intensity of the spectrum of the spot as compared with disk in the violet region, where the intensity is only about one-third to one-fifth as great as in the yellow and green regions, probably indicates the action of powerful general absorption. On the above hypothesis this absorption is assumed to be sufficient practically to obliterate the effects of selective absorption, the vapors producing the latter being assumed to be restricted in depth.

2. Some lines in the less refrangible region of the spectrum seem to be strengthened in the spark at the same time that they are strengthened in the flame and in spots. The principal cases of this sort are the iron lines λ 5430 and λ 5447.

In the same region of the spectrum occurs the titanium oxide fluting, the presence of which in third-type stars is recognized as an evidence of comparatively low temperature. This fluting is found in the spectrum of the strong arc as well as in the flame and the weak arc. Its apparent absence in the spectrum of sun-spots probably indicates a higher temperature for them than for the laboratory sources.[1]

3. The changes in the relative intensities of lines in the flame do not in all cases harmonize in amount with those observed in spots. Examples of this sort are evident from an inspection of the tables. Examination of these tables also furnishes some indication of a greater amount of strengthening in the low-current arc of the strong lines than of the weak lines. The evidence is by no means to be considered conclusive, since the exceptions are numerous and important. That the effect, however, is not due to photographic or physiological causes is shown by its persistence upon plates taken with widely

[1] Since the above was written we have found the titanium oxide fluting, which begins at λ 5597.9, in one of our photographs of a sun-spot spectrum. Two maxima of the fluting are clearly visible. It is now probable that other titanium oxide flutings will be found on our plates.

varying exposure times. If genuine, it is identical with that claimed by Fowler to exist in the case of spot lines, namely, that such lines are in general affected in spots in proportion to their absolute intensities.[1] This conclusion we had occasion in a former communication to oppose,[2] and it certainly is not borne out by the results for at least two of the elements most prominent in sun-spots, titanium and vanadium. The question of the existence of such an effect for lines strengthened in the low-current arc and in the flame we shall hope to settle definitely at an early period in our future investigations.

In this connection, however, it is well to call attention to the fact that in the laboratory work and in sun-spots we are dealing with what are probably considerably different absolute temperatures. Consequently the different increment of change at different absolute temperatures may well account for some of the discrepancies mentioned above. If it were possible to determine by laboratory experiments the temperature of the spot vapors, further work along this line would be much simplified. Assuming the correctness of our hypothesis, such a method of determining this temperature, though perhaps subject to serious practical difficulties, would be by establishing the equality in the laboratory of a pair of sun-spot lines, one of them an enhanced line, which have been observed as of equal intensity. Since enhanced lines diminish in intensity as the temperature falls, while the other lines increase, equality of intensity in the laboratory should serve to determine the temperature in question.

4. As it seems to be the opinion of several observers that enhanced lines may be produced, at low temperatures, by suitable electrical means, this phase of the subject will require further investigation. Our hypothesis does not maintain, however, that temperature differences offer the *only* means of explaining the observed changes in metallic spectra. It merely assumes that variations of temperature afford a simple and satisfactory way of accounting for the changes in the relative intensities of the spot lines, as observed in the laboratory, in sun-spots, and in stars.

5. The hydrogen lines, as far as $H\delta$, are weakened in our photographs of spot spectra. This fact, taken in conjunction with the

[1] *Monthly Notices*, **65**, 205–218, 1905.

[2] *Astrophysical Journal*, **23**, 11–44, 1906.

great strength of the hydrogen lines in the chromosphere and in the first-type stars (where the enhanced lines are strengthened), and their weakness in red stars, might be made the basis of an hypothesis to account for the weakening of the enhanced lines in red stars and in spots. For Hartmann has shown that the enhanced lines are strengthened in an arc in water, because, in his opinion, of the hydrogen produced by its decomposition. A similar effect of hydrogen atmosphere has been observed by Crew with a rotating arc, but he ascribes the production of the spark lines to a more rapid break, caused by the presence of the hydrogen, which introduces an extra electromotive force.[1] We have observed the spectrum of a 110-volt continuous-current arc in hydrogen between fixed metallic poles. As there was no break, Crew's explanation does not seem adequate to account for the apparent strengthening of the enhanced lines on our photographs. If the results of this preliminary experiment are confirmed by later work, the hydrogen hypothesis may require further investigation.

It may be recalled that in his paper on "The Evolution of Solar Stars"[2] Schuster suggests the possibility that the extensive hydrogen atmospheres of first-type stars might account for their high temperatures, if it could be shown that hydrogen, perhaps in some form with which we are unfamiliar in the laboratory, absorbs strongly in the infra-red. We are unaware of any evidence pointing to the existence of such absorption, and therefore do not feel at all certain that the high temperatures of first-type stars depend upon their extensive hydrogen atmospheres. Whether the presence of hydrogen can produce an increase in the temperature of arc or spark may perhaps be determined by laboratory experiments.

AUGUST, 1906

[1] *Astrophysical Journal*, **20**, 280, 1904.

[2] *Ibid.*, **17**, 165–200, 1903.

A 100-INCH MIRROR FOR THE SOLAR OBSERVATORY

By GEORGE E. HALE

I am permitted to announce that Mr. John D. Hooker, of Los Angeles, has presented to the Carnegie Institution of Washington the sum of forty-five thousand dollars, to be used to purchase for the Solar Observatory a glass disk 100 inches (2.54 m) in diameter and 13 inches (33 cm) thick, and to meet other expenses incident to the construction of a 100-inch mirror for a reflecting telescope of 50 feet (15.24 m) focal length. These expenses will include the erection of a building in which the mirror can be ground, figured, and tested; the construction of a large grinding-machine, with crane for lifting the mirror; the provision of a 54-inch (1.37 m) glass disk, to be made into a plane mirror for testing purposes; the purchase of glass disks for the various plane and convex mirrors required in the telescope, etc. The optical work will be done by Professor G. W. Ritchey and the assistant opticians employed under his direction by the Solar Observatory.

In making this gift, Mr. Hooker's desire is to secure the realization of the great possibilities in astrophysical research which a large reflector seems to offer. He has absolute confidence in the ability of Professor Ritchey to make an essentially perfect mirror 100 inches in diameter; no one could ask better evidence of this than his gift affords. He knows, also, that in several classes of work, such as the measurement of the heat radiation of the stars, and the spectroscopic study of the faintest objects, the mirror is sure to yield results fully commensurate with its great size. But he is nevertheless aware that for certain other classes of work, in which the most perfect definition is essential to the highest success, the construction of a mirror of such great aperture must be regarded as an experiment. The immense block of glass will weigh four and one-half tons—four and a half times as much as the disk of our 60-inch (1.52 m) mirror. The difficulty of providing a mounting capable of carrying it with the necessary precision is not slight. The glass is certain

to be distorted by temperature changes, which would ruin its performance if not obviated. The atmospheric conditions, even on Mount Wilson, may not be sufficiently good to permit so great an aperture to be used to full advantage. Of these and other obstacles Mr. Hooker is fully informed, and he does not underestimate their importance. But he perceives and appreciates, with the understanding of one who has himself invented and developed mechanical appliances, that experiment is necessary to progress. He therefore does not hesitate to provide the means for undertaking an optical experiment on a large scale. Let us consider its probable outcome.

In the first place, the question arises whether a sufficiently homogeneous glass disk of the required dimensions can be obtained. Our long experience with the Plate Glass Company of St. Gobain leads us to believe that no insuperable difficulty will be encountered. This old and reliable company has cast for us scores of disks, from which Professor Ritchey has made a large number of plane and concave mirrors, from the smallest sizes up to 60 inches. At present the 60-inch is receiving the finishing touches in our optical shop, and two 36-inch (0.91 m) mirrors, for testing purposes, are well advanced, in addition to several large plane and convex mirrors for the 60-inch reflector. In all of these cases the glass disks furnished by the St. Gobain Company have left nothing to be desired. The 60-inch disk, 8 inches (20.3 cm) thick, and weighing a ton, is fully equal in quality to the smaller ones. We are therefore inclined to believe, since the St. Gobain Company expresses its deliberate opinion that a satisfactory disk, 100 inches in diameter and 13 inches thick, can be produced, that they will be able to carry out the order we have given them.

As for the work of grinding and figuring, no one who has watched the progress of our 60-inch mirror would be likely to doubt Professor Ritchey's ability to accomplish this difficult task. The method of parabolizing which he has recently perfected will apply as well to a 100-inch mirror as to the 60-inch. It eliminates the necessity of any hand-work, and has already yielded a paraboloidal figure so perfect that almost any other optician would be more than contented with it. Professor Ritchey rightly believes, however, that a still higher degree of perfection will be worth attaining, since its advan-

tages will be felt under the most perfect atmospheric conditions. I am confident that he will find no difficulty in bringing the 100-inch mirror, as well as the 60-inch, to this highest order of perfection.

The mounting should offer no great obstacles, especially as it will not be designed until the mounting of the 60-inch has been thoroughly tested on Mount Wilson. Unless I am greatly mistaken, this latter instrument will meet our best expectations. Professor Ritchey has taken the greatest pains with the design, and the co-operation of the able staff of engineers at the Union Iron Works has been most useful. The mechanical execution of the parts is admirable, and with the heavy machinery available, a mounting much larger than that required for the 100-inch mirror should be easily within the bounds of possibility. To a firm which has built some of the most powerful battleships and cruisers in our navy such a mounting would appear much less formidable than to the average instrument-maker, accustomed to a different class of work. Fortunately, the ideas of the Union Iron Works Company, as to the degree of precision required, are entirely in harmony with our own, and appear to have been met in the mounting of the 60-inch, which has just arrived in Pasadena.

The prevention of change of figure due to changing temperature should not prove a very serious problem. During the fine nights of the best observing season on Mount Wilson the temperature remains almost perfectly constant after 9 P. M. It will therefore only be necessary to maintain the mirror (or possibly the entire telescope) at approximately this temperature throughout the day, by means of suitable refrigerating machinery. In the long periods of absolutely cloudless weather the change of temperature from night to night is extremely small, so that little difficulty should be encountered on this score. If the slowly falling temperature during the early evening should prove to give trouble, the observational work might be deferred until after nine o'clock. The dome and building like those designed for the 60-inch reflector, will be so constructed that no air can enter during the day; they will also be protected by louvers from the heat of the Sun. The problem is, of course, altogether different from that encountered in the case of the Snow telescope, where the mirrors are required to give good images in spite of their exposure to direct sunlight.

Assuming that these various difficulties can be successfully overcome, it still remains a question whether the atmospheric conditions on Mount Wilson will be sufficiently good to permit the telescope to give satisfactory images. This cannot be definitely determined until after the 60-inch reflector has been used for some time. Even if it should prove, however, that only a very few nights in the course of a year can be utilized to the fullest advantage, the construction of such a telescope would nevertheless be desirable. For under the ordinary conditions, which are much finer than those in the eastern part of the United States, results of the highest value can be obtained in many classes of work, such as the photography of stellar spectra, the measurement of the heat radiation of the stars, etc. The immense amount of light which this mirror will collect should render it particularly suitable for spectroscopic work of all kinds.

It need hardly be said that the 100-inch mirror, when suitably mounted, will play a most important part in the scheme of research of the Solar Observatory. The investigation of stellar evolution, upon which we are engaged, frequently calls for adequate spectroscopic study of stars beyond the reach of existing instruments. In my work on the red stars of Secchi's fourth type, with the 40-inch Yerkes telescope, I encountered this difficulty, in spite of the great light-collecting power of that instrument. It was impossible to obtain satisfactory evidence as to the transition from solar stars to those of the fourth type. The large number of stars within the reach of a 100-inch reflector should greatly increase the possibility of finding the intermediate types, which are so important in their bearing upon the relationship of solar and red stars. This is only a single instance, but it forcibly suggests itself when considering our program of research. In other fields the large reflector should be equally valuable, especially for the photography of the numerous small spiral nebulae, the details of which should be brought out to good advantage with a focal length of 50 feet; minute study of the large nebulae, in the hope of detecting changes in their form; the study, with very high dispersion, of the spectra of bright stars, etc. The remarkable calm of the summer nights on Mount Wilson should assist materially in all of this work, since vibration of the tube, caused by the wind, would undoubtedly be a serious drawback under less favorable conditions.

No provision has yet been made for the mounting and dome. It is not known from what source funds for this purpose will come, but I believe a donor will be found by the time they are needed. Mr. Hooker's gift is very opportune, because of the fact that it permits us, now that the 60-inch mirror is nearly completed, to retain and use to the best advantage the services of the opticians trained by Professor Ritchey for our present work. The making of the glass disk, and the grinding and figuring of the various mirrors, will probably occupy about four years. Since the Union Iron Works Company will require only a year for the construction of the mounting and dome, it is evident that no funds for this purpose will be needed at present, and that the experience gained with the 60-inch reflector can be utilized in designing them.

PURCHASE OF THE SNOW TELESCOPE BY THE SOLAR OBSERVATORY

As stated in previous papers, the authorities of the University of Chicago were kind enough to loan the Snow telescope to the Solar Observatory for a period of two years. It subsequently became the opinion of all parties concerned that, in view of the very satisfactory performance of this instrument on Mount Wilson, it would be advisable to keep it there permanently. Accordingly, the Snow telescope has been purchased by the Solar Observatory, and will thus form a part of its permanent equipment. I wish to express my sense of obligation to Miss Snow, to Professor Frost, Director of the Yerkes Observatory, and to Acting President Judson and the Trustees of the University of Chicago, for the courtesies shown us in connection with the loan and sale of this valuable instrument.

SEPTEMBER 1906.

Minor Contributions and Notes

THE SNOW FUND OF THE YERKES OBSERVATORY

In view of the very satisfactory performance of the Snow horizontal telescope during the year that it had been in operation at the Solar Observatory on Mount Wilson, the advisability of its permanent retention there suggested itself, some months ago, independently to the directors of the two institutions concerned. A less satisfactory performance had been obtained while the instrument was provisionally installed in its wooden house at the Yerkes Observatory; and it was not to be expected that any great improvement in that respect could be secured without a considerable expenditure in remodeling the building, if indeed the full capacity of the instrument could ever be realized in this climate and under the atmospheric conditions prevailing here.

The donor of the telescope and building, Miss Helen E. Snow, of Chicago, promptly acceded to my suggestion that, under the circumstances, the interests of all parties concerned might be furthered by the transfer of the Snow telescope to the Solar Observatory of the Carnegie Institution, and that the monetary compensation for the telescope be used as a fund for the purchase of auxiliary instruments for the Yerkes Observatory. The Acting President and Trustees of the University of Chicago also took action in accordance with this recommendation, and the Snow Fund has been established for this Observatory. After the purchase of a stereocomparator, the principal of the fund, then amounting to about five thousand dollars, will remain intact, and the income will be added to the regular appropriations for the purchase of equipment at the Observatory. The Snow Fund constitutes the first item of endowment in connection with the Observatory.

The small coelostat used by our party at the eclipse of 1900 will be set up in the Snow building for solar work.

Edwin B. Frost

Yerkes Observatory,
September 1906

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

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A SPECTROSCOPIC STUDY OF THE SPARK SPECTRUM

By W. B. ANDERSON

In the past decade much study has been given to the changes produced in the spark spectrum by varying some of the quantities connected with the spark or its circuit. This work, however, like the earlier researches, consists of many independent investigations dealing with special phases of the subject and conducted with quite dissimilar apparatus. Naturally, different results were obtained and conflicting conclusions reached. Thus one investigator concluded that the main factor in determining the appearance of the spectrum is pressure;[1] another, vapor density;[2] another, temperature;[3] another, nature of discharge;[4] another, inductance;[5] etc. More recent investigators[6] consider pressure, capacity, and inductance to be important factors; but have varied them, with the exception of pressure,[7] through small ranges. The observed displace-

[1] See footnote 4, p. 222. Cailletet, *Comptes Rendus*, **74**, 1282. 1872.

[2] Cazin, *ibid;* **84**, 1151, 1877; see also *Phil. Mag.*, **3**, 153, 1877; Monckhoven, *C. R.*, **95**, 378, 1882.

[3] Plücker and Hittorf, *Phil. Trans.*, **155**, 1, 1865; Salet, *Ann. chim. et physique* **28**, 5, 1873; Secchi, *C. R.*, **70**, 79, 1870.

[4] Stearn and Lee, *Proc. Phil. Soc. Liverpool*, **28**, 1874; also *Phil. Mag.*, **46**, 406, 1873.

[5] Hemsalech, *Journal de physique*, **9**, 437, 1900.

[6] Hale, *Astrophysical Journal*, **15**, 132, 1902; Hale and Kent, *Ibid.*, **17**, 154, 1903; Kent, *ibid.*, **17**, 286; Mohler, *ibid.*, **10**, 202, 1899.

[7] Hale and Kent (*loc. cit.*) took spectra at pressures up to 53 atmospheres.

ments of the spectral lines of certain new stars, and Wilsing's[1] idea that these displacements might be due to very high pressure[2] in the stellar atmosphere, gave the subject added interest.

The present work was undertaken in the hope that some of the questions involved could be answered, and the facts already known correlated, by a study of the effect produced upon the spectrum by separately varying through wide ranges each of the quantities[3] connected with the spark or its circuit.

Work upon the spark spectrum may be divided into two parts: (1) the study of change in the width and character of its lines, as well as other changes in its general appearance, and (2) the study of shift, or change of wave-length of its lines. Recent investigations in which spectroscopic apparatus of high dispersive power was used have been mainly concerned with shift, and have discussed rather incidentally the character of the spectrum. The present research deals with both, but principally with the character of the spectrum and its relation to the character of the spark.

The ultimate cause of the above changes in the spectrum cannot consist in changes of capacity, pressure, etc., *per se*, but in resulting changes in the light-emitting vapor. In all probability the appearance of the metallic spectrum depends solely upon the amount, density, pressure, nature, and temperature of the metallic vapor in the spark; and the capacity,[4] pressure, etc., are of importance only in so far as they affect these conditions of the vapor. The quantities (capacity, etc.) have been varied through the ranges given in Table I, and it has been found that the changes thereby produced in the spectrum may be explained from the above standpoint. Thus the relation between the separate effects of these quantities is shown. It was thought that the potential-drop over

[1] *Astrophysical Journal*, 10, 113, 1899.

[2] Humphreys and Mohler had previously shown (this *Journal*, Vols. 4, 5, and 6, 1896–97) that when the arc is surrounded by a gas at high pressure, its spectral lines are given slight displacement (shift) toward the red relatively to those of the normal arc.

[3] See table, p. 223.

[4] In all work at *pressures* above one atmosphere, the *spark* whose spectrum is being studied takes place between insulated *electrodes* in a *spark chamber* filled with a *gas* under pressure. In this spark circuit there is usually a certain *capacity*, *inductance*, and *resistance*. In this paper the above italicized words will always be used in the above sense, unless otherwise stated.

the spark, the current through it, and the energy developed in it, as well as the resistance of the spark circuit (quantities not previously measured in such work), might be closely related to the character of the spectrum. Accordingly these, too, were investigated.

Since much of the apparatus and all the instruments were made especially for use in the work, a somewhat detailed account of these will precede the discussion of results.

TABLE I

Quantity	Range	Ratio. (Of Max. to Min.)
Pressure	1 to 100 atmospheres.	100
Capacity	0.00145 to 0.2231 microfarads.	150
Inductance	0.0135 to 140 milhenrys.	10,400
Potential-drop	2000 ? to 12,000 volts.	6 ?
Nature of gas	Air, Hydrogen, CO_2.	
Energy in secondary	4, or less, to 125 watts.	30
Resistance	0.2 ohms plus spark to 78 ohms, plus spark.	100 ?
Exposure time	8 seconds to 10 minutes.	75
Oscillation period	1.03×10^{-6} to 1.1×10^{-3} seconds.	1000

APPARATUS

Arrangement.—The arrangement of the apparatus as used is shown diagrammatically in Fig. 1. *T* is the transformer; *C*, the

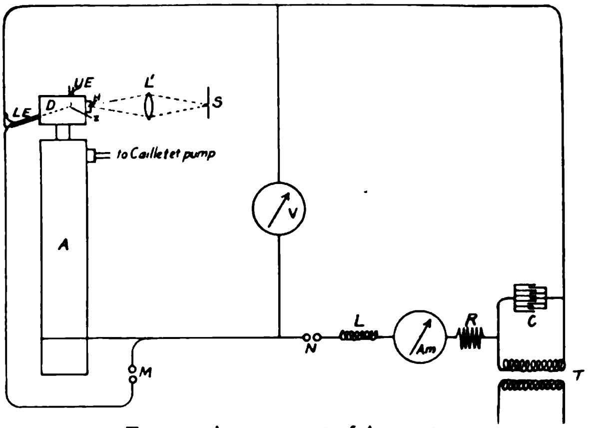


FIG. 1.—Arrangement of Apparatus.

capacity; *L*, the inductance; *R*, the non-inductive resistance (used in only a few cases); *Am.*, the hot-wire ammeter used to measure

the current through the spark Z that takes place between the carefully insulated electrode $L. E.$ and the adjustable uninsulated electrode $U. E.$; V, the static voltmeter[1] for measuring the potential-drop over the spark Z. The light from this spark, which may be called the working spark, passes through the heavy glass window H of the steel compression chamber D, and is concentrated by the lens L' upon the slit S of a Rowland concave grating[2] spectroscope. The adjustable spark-gap N is sometimes used to maintain the disruptive character of the discharge. A similar spark-gap M, in parallel with the working spark, serves to protect the insulation of the lower electrode $L. E.$ from excessive potential differences. R, L, and C can each be varied through wide ranges or thrown out of the circuit entirely.

Compression tube and compression chamber.—To avoid being limited to the pressures obtainable from ordinary commercial gas cylinders, a special form of tube A and attached compression chamber D was made as shown in Fig. 2. As water from the Cailletet pump is forced into tube A at s, mercury is driven from the annular space C into tube B, thereby forcing the gas from B through u (controlled by valve a) into the spark chamber D. If still higher pressure in D is desired, the valve a may be closed and B may be refilled with gas (usually at about 10 atmospheres pressure) through orifice b, which is controlled by a valve c (not shown) similar to a. This additional gas is then also forced into D as before.

The glass window H, protected by a lead washer on each side, is tightly held on D by the screw-cap I. Lead washers are used at all joints not exposed to mercury. For other joints leather or rubber washers are used.

When hydrogen was used it was generated with an ordinary Zn–HCL generator and collected in a large tank, from which it was pumped through drying-tubes into B. When CO_2 was used it was obtained from a commercial compressed gas cylinder.

Pin valves, consisting of a steel cone screwing down into an accurately ground steel seat, and packed with rubber as shown, were found very serviceable at a and at c (not shown). The elec-

[1] Kelvin vertical quadrant type calibrated from 4,000 to 17,000 volts.

[2] 14,438 lines per inch, and 21 feet radius of curvature.

trodes *U. E.* and *L. E.* were likewise packed with rubber to prevent leakage. Nevertheless, considerable trouble with leakage was

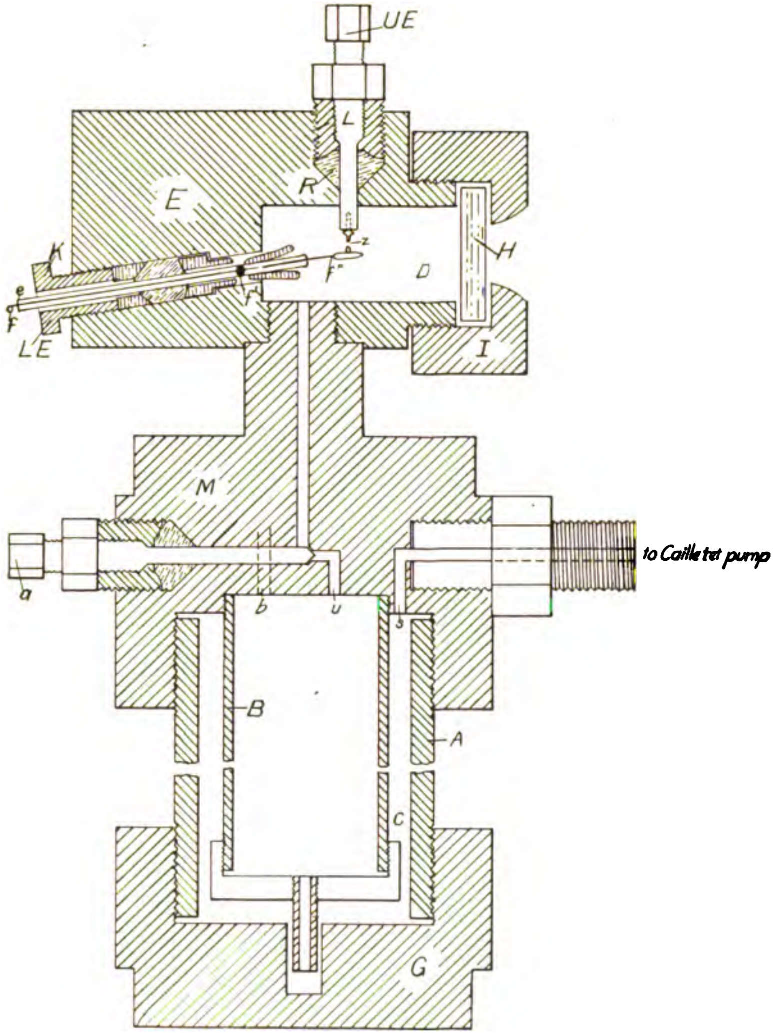


FIG. 2.—Compression Tube and Spark Chamber.

experienced at pressures from 80 to 100 atmospheres, especially with hydrogen.

The electrodes.—The upper electrode, *U. E.*, which is in electrical contact with the compression chamber, can be screwed up or down

to alter the length of the spark-gap and consequently the potential fall over the spark. With larger capacity, and at high pressures, the spark-gap must be very short, or the spark will either not pass at all or else it will pass through the parallel spark-gap M (Fig. 1), even though the latter be 6 to 10 mm in length. The intense heat developed under these circumstances readily fuses the electrodes together, while a slight turn of $U. E.$ usually separates them so far that no spark passes; thus continual adjustment is necessary. For this reason, what might be termed an hour's exposure really consists of several thousand exposures varying from one-twentieth to one-half second. At lower pressures, say 50 atmospheres or so, the spark often runs steadily for several minutes.

For the lower electrode $L. E.$ various forms were tried, and many glass tubes were exploded before one was made that could long withstand the great heat, and the electrical and mechanical stresses. In the final form ($L. E.$, Fig. 2) a copper wire f has a short piece of platinum wire tightly fitted into a hole bored in one end. To this platinum wire is fused a small globule of fusible glass, which is then inserted into the tube e, and sealed to it at f' as shown. With this method of sealing, it will be seen that the gas pressure on the tube is from without, and hence easily withstood. To prevent the wire f from locally heating and thereby fracturing the tube e, mercury is put in around the wire and kept there by a vulcanite cap (not shown) containing soft wax. This cap is connected with the nut K by a vulcanite clamp (not shown), and thus prevents e from being forced out by the pressure in D. The wire f'', making electrical contact with the platinum wire, and hence with f, through a drop of mercury in e, completes the lower electrode. (K to $e = 3$ in.)

The transformer T (Fig. 1) has its primary connected with the city circuit of 108 volts and 60 cycles per second. The secondary gives a potential of 15,000 to 20,000 volts, and can furnish a large current, which is necessary when large capacity is used.

The condenser consists of thin (No. 33) sheet brass 9×18 in., shellacked to glass plates 12×22 in. Eighty of these are arranged on edge in an oil tank filled with kerosene of high flashing-point. The brass sheets, properly grouped, are connected with the mercury cups of a switch-board by which the various groups may be con-

nected in parallel or series, and the capacity varied from 0.00145 to 0.2231 microfarads. This condenser is efficient and serviceable, and was built after it was found that the heavy discharges and continuous use burned up the tinfoil and melted the paraffin of a tinfoil-on-glass condenser incased in paraffin.

The variable inductance is composed mainly of 2,000 ft. of No. 14 rubber-covered wire. By connecting to different terminals, inductances from 0.14 to 140 milhenrys are obtained. For smaller inductances special coils were used.

Non-inductive resistance.—Since resistance causes a damping of the oscillations, and may even altogether prevent them, it seemed important to study its effect upon the spectrum. To avoid simultaneously introducing considerable inductance, this resistance is made nearly non-inductive by using high resistance (IA) wire which is wound back and forth over two rows of pegs about 4 inches apart. Computation gives 0.03 milhenrys for the 78 ohms resistance.

The ammeter.—In the hot-wire ammeter shown in Fig. 3 a strip of sheet brass, *D*, 0.5×5×500 mm, stretched between the binding

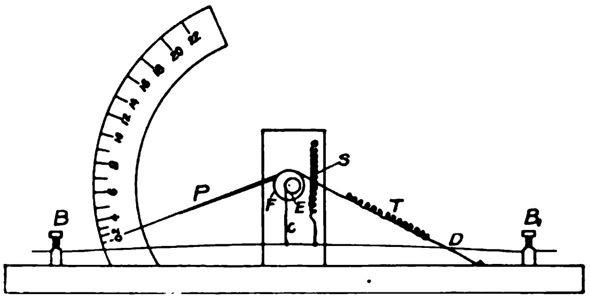


FIG. 3.—Hot-wire Ammeter.

posts B, B_1, carries the current. As the current heats this strip it elongates, and the spring *s* contracts, thus permitting spring *T* to rotate *F* and thereby raise the pointer *P*, at the same time winding the thread *C* upon the eccentrically placed pulley *E*. *E* was so placed as to give a fairly uniform scale, as shown. This instrument was calibrated from 2 to 22 amperes, while a similar one with a smaller conductor was used for currents from 0.25 to 5 amperes.

The wattmeter.—Various devices were tried for ascertaining the amount of energy developed in the spark. In the first, immediately

after taking the spectrum, the spark, having all other conditions connected with it kept unchanged, was made to take place inside a special calorimeter. From the rate of temperature rise of the calorimeter, the watts developed by the spark could be computed. However, the spark as used occurred in a gas at high pressure (in the spark chamber), while in the calorimeter it occurred in air at one atmosphere pressure. This makes a great difference in the amount of energy developed, so this arrangement was abandoned.

The spark calorimeter was next used simultaneously with the working spark, and in series with it. This insured the same current in each spark; so that the energy in each was proportional to the potential drop across it. Uncertainty as to the measurement of these potentials was here a fatal objection.[1]

Finally the compression chamber itself was made to serve as spark calorimeter, and was calibrated as a wattmeter. For the calibration a spiral of wire heated by a direct current, and using a measured number of watts (say 16.5) was placed in the compression chamber, and temperature[2] readings were taken every minute. Curve *A*, Fig. 4, is obtained by plotting these temperature readings as ordinates and the corresponding times as abscissae. The other six curves were obtained in the same way. In all cases the calibration was begun with the chamber at room temperature; so this temperature was used as an arbitrary zero in plotting.

To illustrate the use of these curves, suppose in 20 minutes the spark were to heat the spark-chamber from room temperature to 12° above. The point whose abscissa is 20, and whose ordinate is 12, is about half-way between curves *B* (19 watts) and *C* (31.3 watts); hence about 25 watts would be the power expended in the spark.

Though the compression chamber radiates heat freely, and also imparts it to the compression tube by conduction, errors from these causes are eliminated because the same heat losses occurred during the calibration. Indeed, for fifteen or more watts, it seems to be a sufficiently sensitive and accurate wattmeter. The fact that the

[1] See multiple spark-gap, footnote 3, p. 231.

[2] The thermometer was inserted into a hole bored into the wall of the compression chamber and containing some mercury to make good thermal connection.

curves produced cut the x axis within 20 or 30 seconds of the origin, and also that after breaking the circuit (as at z, curve F) the indicated temperature reaches a maximum in less than a minute, show that the heat is quickly transmitted from the spark to the thermometer.

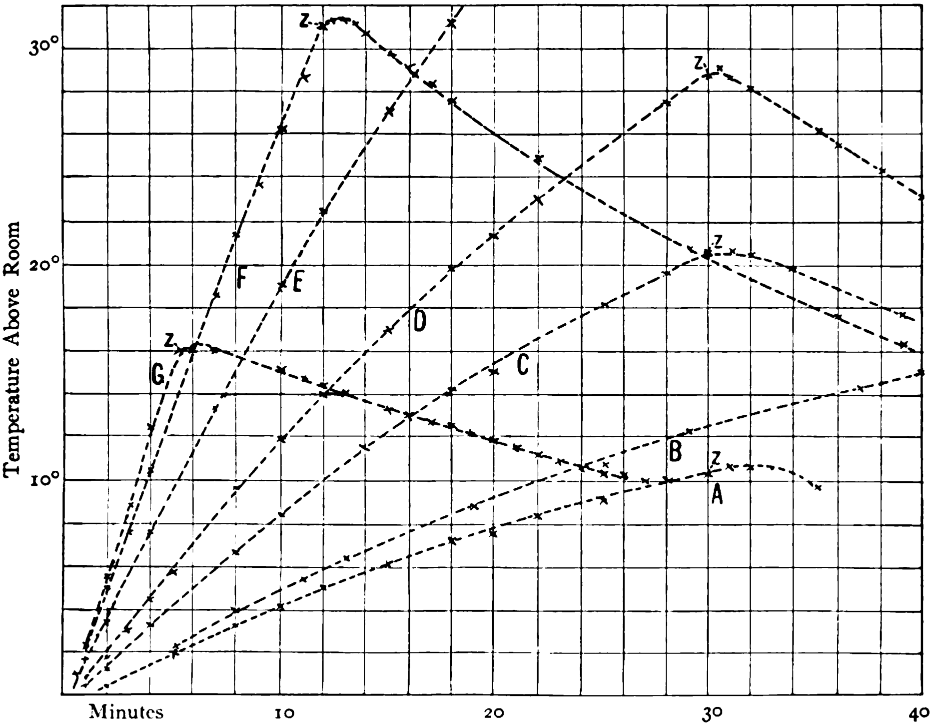


FIG. 4.—Calibration of Spark-Chamber as Wattmeter.

Z marks the point where the circuit was broken.
A=16.5 Watts; B=19; C=31.3; D=47; E=70.5; F=100; G=112 Watts.

The camera.—The camera, of the usual form for gratings, may be raised and lowered, by which motion, and the use of a suitable screen, a dozen or so spectra may be taken on the same plate (see No. 174 and footnote 2, p. 235). This method can be used in comparing the general appearance of the lines of successive spectra as some one of the quantities, capacity, inductance, etc., is varied. To determine shift, the method used by Humphreys and Mohler[1] was adopted.

[1] *Loc. cit.*, p. 222.

RESULTS

Electrical conditions in the spark.—The true value of the current through the spark, the watts used in it, and the potential-drop over it manifestly cannot be directly inferred from the observed values obtained by the ammeter, the wattmeter, and the voltmeter, respectively, because the current through the spark is flowing a very small fraction of the time. Though the spectrum varies much less with these observed values than with change of capacity or pressure, etc., it is believed that the actual values of the current and energy during the brief intervals that the current flows, are of prime importance, and that it is through changes in these values mainly that change of capacity, pressure, inductance, etc., affect the spectrum.

These instruments were calibrated either with direct current circuits or alternate current circuits of moderate frequency. The current and potential variations in the case of the spark are of very high frequency and are also exceedingly irregular; so that direct comparison between measurements of them and those of low frequency cannot be made without serious error.

As this is, to my knowledge, the first attempt[1] at measuring the current, the potential-drop, and the energy of the spark in such work, I shall endeavor to interpret the readings obtained by showing to what extent the various actual values[2] of the current, the potential, etc., contribute toward producing the observed deflections. This can be best shown by curves.

In Fig. 5, curve I is intended to represent the current in the secondary of the transformer; while curve II, it is believed, represents the potential-drop over the spark, which is practically the potential difference between the needle and the quadrants of the voltmeter. Although the condenser when fully charged produces an opposing *E. M. F.* of several thousand volts, the inductance of the secondary of the transformer (80,000 turns) is too great to permit of sudden variations of the current; so that curve I should be a nearly smooth sine curve as shown.

[1] Battelli and Magri (*Phil. Mag.*, 5, 620, 1903) in an investigation of the oscillation period measured the potential and the energy of the spark in air at one atmosphere.

[2] Mean square values for various intervals in each cycle.

With curve II, suppose at the time represented by a_2, the condenser is completely discharged. The value of the current at that instant is aa', which rapidly charges the condenser to a potential $b_2b'_2$, whereupon the spark-gap breaks down and a group of oscillations follows. For several oscillations the potential-drop over the condenser is but little less than $b_2b'_2$, but the drop over the spark is a small part of this (as shown), because the inductance of the spark is very small compared with the total inductance, and because

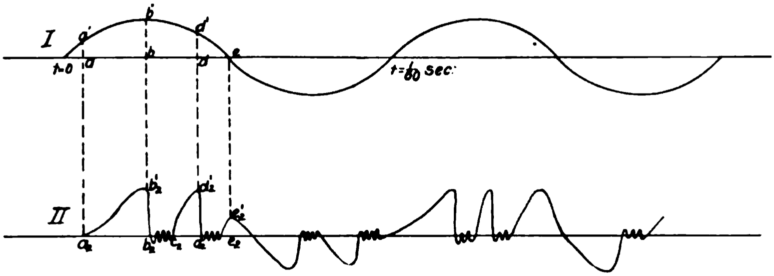


FIG. 5.—Curve I: Current in the Secondary of the Transformer.
Curve II: Corresponding Potential-drop over Spark.

its resistance during oscillations also is probably very small (p. 233). These oscillations continue from b_2 to c_2,[1] after which the above cycle is repeated.[2] At e_2 the condenser is not charged to the sparking potential, and, as the current reverses at e (curve I), no spark occurs until it is charged in the reverse sense to the potential $f^2f'_2$. From the curve it is evident how insignificant[3] a part the potential

[1] The distance b_2c_2 is greatly exaggerated in the curve. From photographs of the oscillations obtained by the use of the rotating-mirror method and from other calculations, it seems that ordinarily the oscillations last about 1/100 of the time; i. e., $b_2c_2 = 1/100\ b_2d_2$, roughly.

[2] Here the special case has been taken in which the secondary of the transformer furnishes in one-half cycle (1/120 sec.) enough electricity to charge the condenser twice to the potential required to break down the spark-gap. From the photographs of the spark taken with a rapidly moving plate this number was found to vary from one to twelve or more and to be different even for successive half-cycles.

[3] In some allied work seven brass knobs, placed in a row and separated by millimeter spaces, were used as a spark-gap. It was found that the potential over four of these sparks (exclusive of the two end ones), instead of being 4/6 of the total, was quite too small to be measured or even detected. Doubtless this was due to the fact that only from b_2 to c_2 (i. e,. during oscillations) was there any potential-drop over the voltmeter, except through slight charging by the streamers that precede the spark.

during the interval b_2c_2 contributes to the resulting deflections of the volt-meter needle, and hence how hopelessly masked its value is.

It is just at this time, too, when the current is surging through the spark, that the potential-drop over it might be most expected to affect the spectrum. The potential read is probably about two-thirds the discharge potential[1] $b_2b'_2$ (see curve II).

This discharge potential may affect the character of the light in three ways. First, through disintegration of the electrodes by the enormous potential gradient,[2] particles may be thrown into the spark and by their momentary incandescence form the continuous spectrum. Second, through the effect the initial electrostatic stresses may have upon the light-vibrations, if they result from direct electrical action and not from heat. My results throw no light upon this question, but I simply suggest the possibility of such effects.

The third way, through its effect upon the initial discharge, is discussed on p. 248.

The hot-wire ammeter (Fig. 3) integrates the instantaneous values of C^2R, C being the current through the brass strip D, and R its resistance. Evidently the current is practically zero except during oscillations, i. e., from b_2 to c_2, curve II; so that during these brief intervals, which probably constitute about 1/100 of the time, the current must be very strong to produce as great heating effects as were observed.

Some computations as to the probable current, potential-drop, watts, and resistance of the spark when taking spectrum No. 71*c* will be given. This particular case is chosen because many other spectra were taken under conditions not greatly different. The indicated current was 20 amperes, voltage, 7,000. (See No. 71 *c*, Table II.) To produce this heating effect with an intermittent current flowing 1/100 of the time requires of course $\sqrt{100}\times 20$ or

[1] I find this estimate in accord with the discharge potentials given by Battelli and Magri (*loc. cit.*, p. 230) for various spark-lengths. Of necessity, since their work was at one atmosphere, I compare the length of their spark ("explosive distance") with that of the parallel spark M, (Fig. 1), which was so adjusted that the spark occasionally took that path.

[2] Judging from the slight motion (p. 226) of the electrodes required to short-circuit the spark at pressures from 80 to 100 atmospheres, it seemed that the spark-gap was less than 0.05 millimeter. With 10,000 volts over the spark, the potential gradient would then be more than 2×10^6 volts per centimeter.

200 amperes.[1] Again from the capacity, voltage, and frequency, we obtain by computation (as on p. 243) 218 amperes.

Both of these values of the current are necessarily rough approximations because of the uncertainty (1) as to the fraction of the time that the oscillations exist, and (2) as to the value of the charge on the condenser during these oscillations. The current, however, is very probably between 100 and 200 amperes in many cases.

The wattmeter (calorimetric—see p. 228) probably gives the average value of the watts within 10 per cent. More energy is radiated through the glass window of the compression chamber by the spark than by the hot wire with which the calibration was made, but this is a small fraction of the energy in either case, so that the error introduced is slight.

When taking No. 71 (see above), the wattmeter indicated about 100 watts. Assuming the oscillations to persist 1/100 of the time, the average value of the watts during oscillations is 10,000, which with 200 amperes would require 50 volts drop over the spark, showing the resistance of the spark to be 0.25 ohm. This is a very low resistance,[2] but the spark is exceedingly short, and the great amount of energy in it must develop considerable metallic vapor.

The aim has been to vary, one at a time as far as possible, the quantities given in Table I, and study the changes thereby produced in the spectrum. The capacity, inductance, resistance, and pressure may be varied independently; but the current and energy depend very much upon these values and upon the amount of energy furnished to the primary. Indeed, I believe the effect produced upon the spectrum by varying the capacity, inductance, and resistance is due almost entirely to the resulting change in the energy of the spark, which in turn determines the amount of metallic vapor present. We may then consider pressure, capacity, inductance,

[1] This is without taking into account the increase of R due to "skin effect" (see Drude's *Physik des Aethers*, p. 245). Adapting the formula (Rayleigh's $R = R_1 \pi r \sqrt{\frac{\mu\sigma}{T}}$) which is designed for round conductors, to apply to the brass strip (D) of the hot-wire ammeter (p. 227), I find that with the oscillation period used for No. 71, the skin effect just about doubles the resistance. This doubles the heat effect, so instead of 200 amperes it would require but $200/\sqrt{2}$ or 140 amperes.

[2] Battelli and Magri, *loc. cit.*, p. 230, obtain less than an ohm for very much longer sparks (at one atmosphere pressure).

resistance, potential-drop (i. e., length of spark) as the independent variables; and current, energy, and oscillation period, as the dependent variables. The energy and also the current may, however, be varied by varying the energy in the primary of the transformer (see No. 91, Table II), leaving the above independent variables unchanged.

In studying the effect of each quantity, fifteen or twenty spectra have, as a rule, been taken; so that it is believed that errors arising

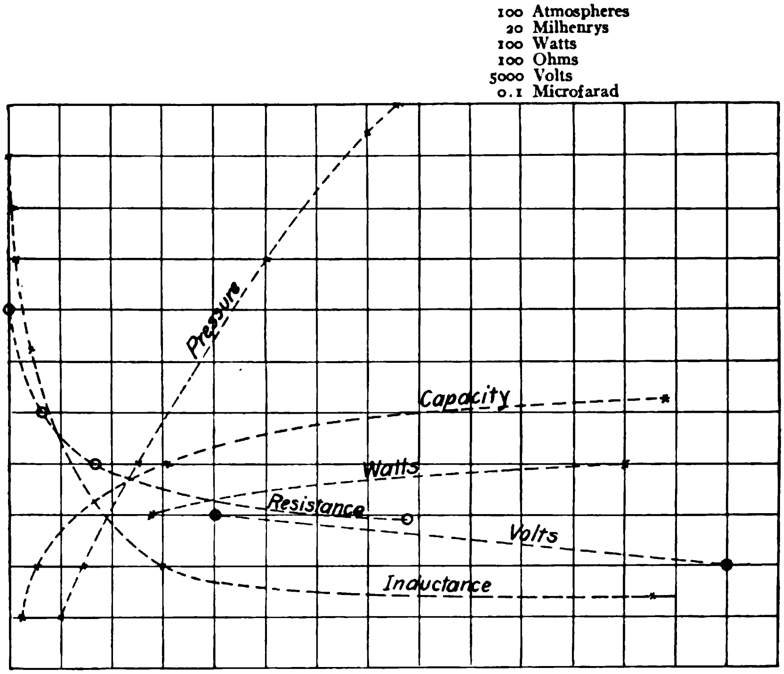


FIG. 6.—Ordinates: Widths of Lines (arbitrary scale).
Abscissae: Pressure, Capacity, Inductance, etc. (on different scales).

from accidental variations have been avoided. In Fig. 6, several curves have been plotted having relative width of reversed lines as ordinates and the corresponding values of the given variable for abscissae. The ordinates are plotted to a different (arbitrary) scale for each curve, since in separately studying the effect of each variable, it is obvious that nothing would be gained by having the scale uniform. The relative widths are averages based upon estimates from several

lines in each case. Frequent references to these curves will be made in the following pages.

Effect of pressure.—The general effect of increase of pressure[1] is to cause the bright lines to increase in width, then reverse, and finally disappear entirely, leaving only, or mainly, reversed lines on a strong continuous spectrum. This is shown, for example, in No. 170,[2] in which the pressure (see pressure-curve, Fig. 6) is varied as shown on the margin, while the capacity, inductance, etc., were kept constant (see Table II). This effect has the same trend whether the gas used is carbon dioxide (No. 170), air (No. 48), or hydrogen (No. 85); and also whatever value the capacity, inductance, etc., may have, provided these values be kept constant for the series.

If the capacity is small and the inductance large, the pressure must be much higher to produce as wide lines as if the reverse were true; thus No. 120 *a* (70 atmospheres) has not so wide absorption lines as No. 103 *d* (20 atmospheres), Nevertheless, the pairs Nos. 112*a* and 120*a*, or Nos. 112*b* and 120*b*, or Nos. 112*c* and 120*c*, show that in this case also increase of pressure widens the lines if the capacity and inductance are kept constant. These few spectra demonstrate the necessity for stating all the conditions under which such spectra are taken; and also show how difficult it is to draw any conclusions from many previous researches in which this fact has been overlooked.

In case the lines of the different spectra on a plate—e. g., *a*, *b*, *c* and *d* of No. 170—appear slightly displaced relatively to each other, it must not be construed as shift (i. e., change of wave-length), since it is probably due simply to motion of the camera. To determine shift great care was taken not to jar the camera, and a different method was employed (see pp. 229 and 249).

Effect of capacity.—Increasing the capacity has very much the same effect upon the spectrum as increasing the pressure, as shown

[1] Compare work by Hale and Kent, *loc. cit.*

[2] Hereafter the spectra will be referred to simply by number, as No. 170*a*, No. 170*b*, etc. The plates, together with the conditions under which they were taken, are given in numerical order in Table II, which also shows, in column 2, where the photographic reproductions may be found, if given.

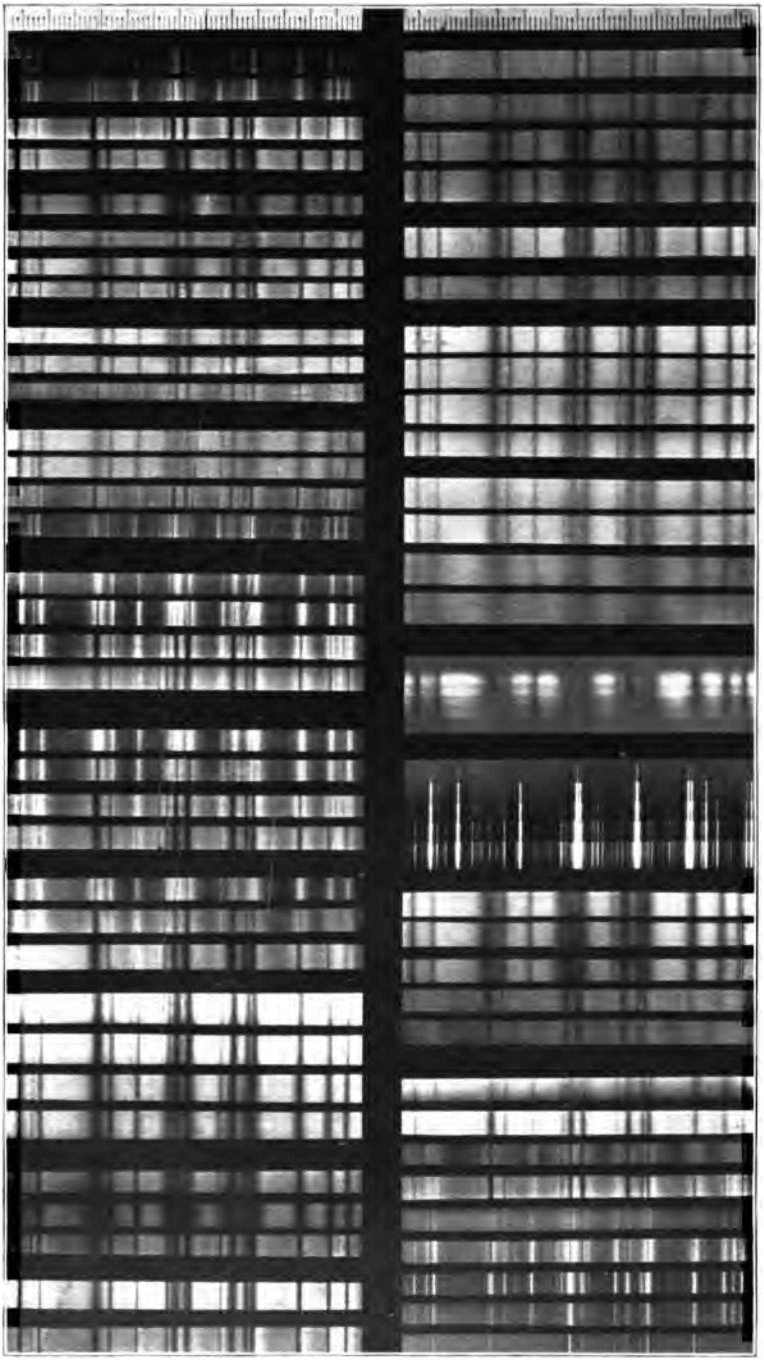
by No. 112, No. 116, and No. 120, in which capacity[1] alone is varied. No. 112*a* is essentially a bright-line spectrum with but one line (λ 3720.08) reversed; while No. 112*d*, with 60 times as much capacity, has many and very wide reversals—in fact, but two lines that are not reversed. These are $\lambda\lambda$ 3709 and 3765, the middle lines of two similar and interesting triplets. In the latter triplets I find changes produced by capacity and pressure similar to those observed by Hale[2] with the spark under water. The changes of intensity of the different lines of such a triplet relatively to each other is easily noticed because of the close proximity of the lines; so that they are among the best lines to observe throughout the various spectra.

Naming the lines of the former triplet, α, β, and γ (α being λ 3706), their relative intensities may be expressed by 5, 1, and 7, for the arc No. 149*a*); by about the same numbers for No. 174*k* (spark in CO_2 at 1 atmosphere); by 8, 1, and 5, for No. 112*a*; by 5, 1, and 5, for No. 112*b;* and by 1, 1, and 5, for No. 112*c*. β is not reversed in No. 112*d*. γ is, but also shows faintly the bright emission line on each side of the reversal, while α shows no trace of emission line. The intensity of a bright line diminishes very rapidly with increase of capacity (or any other cause of reversal), when the capacity is such as to produce barely perceptible reversal of the line, because the reversal is just at that part of the emission line that would otherwise be most intense. This accounts for the above-cited rapid diminution in intensity of α, which reverses more readily with increase of capacity than does β or γ. The increase in intensity of the middle line relatively to the outer two of the triplet observed by Hale may be explained in the same way. For this middle line (λ 3765.69) has just been shown to be one of the most difficult to reverse in that region of the spectrum. In No. 55, No. 75, and many others α, β, and γ are all reversed, and their relative intensities are about 6, 1, and 4, respectively; while in the solar spectrum (No. 199) they are 2, 1, and 3.

[1] The capacity-curve (Fig. 6) shows the relation between width of lines and capacity change under certain other constant conditions (inductance 140 milhenrys, pressure 50 atmospheres, etc.), and might have a somewhat different form under other conditions. The same may also be said of the other curves.

[2] *Loc. cit.*

PLATE IV



3720 3740 3760
3720 3740 3760
PLATE PRES.
No. 85a 10
85b 15
85c 30
85d 50
85e 70
79d 30
79e 60
79f 25
48a 60
48b 40
48c 20
170a 50
170b 40
170d 15
170e 10
CAPACITY
116a 0.002
116b .0054
116c .0300
116d .1280
112a .002
112b .0054
112c .0300
112d .1280
120a .002
120b .0054
120c .0300
INDUCTANCE
55a 0.868
55b .594
55c .278
55d .155
PR. IND.
103a 50 25
103b 50 6.5
103c 20 0.014
95a 40 25
95d 20 0.014
PLATE RES.
No. 63a 78
63b 16.9
63c 4.69
63d 0.25?
WATTS
91a 27
91b 120
VOLTS
71a 2000?
71b 4100
71c 7000
71d 2000?
Exp. Time
75a 600
75b 120
75c 20
75d 5
167a 180
167b 66
167c 20
167d 6
167e 2
Iron Arc
2a 1
2b 6
2c 30
2d 120
2e 600
IND. CAP.
161a 75 0.2231
161b 23 .073
161c 8.8 .073
161d 0.7 .0054
161e 0.7 .002
PRES. GAS
174a 50 CO2
174b 50 H
174c 25 CO2
174d 25 H
174e 15 CO2
174f 15 H
174g 10 CO2
174h 10 H
174i 5 CO2

Effect of inductance.—Increase of inductance renders the discharge less violent and produces an effect upon the spectrum directly opposed to that due to increase of capacity or pressure. The reason for this will be discussed later (p. 248). Small variations do not produce a very marked effect (see No. 55); but No. 71*c* (0.0135 milhenry), and No. 112*c* (140 milhenrys) are radically different, the latter having several bright lines, the former none. Compare also No. 186 with No. 187.

Effect of resistance.—Resistance affects the spectrum[1] in the same way that inductance does, as shown by No. 63.

Energy in the spark.—The effect of increasing the energy in the primary from 120 to 720 watts (capacity, etc., kept unchanged), and thereby increasing the energy in the spark from 27 to 120 watts (see No. 91), is similar to that produced by increase of capacity, the absorption lines becoming, in this case, about one-third wider, and the continuous spectrum more intense. This effect is less than was expected (see p. 247).

Effect of potential.—As the potential is increased, by lengthening the spark-gap (see Fig. 2), the lines become slightly narrower (see No. 71), though just the opposite effect[2] might be expected (p. 248). The lowest potential used in this case is lower than that for which the voltmeter was calibrated, and is therefore uncertain; but this affects the result only in a quantitative way.

Nature of the gas.—In No. 149 several spectra were taken, using alternately air and hydrogen in the spark-chamber at the pressures shown, in order to compare the effects of these two gases.

A similar series (No. 174) was taken to compare carbon dioxide with hydrogen and hence with air. The pair *a* and *b* cannot be directly compared because *b* was much more exposed[3] than *a*, and consequently has narrower lines.

Equal exposure for such a series of spectra is very desirable,

[1] See resistance and inductance curves, p. 234. It will be observed that both these curves are roughly of the exponential type, $y=e^{-x}+b$, in which b is the limiting (minimum) width approached as inductance or resistance, x, is increased. The capacity-curve is of the form $y=a-e^{-x}$, in which a is a limiting (maximum) width of line.

[2] Perhaps the opposite effect could be obtained under other conditions of pressure, capacity, etc.

[3] See effect of exposure, p. 238.

but not easily attained, because different amounts of "smoke" on the compression-chamber window, and various other conditions, cause the proper time of exposure to vary from a few minutes to several hours. A small test-plate, slipped into an attachment at one side of the camera, which could be taken out and developed when it was thought the main plate was sufficiently exposed, was found very useful, but did not completely solve the problem of equal exposures. From these plates, however, from No. 55*a* and No. 186, and from plates taken for other purposes, I estimate the relative widths of the reversed lines to be 4, 5, and 6, respectively, for air, carbon dioxide, and hydrogen. The spark runs more steadily in hydrogen than in air or in carbon dioxide, which would probably cause more vapor to be produced and hence wider lines.

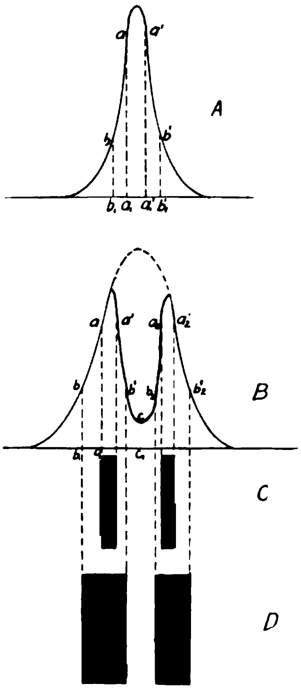


FIG. 7.—Intensity-curve for spectral line.

Duration of exposure.—In some previous work it was observed that the width of the bright lines increased markedly with the time of exposure (No. 2); and it was thought that for the same reason the reversed lines should be narrowed by increased exposure. This is shown to be the case by No. 167 and No. 75. No. 75*c* (20 sec.) has nearly twice as wide lines as No. 75*a* (10 min.).

The reason for this change in width is evident from a consideration of the intensity-curve in the immediate vicinity of a bright line[1] (*A*, Fig. 7), as given by Rayleigh[2] and Michelson[3]. If the time of exposure is so brief that intensities of less value than *a a* fail to noticeably darken the negative, a line of width *aa'* results; whereas a longer

[1] For a different application of the same idea see Kayser's *Spectroscopie*, II, p. 207.

[2] *Ibid.*, p. 343; *Phil. Mag.*, **27**, 298, 1889.

[3] *Astrophysical Journal*, **2**, p. 251, 1895.

exposure, which enables intensity b_1b to darken the negative, gives a line of width bb'.

Similar reasoning will show that if B, Fig. 7, represents the intensity curve[1] for a bright line partly reversed, then the short exposure will give a bright line of width a_1a_{11} (aa'_2), having a reversal of width $a'a_2$ as shown by C; while the longer exposure gives the line shown at D.

Order of reversal.—In general it may be said that certain lines reverse more readily[2] than others, and that any series of changes that increases the number and width of the reversals, will do so in approximately the same order, whether it be through increase of capacity or pressure, or through decrease of inductance or resistance. There are, however, cases in which the order of reversal seems to differ for some of the weaker lines. For example, No. 143*a* and No. 170*e* (Table VI) have each twenty-seven reversals, but not of the same lines throughout. Duration of exposure might cause such differences; for if certain lines have a relatively high value c_1c (B, Fig. 7), it will be clear that such intensity will suffice to darken the plate with long exposure so that no reversal occurs, even though with short exposures they do occur.

Increase of capacity and pressure tends to reverse more lines and widen those already reversed, while increase of inductance tends to narrow and also to suppress reversals. It appears, however, that capacity and pressure affect most strongly the prominent, easily reversed lines, and inductance, those difficult to reverse. Thus when both capacity and inductance (No. 134 and No. 120*c*), or pressure and inductance (No. 127), or all three (No. 137), are large, the strong lines have very wide reversals, while some of the weaker lines e. g., λ 3765, do not reverse at all. Compare these with No. 186 and No. 103*d* of low pressure and small inductance. No. 137 and No. 103*d* show the most striking difference. In No. 103*d*, λ 3765 shows as a reversed line and is not reversed in No. 137, although in the latter λ 3720 is three times as wide as in the former. I cannot explain this peculiar behavior of the lines. Inductance

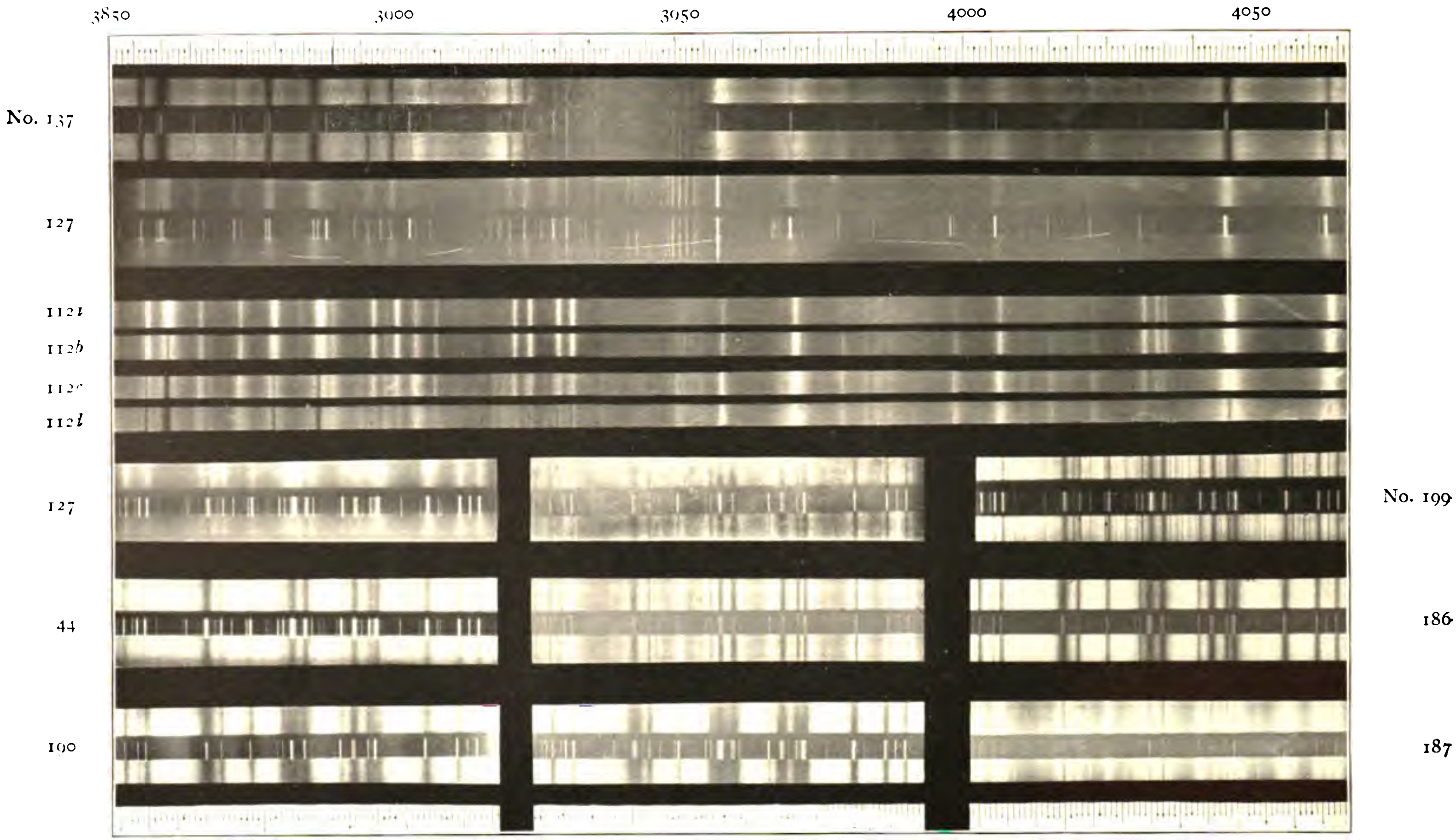
[1] See curve by Jewell, *Astrophysical Journal*, 3, 89, 1896; also Kayser's *Spectroscopie*, II, p. 360.

[2] Kayser's *Spectroscopie*, II, pp. 518, 296.

TABLE II

Spectrum No.	Plate No.	Pressure in Atmospheres	Inductance in Milhenrys	Capacity in Mfds.	Secondary Volts	Secondary Amperes	Secondary Watts	Gas	
44	V	80	0.0135	0.00152				*H*	
48*a*	IV	20						Air	
48*b*	IV	40						Air	**Watts in Primary,**
48*c*	IV	60						Air	
55*a*	IV	50	0.868	0.0309	5200	5.75	100.0	*H*	**740**
55*b*	IV	50	.594	.0309	5200	—	80.0	*H*	**740**
55*c*	IV	50	.278	.0309	5200	8.0	—	*H*	
55*d*	IV	50	.155	.0309	5200	9.0	—	*H*	**Res. in Secondary, 78 ohms**
63*a*	IV	50	.045	.0309	6800	2.35	4.1	*H*	
63*b*	IV	50	.045	.0309	7000	4.5	9.6	*H*	**16.9 ohms**
63*c*	IV	50	.045	.0309	6200	9.5	40.0	*H*	**4.69 "**
63*d*	IV	50	.045	.0309	6200	16.	115.0	*H*	**0.5 "**
71*a*	IV	50	.0135	.0309	2000 ?	18.	70.	*H*	**(leads and spark)**
71*b*	IV	50	.0135	.0309	4100	18.	100.	*H*	
71*c*	IV	50	.0135	.0309	7000	20.	90.	*H*	
71*d*	IV	50	.0135	.0309	2000 ?	22.	75.	*H*	
75*a*	IV	60	.0135	.0309	5000	18.	80.	*H*	
75*b*	IV	60	.0135	.0309	5000	—	75.	*H*	
75*c*	IV	60	.0135	.0309	5000	—	—	*H*	
75*d*	IV	60	.0135	.0309	5000	—	—	*H*	
79*d*	IV	30	.0135	.0309	5000	15.	—	*H*	
79*e*	IV	60	.0135	.0309	5000	—	—	*H*	
79*f*	IV	25	.0135	.0309	5000	15.	—	*H*	
85*a*	IV	10	.0135	.00543				*H*	**Watts in Primary,**
85*b*	IV	15	.0135	.00543				*H*	
85*c*	IV	30	.0135	.00543	2700	11.5	85	*H*	**520**
85*d*	IV	50	.0135	.00543	—	—	—	*H*	
85*e*	IV	70	.0135	.00543	—	—	—	*H*	
91*a*	IV	50	3.0	.0388	—	2.8	27.	*H*	**140**
91*b*	IV	50	3.0	.0388	4100	6.8	120.	*H*	**720**
95*a*	IV	40	25.0	.170	—	2.0	—	*H*	**Res. in Secondary, 4.9 ohms**
95*d*	IV	20	0.0135	.170	—	16.	—	*H*	
99*a*	—	50	25.0	.0388	4100	2.7	60.	*H*	**4.9**
99*b*	—	50	0.0135	.0388	4100	22.0	125.	*H*	
99*c*	—	50	0.0135	.0388	4100	—	150.	*H*	
99*d*	—	50	25.0	.00543	—	1. ?	45.	*H*	**4.9**
103*a*	IV	50	25.	.1286	—	1.5 ?	38.	*H*	**4.9**
103*c*	IV	50	6.46	.1286	—	—	—	*H*	**2.59**
103*d*	IV	20	0.0135	.1286	—	—	—	*H*	
107*a*	—	100	25.0	.00543	3200	1. ?	78.	*H*	**4.9**
107*b*	—	100	25.0	.0309	4000	2. ?	58.	*H*	**4.9**
107*c*	—	100	6.46	.00543	3800	2.	68.	*H*	**2.59**
107*d*	—	100	0.0135	.00543	4100	14.	95.	*H*	
107*e*	—	100	.0135	.002	4100	10.5	100.	*H*	
112*a*		50	140.0	.002	—	1.4	55.	*H*	**9.30**
112*b*		50	140.	.00543	—	—	30. ?	*H*	**9.30**
112*c*	IV	50	140.	.0309				*H*	**Res. in Secondary, 9.3**
112*d*	IV	50	140.	.1286				*H*	
116*a*	IV	40	140.	.002		1.0	35.	*H*	**9.3**
116*b*	IV	40	140.	.00543		0.45	35.	*H*	**9.3**
116*c*	IV	40	140.	.0309		.4	50.	*H*	**9.3**

PLATE V



No. 127 in H at 100 atmos. Ind. 140. Cap. 0.0054
44 in H at 80 atmos. Ind. 0.0135. Cap. .0015
100 in CO_2 at 100 atmos. Ind. 0.0135. Cap. .0054

No. 132 in H at 30 atmos. 500 Volts. *Fe* Arc.
134 in H at 30 atmos. Spark. Ind. 140. Cap. 0.223
137 in H at 70 atmos. Spark. Ind. 140. Cap. 0.223

No. 199 Solar Spectrum
186 in CO_2 at 53 atmos. Ind. 0.87. Cap. 0.027
187 in CO_2 at 50 atmos. Ind. 75. Cap. 0.027

TABLE II.—*Continued*

Spectrum No.	Plate No.	Pressure in Atmospheres	Inductance in Millihenrys	Capacity in Mfds.	Secondary			Gas	
					Volts	Amperes	Watts		
116*d*	IV	40	140.	0.1286		—	—	*H*	9.3
120*a*	IV	70	140.	.002		.6	55	*H*	9.3
120*b*	IV	70	140.	.00543		.4	68	*H*	9.3
120*c*	IV	70	140.	.0309		.45	60	*H*	9.3
126	—	100	140.	.00543		.4	55	*H*	9.3
127	V	100	140.	.00543		—	55	*H*	9.3
132	V	30	500 volt arc					*H*	
134	V	30	140.	.2231			37	*H*	9.3
135	—	50	140.	.2231			—	*H*	9.3
137	V	70	140.	.2231			—	*H*	9.3
143*a*	—	1	Iron arc					Air	Watts in Prim-
143*b*	—	10	0.0135	.0309		9	52	Air	ary,330.
143*c*	—	15	.0135	.0309		15	54	Air	550.
143*d*	—	20	.0135	.0309	3000 ?	15	58	Air	560.
143*e*	—	30	.0135	.0309	3200 ?	20	—	Air	—
143*f*	—	50	.0135	.0309	—	—	—	Air	—
143*g*	—	5	.0135	.0309	—	—	—	Air	—
143*h*	—	1	Iron arc					Air	—
									Watts in Prim-
149*c*	—	30	0.0135	.00543	3000	11	50	Air	ary, 500.
149*d*	—	30	0.0135	.00543	2700	11.5	85	*H*	520.
149*e*	—	50	0.0135	.00543	3000	8.5	50	Air	320.
149*f*	—	50	0.0135	.00543	—	—	—	*H*	
149*h*	—	70	0.0135	.00543	—	—	—	Air	
149*i*	—	70	0.0135	.00543	—	—	—	*H*	
161*a*	IV	70	75.0	.2231	—	—	—	*H*	
161*b*	IV	70	23.	.0734	—	—	—	*H*	
161*c*	IV	70	8.85	.0734	—	—	—	*H*	
161*d*	IV	70	0.703	.00543	—	—	—	*H*	
161*e*	IV	70	.703	.002	—	—	—	*H*	
167	IV	70	0.0135	.0309	Exposure time of *a*, *b*, *c*, *d*, and *e*, respectively, 2, 6, 26, 66, and 180 seconds.				
170*a*	IV	50	.0135	.0309	—	—	—	CO_2	
170*b*	IV	40	.0135	.0309	—	—	—	CO_2	
170*d*	IV	15	.0135	.0309	—	—	—	CO_2	
170*e*	IV	10	.0135	.0309	—	—	—	CO_2	
174*a*	IV.	50	.0135	.00543	—	—	110	CO_2	
174*b*	IV	50	.0135	.00543	—	—	—	*H*	
174*c*	IV	25	.0135	.00543	—	—	—	CO_2	
174*d*	IV	25	.0135	.00543	—	11	—	*H*	
174*e*	IV	15	.0135	.00543	—	—	—	CO_2	
174*f*	IV	15	.0135	.00543	—	—	—	*H*	
174*g*	IV	10	.0135	.00543	—	—	—	CO_2	
174*h*	IV	10	.0135	.00543	—	—	—	*H*	
174*i*	IV	5	.0135	.00543	—	—	—	CO_2	
186	V	53	.867	.0270	—			CO_2	
187	V	50	75.0	.0270	—			CO_2	
190	V	100	0.0135	.00543	—			CO_2	
199	V	Solar spectrum, same region.							

slows the discharge and causes conditions in the spark to approach more those of the arc. But it is just in the arc that numerous but narrow reversals occur (column 2, Table VI). To be sure, in the case of the arc, the briefness of exposure, and the fact that the spectrum is not a composite of numerous spectra as with the spark (p. 245), accounts largely for the more numerous reversals.

Wave-length.—In Plate V a portion of the spectrum toward the region of longer wave-lengths is shown for Nos. 112, 127, and 137. From these, and from the portions of the same spectrum numbers (also Plate V) in the vicinity of λ 3700, it will at once be seen (as many have observed) that the lines of short wave-length, in general, reverse much more readily than those of longer wave-length. As the character of the discharge becomes more disruptive and less like the arc, the lines of shorter wave-length become more prominent, and new "spark lines" appear. This is generally supposed to be accompanied by more intense heat in the spark. The light producing the more readily reversed lines may, then, originate near the center (hottest part) of the spark, and the surrounding vapor causes the reversal, while for the less readily reversed lines, the surrounding vapor is sufficiently hot to give them as bright lines, hence they do not ordinarily reverse (compare Lockyer's "long" and "short" lines of the arc).

Oscillation period.—Not much can be gained by comparing spectra taken with different oscillation periods, for the reason that in changing the period either capacity (C) or inductance (L), or both, must be changed; since $T=2\pi\sqrt{LC}$. It was thought, however, that with an exceedingly slow period the current would resemble that of the alternating arc, and so might also the spectrum resemble the arc spectrum. Accordingly, a very large capacity and inductance were used, giving a period of 1/900 second, which does not differ greatly from that of ordinary alternating circuits. Under these conditions No. 133, No. 134, No. 135, and No. 137 were taken at 20, 30, 50, and 70 atmospheres pressure, respectively.

It was next attempted to take arc (500 volts with 100 ohms in series) spectra at these same pressures for comparison, but the apparatus was not well adapted to the use of the arc, so that the best spectrum obtained, No. 132 (30 atmospheres), was hardly good

enough to reproduce. However, by comparing negative No. 132 and No. 134, which was taken at the same pressure but with the spark, it was found that the latter gave somewhat wider lines. The reason for this difference is not apparent, since the value of the current seems to be about 4 or 5 amperes in each case. For with the spark, the frequency multiplied by the capacity in farads (see No. 134, Table II), by the charging potential in volts, and by 4,[1] gives 4.9 amperes;[2] while from the resistance and voltage used, the current through the arc must have been about 5 amperes. This current was too intermittent to be measured.

Direct comparison of effects.—In some cases two spectra show that the effect upon the spectrum of a change in one variable is just about compensated for by a simultaneous change in another variable. Thus No. 95*d* and No. 95*a* are very much alike, showing that the widening effect upon the lines, caused by doubling the pressure, is balanced by the narrowing effect of increasing the inductance two thousand fold. With one-fourth as much capacity, but seven-fifths as high pressure, No. 120*c* is similar to No. 112*d*. Also the effect of increasing the inductance from 0.045 to 25 milhenrys appears to be offset by decreasing the resistance from 78 to 4.69 ohms, as shown by No. 63*a* and No. 99*a*. No. 161*e* and No. 161*a* show that increasing both capacity and inductance a hundredfold gives two and a half times as wide lines, hence capacity has the greater effect. Other similar cases might be cited, but the relative importance of these various quantities in affecting the spectrum can perhaps be better discerned from certain particular cases of maximum effects.

Maximum effects.—In studying the effect of capacity four different spectra were taken with the four different values given (see No. 116), first at 40 atmospheres, then at 50 (No. 112), then at 70 (No. 120). The inductance for all twelve spectra was 140 milhenrys. These spectra show that for a certain value of the capacity a given percentage change of the variable (capacity) produces a maximum percentage change in the width of the lines. This value of the

[1] The factor four is introduced because the condenser not only discharges but charges again in the opposite sense in one-half cycle.

[2] Not mean square, but average current during each group of oscillations.

capacity may then be said to be that at which the maximum effect of capacity change occurs (other conditions being as given). With different inductance, etc., this value of the capacity would probably be somewhat different. The percentage change in the variable (e. g., capacity) divided into the resulting percentage change in line-width may be termed a.

In Table III, column 6, the maximum value of a, found in the above way for the different variables is given, and the variable having the largest value for a, namely pressure, may be called the most important variable.

TABLE III

Spectrum No.	Reproductions Plate	Gas Used	Quantity Varied		Relative Width of Reversed Lines	a	No. Reversed Lines	Capacity	Pressure	Inductance
174e	IV	CO_2	Pressure	15 Atmos-	2	2.0	25	0.00543	15	0.0135
174g				10 phere	1		8	.00543	10	.0135
143c	—	Air	Pressure	15 Atmos-	3	1.0	35	.0309	15	.0135
143b	—			10 phere	2		27	.0309	10	.0135
79e	IV	H	Pressure	60 Atmos-	2	1.0	61	.0309	60	.0135
79d	IV			30 phere	1		61	.0309	30	.0135
112b	IV	H	Capacity	.00543	2	0.4	14	.00543	50	.140
112a	IV			.002	1		2	.002	50	.140
120b	IV	H	Capacity	.00543	3	0.2	19	.00543	70	.140
120a	IV			.002	2		7	.002	70	.140
55b	IV	H	Inductance	.58	2	−.11	62	.0309	50	.58
55d	IV			.14	3		69	.0309	50	.14
103a	IV	H	Inductance	25.	3	−.08		.1286	50	25.
103c	IV			6.5	4			.1286	50	6.5
71c	IV	H	Volts	2000 ?	3	−.13		.0309	50	0.0135
71a	IV			7000	2			.0309	50	.0135
174d	IV			H	4			.00543	25	.0135
174c	IV			CO_2	3			.00543	25	.0135
149d	—			H	3			.00543	30	.0135
149c	—			Air	2			.00543	30	.0135
91b	IV	H	Watts	120	4	.1		.0388	50	3.0
91a	IV			27	3			.0388	50	3.0
63b	IV	H	Ohms	16.9	4	−.08	61	.0309	50	0.0450
63c	IV			4.69	5		68	.0309	50	.0450
75b	IV	H	Sec. expos'd	120	$\frac{2}{3}$ to $\frac{1}{2}$			.0309	60	.0135
75c	IV			20				.0309	60	.0135

DISCUSSION OF RESULTS

Having shown the effect of these variables upon the width[1] of the lines, etc., it remains to correlate these various effects by showing

[1] All causes that widen the lines also increase the intensity of the continuous spectrum.

how the energy in the spark depends upon each variable. But the energy in the spark determines the amount, and largely the condition, of the vapor in the spark, which in turn determines the character of the spectrum.

Widening of spectral lines.—I wish first to refer briefly to some hypotheses that have been advanced to account for widening of lines. Among these may be mentioned (1) change of wave-length due to Doppler effect caused by motion of the light-emitting particles[1] (kinetic gas theory); (2) modification of the vibration period due to the presence of neighboring molecules;[2] (3) limitation of the number of regular vibrations by sudden changes of phase[3] resulting from collisions. To these I will add (4) a one-sided widening due to shift caused by change in the average dielectric constant (p. 253).

Of these, (1) is of first importance in rarefied gases, as pointed out by Michelson[4], and of little importance in my work. For the molecular speeds increase with the temperature, which presumably is not vastly higher in the spark under pressure than in the case of incandescent gases at low pressure, which latter give narrow lines. The other hypotheses would account for the great increase in the width of the lines through the crowding together of the molecules as the pressure, or the rate of generation of the metallic vapor, is increased. Especially important in the present work seems (3), by which Michelson has been able to account fairly well for the widths he obtained experimentally.

With the spark the light may be considered as originating mainly in the hot central "core" of the spark, and that in passing through the less heated vapors enveloping it certain wave-lengths are absorbed, giving rise to the absorption lines, just as in the case of the arc.[5] With the direct-current arc, however, we have an invariable source of light whose spectrum is the same from instant to instant, but with the spark I believe this is far from true.[6] The light pro-

[1] Lippich, *Pogg. Ann.*, **137**, 465, 1870; Lord Rayleigh, *loc. cit.*; Michelson, *loc. cit.*, p. 238.

[2] A. Schuster *Astrophysical Journal*, **3**, 292, 1896.

[3] A. A. Michelson, *loc. cit.*

[4] *Loc. cit.*

[5] Jewell, *loc. cit.*, p. 239.

[6] Schuster and Hemsalech, *Phil. Trans.*, **193**A, 189, 1900.

duced during the first oscillations of a group must pass through less cooler vapor than that produced by the later ones after the vapors have had time to accumulate;[1] and should therefore give narrower absorption lines, and also tend more to give bright lines. The spectrum obtained is then a composite of an infinite number of superposed spectra, giving varying degrees of reversal, and for some lines perhaps even superposing the bright line and the reversed line. The latter would account for the fogging of the reversed lines (on the negative) which makes determination of shift so difficult, and also would account in part for the failure of certain lines to reverse which do reverse in the arc. That the first oscillations of a group produce narrower reversals is proven, I believe, by No. 63*a*, for which a large non-inductive resistance was used to damp[2] the oscillations and cut out the later ones. To be sure, the energy of the spark is simultaneously decreased with increase of resistance, which also tends to narrow the lines.

According to the preceding hypotheses, it follows that the width of the metallic absorption lines should increase with the density of the metallic vapors, whether this density is produced by the pressure of the surrounding gas, by rapidity of evolution of vapor, or by both. The thickness of the absorbing layer—or, in other words, the amount of vapor—is important as regards absorption lines, which are the only lines present in many cases. I shall therefore limit myself to showing how increase of pressure, capacity, etc., influences the amount and density of the vapor.

Clearly the amount of vapor about the spark depends not only upon its rate of production, but also upon its rate of dissipation.

[1] H. Crew has recently shown that the alternating arc in different phases produces different spectra.

[2] From the logarithmic decrement, the relative charge on the condenser at the *n*th oscillation compared to the first is tabulated (Table IV) for various resistances up to 76 ohms. The capacity, inductance and some of the resistances are the same practically as those used for No. 63. It will be seen that with only 10 ohms, less than half (0.44) of the charge is left on the condenser after one complete period, although 76 ohms is necessary to make the discharge aperiodic. The table shows that the very similar spectra, No. 63*a* and No. 63*b*, must have been produced almost entirely by the first oscillation; and No. 63*c*, by the first five or six oscillations; because 16.9 ohms was used with No. 63*b*, and the table shows that after one full period but 0.25 of the charge remains, after four periods but 0.063.

TABLE IV

RELATIVE CHARGE ON THE CONDENSER AT THE nTH VIBRATION

(Inductance and capacity the same as used for No. 63. Period $= 7 \times 10^{-6}$.)

Resistance	When $t=T$, $n=2$	$t=2T$, $n=4$	$t=\frac{nT}{2}$
76.0 ohms	0.000		
50.0 ohms	.004	0.000016	
16.9 ohms	.25	.063	
10.0 ohms	.44	.194	
5.0 ohms	.67	.45	$n=8$ 0.002
1.0 ohms	.92	.846	$n=16$ 0.5
0.1 ohms	.993	.986	$n=160$ 0.5

The former increases with the energy; the latter increases with the temperature, with the spark-length, and with *decrease* of pressure (increase of free path). Increase of the energy in the spark through increase of the energy of the primary circuit should, then, increase the width of the reversed lines. Increase of capacity causes a greater amount of electricity to surge through the spark at each oscillation, thus developing more energy and more vapor, and consequently producing wider lines. The increase of energy in the former case is probably brought about mainly through causing more groups of oscillations to occur during each cycle (see footnote 2, p. 231), with but little change in the energy of each group. It is also probable that vapor formed by one group of oscillations is pretty well dissipated before the next group (about 1/1000 sec. later) occurs, while with increase of capacity the increase in energy comes with each group and each oscillation. This difference accounts for the fact that capacity change affects the spectrum more than (direct) energy change does (p. 237).

Increase of pressure not only increases the energy, but also decreases the free path. At the same time, the spark-gap must be made much shorter (p. 226); so that the vapors are more confined and hence the light from the "core" must pass through a much thicker stratum, as may be seen by comparing *A* and *B*, Fig. 8. For these reasons, pressure is the most important variable in affecting the spectrum.

Since increase of inductance (L) increases the period ($T = 2\pi\sqrt{LC}$), it will be seen that this decreases the current, as the oscillating charge is the same, and hence decreases the energy; and narrower lines should result, as found. As the spark-gap must be lengthened to increase the potential-drop over the spark, the escape of the vapor is thereby facilitated (see Fig. 8) so that narrower lines result with increase of potential, as shown in No. 71. On the other hand, increase of potential causes an increase in the charge, and hence in the energy, and therefore might be expected to produce wider lines. Under some conditions I think this might occur, but I have not observed it.

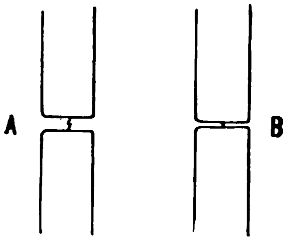


FIG. 8

The effect of the nature of the gas, duration of exposure, and oscillation period have already been discussed on pages 237, 238, and 242, respectively. The actual value of the current during oscillation is too uncertain, and its effect is also too intimately connected with that of energy, to warrant, or require, separate discussion, further than given on pp. 232 and 243.

SHIFTS OF SPECTRAL LINES

Humphreys and Mohler[1] showed that if the gas surrounding the arc is subjected to several atmospheres pressure, the arc lines are slightly displaced relatively to those of the arc at atmospheric pressure.

This displacement, or shift, was determined by them for several lines of nearly all the metals. Since then several investigators[2] have studied the shift of the spectral lines of the spark in compressed gases. Of these Hale and Kent used the highest pressures (1 to 53 atmospheres) and found, for some of the iron lines, shifts as high as 0.22 tenth-meters.

Whether other causes than pressure may produce shift in the case of the spark spectrum is a much-mooted question. Eder and

[1] *Loc. cit.*

[2] J. F. Mohler, *Astrophysical Journal*, **10**, 202, 1899; Hale and Kent, *ibid.*, **17**, 154, 1903; and others.

Valenta[1] found no such shifts, while Kent[2] and others have found them. Kent has recently repeated his work with great care and is satisfied that such shifts exist. In general, previous investigations have shown that shift increases with increase of capacity and decreases with increase of inductance.[3]

The shifts of about thirty lines have been determined from No. 187, which has fairly well-defined reversed lines, to see if any grouping of lines could be made upon the basis of equal shifts. The shifts of a few lines at pressures of 80 and 100 atmospheres have also been measured.

Measurement of shift.—The shift of the lines of the spark spectrum with reference to those of the arc comparison spectrum taken on the same negative was measured with the dividing engine, upon which was mounted a low-power microscope. In lieu of cross-hairs, a system of fine lines ruled on glass, as shown in Fig. 9, was finally used. The adjustment was so made that some one of these lines was near one edge of the spectral line, while some other one was equally distant from the other edge. By this means more accurate settings could be made than with a single cross-hair, set on the middle. A broken line is far superior to a continuous line, as it permits a nearly unobstructed view of the edge of the spectral line, which is essential with lines that shade off so gradually in intensity.

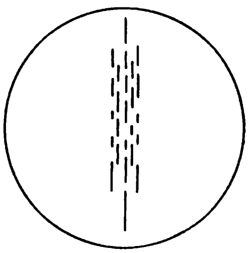


FIG. 9.—Reticle of microscope.

RESULTS

In Table V the shift and width of thirty lines of No. 187 and No. 186 (see Table II) are given, together with shifts obtained by Hale and Kent[4] for some of the same lines at about the same pressure and in the same gas (CO_2, 53 atmospheres). In column 4 *r* and *v* refer to the relative widths of the red and violet components of the bright line on each side of the reversal, *r* (red) being toward the longer wave-lengths. It will be seen that there is but one exception to the rule that the red component has the greater width. This

[1] *Astrophysical Journal*, **19**, 251, 1904.

[2] *Loc. cit.*, p. 221.

[3] Hale and Kent, *loc. cit.*, p. 221.

[4] *Loc. cit.*, p. 221.

is due partly to shift and partly to unsymmetrical widening of the bright line. In all cases the shift refers to the reversed (i. e., absorp-

TABLE V

Wave-Length of Line	Width in Tenth-Meters			Shift									
				For Plate No. 187 Determinations						Hale and Kent	Humphreys and Mohler	For Plate No. 186	
	Emission Line	Absorption Line	r* and v*	I	II	III	IV	V	Average			Shift	Width of Absorption Line
f3687.62	0.53	0.03	$r=1$ $v=1$	0.089	0.121	0.138	0.116	0.071	0.107			0.123	0.20
g3709.37	0.67	.11	$r=3$ $v=2$	.126	.112	.133	.106		.119				
f3722.71	0.47	.13	$r=4$ $v=1$	.064	.035								
e3733.47		.09		.04	.040		.040						
c3758.38	1.33	.212	$r=3$ $v=2$	.094	.076	.093	.082	.102	.089			.127	.60
d3763.94	0.93	.13	$r=2$ $v=1$	.084	.064	.085	.085	.093	.082			.135	.35
h3765.71	.27	0		.095	.090	.116	.102	.080	.097	0.12		.180	.18
e3767.33	.80	.08		.118	.104	.108	.111	.133	.115			.143	.25
3805.49	.27	0		.076	.093		.058	.061	.068				
h3813.17	.27	0			.076	.065	.089	.065	.074			.110	.20
e3815.99	1.20	.13	$r=2$ $v=1$	.091	.109	.112	.102	.093	.104	.14		.158	.35
c3824.63	0.66	.13	$r=2$ $v=1$	.058	.53	.054	.038	.055	.052				
e3827.97	1.10	.13	$r=3$ $v=2$	.100	.121	.136	.129		.126	.20		.175	.22
d3834.42	1.00	.16	$r=3$ $v=2$	.092	.096	.90	.105	.092	.095			.149	.32
c3856.52	0.66	.08	$r=3$ $v=2$	.043	.052	.053	.066	.061	.055			.085	.25
h3865.70		0		.076	.064	.081	.093	.060	.075			.160	.23
g3969.39	.50	0		.110	.080		.085		.092			.149	.26
h3977.91	.22	0				.082	.090	.054	.075				.24
h3997.55	.27	0				.081	.085	0.71	.079		0.129		.22
h4022.02	.20	0		.077	.090	.085	.097	.118	.093			.180	.64
h4030.89	.22	0					.106						
h4033.22	.18	0				.074	.077		.075				
c4045.98	1.60?	.18	$r=3$ $v=2$			.104	.113	.102	.106		.089	.170	.40
e4063.75	1.	.1		.100	.118	.112		.090	.105	.18	.102		
e4071.90	0.9	?			.128	.134			.131				
h4118.72	.27	0				.106							
h4132.20				.103	.112				.107			.195	.22
h4156.93	.20	0		.057	.087	.076		1.	.073				
h4199.26	.70	0		.085	.094	.092		.088	.090		.120	.180	.20
h4219.52	.27	0		.088	.114	.106		.079	.097		.085		

(* When a bright line has superposed upon it an absorption line, r and v, respectively, refer to the relative widths of the red and violet components of the bright line.

tion) line, excepting for those lines of No. 187 whose width of reversal (column 3) is marked zero.

Column 12 gives the calculated shift for the arc at 50 atmospheres, based upon shifts obtained by Humphreys and Mohler[1] at 10 or 12 atmospheres and upon a linear relationship between shift and pressure which they believe to exist. There is fairly good agreement between these calculated values and those determined by me with the spark at 50 atmospheres (column 10). The large capacity and inductance of No. 187 gives a frequency of only 3570, which is not vastly higher than sometimes used with the alternating arc, so that similar shifts might be expected (see p. 242).

It will be seen that the shift obtained by Hale and Kent (column 11) is uniformly larger than that of No. 187, and smaller than that of No. 186. When taking No. 186, a considerably larger capacity was used than that employed by Hale and Kent, which would account for the greater shift (due to unsymmetrical widening of lines); while No. 187, likewise with larger capacity, had also several thousand times as large inductance, which may account for the smaller shift (large inductance produces narrow lines, hence less of false shift due to unsymmetrical widening).

In Table VI the lines marked with a (+) are reversed. It will be seen that in No. 112*a* but two lines, $\lambda\lambda$ 3720.08 and 3737.28, are reversed; so that these may be termed the most easily reversed and are designated by *a* (column 1). Those reversed in No. 143*h* (the arc) and not in any others are designated by *g*. Those not reversed in the arc may be grouped as class *h*. It was thought that similar lines might be similarly shifted. In column 10, Table V, there are five lines with shifts from 0.073 to 0.075, which all belong to class *h*. To determine whether or not this accords to some law of shift of the lines of a series of Kayser and Runge, will require more experimental evidence than is at present available.

In Table VII are given shifts for some of the lines of plates Nos. 137, 44, 127, and 190, at pressures of 70, 80, 100, and 100 atmospheres, respectively. With No. 190, CO_2 was the gas used. At 100 atmospheres CO_2 is a liquid, in which it was found impossible to run the spark long enough to obtain a good spectrum. By heating

[1] *Loc. cit.*, p. 222.

TABLE VI

Wave-Length	Arc. No. 143h	No. 112c	Arc. No. 143a	No. 170e	No. 120b	No. 112b	No. 120a	No. 112a	Wave-Length	Arc No. 143h	No. 112c	No. 143a	No. 170e	No. 120b	No. 112b	No. 120a	No. 112a
f3680.00	+	+							d3834.42	+	+	+	+	+			
f3687.62	+	+							e3841.20	+	+		+				
d3705.00	+	+	+		+				c3856.52	+	+	+	+	+	+		
g3709.37	+		+						b3859.50	+	+	+	+	+	+	+	
a3720.08	+	+	+	+	+	+	+	+	h3865.70								
f3722.71	+	+	+						d3878.15	+	+			+			
e3727.79	+	+		+					c3886.43	+	+	+	+	+	+		
e3733.47	+	+	+	+					f3899.85	+	+	+					
b3735.01	+	+	+	+	+	+	+		h3922.5		+						
a3737.28	+	+	+	+	+	+	+	+	g3969.39	+							
b3745.50	+	+	+	+	+	+	+		h3977.91								
c3748.41	+	+	+	+	+	+			h3997.55								
b3749.63	+	+	+	+	+	+	+		h4022.02								
c3758.38	+	+	+	+	+	+			h4030.89								
d3763.94	+	+	+	+	+				h4033.22								
h3765.71									c4045.98	+	+	+	+	+	+		
e3767.33	+	+	+	+					e4063.75	+	+	+	+				
h3813.17									c4071.90	+			+				
e3815.99	+	+	+	+					h4118.72								
b3820.59	+	+	+	+	+	+	+		h4132.20								
c3824.63	+	+	+	+	+	+			h4156.93								
c3826.02	+	+	+	+	+	+			h4199.26								
e3827.97	+			+					h4219.52								

the compression tube to 38°, which is well above the critical temperature for CO_2, a fairly good spectrum was finally obtained after about two weeks' work. Only a few of the lines on these negatives are well enough defined to permit of shift determinations, but they were the best obtainable at such high pressures.

The shifts on No. 190 are larger than those of No. 127, although both at 100 atmospheres. For No. 127 hydrogen was the gas used,

TABLE VII

λ	Plate No. 190		Plate No. 127		Plate No. 44		Plate No. 137		Plate No. 187	
	Shift	Width	Shift	Width	Shift	Width	Shift	Width	Shift	Width
3763.04...	0.231	0.75			0.131	0.40			0.082	0.13
3767.33...	.264	.80	.44	0.70	.150	.60			.115	.13
3856.52...			.116	.60	.100	.35	.088	.75	.055	.08
3969.39...	.292	60								
4063.75...	.257	.65					.124	.50	.105	.1

Shift and Width are expressed in tenth meters

and the shifts are about the same as those of No. 187 at 50 atmospheres in CO_2. Probably as a result of greater shift of the emission line than of the superposed absorption line, the violet component of the emission line is sometimes wanting.[1] The following lines of No. 170*b* are of this class:

3977.89	4107.65
4009.86	4156.97
4014.68	4175.81
4022.02	4219.52

To account for shift, Schuster[2] has suggested that the light-emitting particle may have a period slightly greater than its free period because of the proximity of other particles of like period. Also, according to the electromagnetic theory of light, the atom may be viewed as an infinitesimal Hertz oscillator whose capacity will increase if the dielectric constant ϵ, of the surrounding medium is increased.[3] An increase of ϵ may be brought about by increase of pressure as well as by changing from hydrogen[4] ($\epsilon = 1.000264$) to CO_2 ($\epsilon = 1.000946$); for if

$$\begin{aligned} \epsilon &= 1 + a\,, \\ \epsilon_1 &= 1 + Pa\,, \\ &= 1 + P(\epsilon - 1)\,, \end{aligned}$$

where ϵ and ϵ_1 are the dielectric constants of a gas at one, and at p atmospheres pressures, respectively. Calculating upon this basis and from $T = 2\pi\sqrt{LC}$, the shift of the lines of No. 127 (100 atmospheres pressure) for λ 4000 should be about 72 tenth-meters. The observed shifts are only about 0.15 tenth-meters.

If, however, we postulate a certain region B about the atom into which no other atoms can enter, we thus preclude variation of ϵ in just that region whose ϵ most affects the capacity of the atom. This might account for the above great discrepancy between the observed and the calculated shifts. A metal for which B is large would give correspondingly small shifts. My results point to greater shifts with the gas of greater ϵ, namely CO_2; still it seems that in the dense metallic vapors in the spark, few CO_2 atoms will

[1] Hale and Kent, *loc. cit.*, p. 221.

[2] *Astrophysical Journal*, **3**, 292, 1896. [3] *Ibid.*, **5**, 210, 1897.

[4] L. Boltzmann, *Pogg. Ann.*, **155**, 403, 1875.

mingle, so that the value of ϵ_1 will depend rather upon the proximity of neighboring metallic atoms.

In applying what we know of finite condensers to a supposed one of atomic dimensions, we verge on the purely speculative. This theory, however, may yet be shown to be quite tenable.

SUMMARY

It has been shown that the lines become wider with increase of capacity,[1] pressure,[1] and energy, and narrower with increase of inductance,[1] resistance, voltage, and time of exposure. The probable cause of widening in all cases is an increased amount of vapor about the spark, which depends upon the rate of production of the vapor (which in turn depends upon the watts expended in the spark during oscillations), and upon its rate of dissipation. When CO_2 is used in the compression chamber, wider lines are obtained than with air, but narrower than with hydrogen. Estimates of the probable value of the current, watts, potential-drop, and resistance of the spark during oscillations have been made, based upon the observed (average) values.

The shifts for thirty different lines have been determined and found to be about the same as those measured by Humphreys and Mohler (with the arc), if reduced to the same pressure. Shifts have been determined for pressures up to 100 atmospheres in CO_2 and hydrogen, the latter giving the smaller shifts.

In conclusion I wish to thank Professor Mendenhall for the interest he has taken in the work, and Professor Snow for placing at my disposal all the necessary material and apparatus. I also take great pleasure in acknowledging my indebtedness to Mr. H. W. Kircher, for faithful assistance and valuable suggestions during the early part of the work; and to Mr. R. J. Wallace, photophysicist at Yerkes Observatory, for numerous suggestions in preparing the positives for the photo-engraver.

PHYSICAL LABORATORY, UNIVERSITY OF WISCONSIN,
August 6, 1906.

[1] Hale and Kent, and others, had already observed these effects of capacity, pressure, and inductance, but with smaller values and through much smaller range.

PHOTOGRAPHIC OBSERVATIONS OF GIACOBINI'S COMET (1905 *c*)

By E. E. BARNARD

When near perihelion this comet was unfortunately situated for photographic observation. It was low in the east and was involved in dawn and daylight; when in a very active condition it disappeared in the direction of the Sun. In the latter part of December and the first part of January the comet began to show a strong activity and shot forth a tail 8° or 10° long. To the naked eye it appeared like a hazy star of the fourth or fifth magnitude. If the position had been favorable for observation, the comet would doubtless have become one of the most interesting yet observed. Despite the unfavorable situation, if the morning skies had been clear, a valuable series of photographs would have been obtained. The weather here, however, was unusually unfavorable, and the mornings, with but few exceptions, were overcast. Several photographs were secured, however, on the few mornings the comet could be seen. These pictures are extremely interesting, and I believe they are important. If photographs of the comet were made elsewhere on the same mornings, sufficient distinctive features were visible to give good data for determining the motion of the particles of the tail. Some of these phenomena were much like those of Swift's comet of 1892, while others seemed peculiar to this comet.

On December 7, 1905, the night following the discovery of the comet by Giacobini, it was photographed here with the 10-inch Brashear doublet, but it did not show any features of interest; a faint tail was visible for about ½°. The comet at this time was faint in the 5-inch guiding telescope.

The next photograph was made on December 25. On this plate the tail was about 4° long, and was faint and slender.

On December 29 the tail was remarkably and beautifully developed. The photograph on this date (Plate VI) is in some respects quite unique. From a rather large head and a slender neck the tail widens out on each side in a graceful curve, which partly closes

in again and gives a strong convexity to the tail. The edges of these convexities are sharply defined and are outlined by a rather narrow bright rim or border. The appearance of the tail strongly suggests a hollow convex transparent cone with a sensible thickness. A straight-edge placed along the sides of the tail shows this convexity strikingly, as may be seen on an examination of the plate. I have not noticed quite this appearance before in a comet's tail. Besides this peculiarity of convexity, there is considerable structure in the tail. Several very faint threadlike streams diverge backward from the head on each side at angles of approximately 15°, 20°, and 30° to the axis of the main tail. On the south side, about 1° back from the head, is a long diffused strip running out from the edge of the tail at a very slight angle. This strip seems to cross the bright rim on to the body of the tail, as if it were nearer to us. The entire length of the tail in the picture is about 4½°.

On the night following this, December 30, the head is small, and the first part of the tail seems to be made up of a great number of threadlike strands which diverge from the sides near the head; while the main part of the tail, at first narrow, spreads out in a diffused manner at a large angle from about 1° back from the head. The tail is almost bifurcated in places. The entire comet in this picture differs widely from its appearance on December 29.

No other photograph was possible until January 4, 1906, on which date the plate was much affected by daylight. The tail consists of one main strand, which slightly widens as it recedes from the comet. On the east side there is a slender lateral tail diverging only slightly from the main tail, which is about 8° long.

January 5.—The tail can be traced to the edge of the plate, a distance of 10¼°. In this picture the brightness of the central part of the tail is irregular and wavy. There is at least one slender streamer on the west side of the comet. The exposure was closed in this picture when the Sun's center was 16¼° below the horizon. Even under these conditions the photograph was affected by the dawn.

January 7.—This is a very interesting photograph (Plate VII). The head is very slender; the tail widens out rapidly and seems to be made up of a great number of strands. About 1° from the head is a great deal of beautiful thread-like structure. At this point the

PLATE VI

GIACOBINI'S COMET (1905 *c*)
December 29, 1905, at $23^h\ 20^m$ G. M. T.
Exposure $1^h\ 38^m$. Enlarged 2.5 times. Scale: $1° = 63.5$ mm.

tail is separated into three streams, the central one being very long with irregular masses on it.

January 8.—This plate is badly fogged by daylight. The tail is at least 10° long and irregular; the central brightness alone shows; there is a slender thread on the east side which does not diverge much.

January 9.—This picture was killed by daylight. Very little of the tail shows.

When in these notes reference is made to the length of the tail, it refers to the photograph taken with the 6¼ inch lens.

Following is a list of all the exposures made here with the 10-inch and 6¼-inch lenses of the Bruce telescope:

1905 Dec.	7	
	25	
	29	
	30	
1906 Jan.	4	
	5	
	7	
	8	Injured by daylight
	9	Ruined by daylight
Feb.	21	

The exposure on February 21 was after the comet had passed perihelion, and shows only a suggestion of a tail.

An inspection of these photographs shows that, though the tail was subject to great physical changes, there were no deflections of its general direction. This will be seen by the following table of position angles. These values were obtained by locating the direction of the tail on the *B.D.* charts. I have also taken from these charts the position of the comet's head at the time of each photograph. They refer to the epoch of 1855.0.

TABLE OF POSITIONS OF THE COMET, AND OF THE POSITION ANGLES OF ITS TAIL

Central Standard Time		α	δ	P. A.
1905, December 25........	17^h 6^m	16^h 7^m	+9° 0′	317°
" " 29........	17 20	16 34	+5 12	312½
" " 30........	16 58	16 41	+4 24	312
1906, January 4........	18 2	17 18	−1 11	312
" " 5........	17 37	17 25	−2 20	312
" " 7........	17 45	17 42	−4 30	312½

I think there is no question that all the phenomena of this comet were due entirely to the action of the Sun. There does not seem to be any evidence of any outside influence to distort and rupture the tail, as was so evident in the case of Brooks' comet (1893 IV). Indeed, the more I see of comet photographs and of the changes they show in a comet's tail, the more I am convinced that the phenomena of the tail of Brooks' comet were unique, and that they were due to a disturbing influence foreign to the comet and the Sun.[1]

It is a very serious question, in photographing a comet near the Sun, as to when the exposure can begin, if in the evening sky, and when it must cease, if in the morning. One's judgment is rather fickle, for there is always the desire to get as much exposure on the comet as possible. This sometimes leads the observer to carry the exposure too far, and he not only does not get more on his picture, but he may lose that which he has already secured. I have thought that a table might be constructed with the Sun's distance below the horizon as an argument to guide one in closing the exposure in the morning, or beginning it in the evening. But this would probably do more harm than good, for everything would depend on the proximity of the comet to the position of greatest dawn effect, and also on the purity of the sky. Each succeeding morning would bring the comet on a brighter background, if approaching the Sun, at corresponding moments of time, and hence dependence on such a table would be apt to result in ruin of the plate by overexposure of the bright sky. The only guide would therefore seem to be the observer's judgment of the appearance of the sky at the time. This can be fairly relied on, if much work of this kind is done, which we know is not often the case—for the want of opportunity. The best plan to pursue is to so regulate the exposure as to give rather less than the plate will bear, otherwise what has already been obtained is likely to be lost.

YERKES OBSERVATORY,
August 1, 1906

[1] See *Astrophysical Journal*, **22**, 249–255, November 1905.

PLATE VII

GIACOBINI'S COMET (1905 *c*)
January 7, 1906, at $23^h\ 45^m$ G. M. T.
Exposure 50^m. Enlarged 3.4 times. Scale: $1^\circ = 80.4$ mm.

THE PERIOD OF β *CEPHEI*

By EDWIN B. FROST

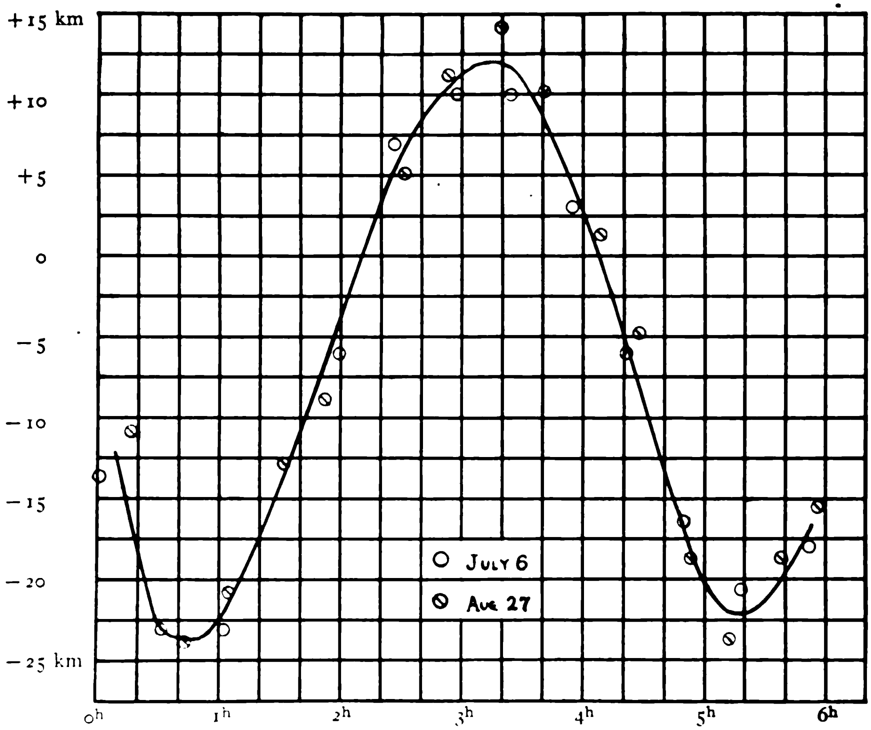
The period of the spectroscopic binary β *Cephei* has been under investigation here, intermittently, for the past four years. The results of the measurement of the first eleven spectrograms, taken between December 18, 1901, and May 23, 1902, were given in this Journal for June 1902 (**15**, 340). Although I inferred from two plates taken on the night of May 14, 1902 that the period was short, the subsequent scattered observations could not be reconciled with periods of 3/4 day, or 3/2 day, or 3 days, which seemed to be indicated by certain plates.

During 1903, 1904, and 1905 the star was seldom observed, a total of only twelve plates being secured. On resuming the observation of the star this season with a view to the settlement of the question, I arranged to have several plates taken on each night, and we were fortunate in obtaining a total of eight plates on the successive nights of May 18, 19, 20, and 21. Approximate measurements of these plates confirmed my suspicion that the period was very short, and indicated that it was very close to 3/16 day or $4^h 30^m$.

I then decided to have the star observed continuously through several nights, and the result has been that we (Mr. Barrett or the writer, with the assistance of Mr. Sullivan) secured on May 28, sixteen one-prism plates, with an average exposure of about 12 minutes; on July 6, fourteen two-prism plates, with an average exposure of about 22 minutes; on August 27, twenty two-prism plates, with an average exposure of about 20 minutes. Less complete sets of two-prism plates were obtained on June 18 and June 22.

These plates show conclusively that the star's radial velocity passed through its whole cycle even in a short summer's night. As I have at present no regular assistance in the measurement of spectrograms, a considerable time will elapse before I can measure and discuss our plates, now numbering 110, and I have therefore thought that this preliminary statement might be of interest in view of the exceptional shortness of the period.

In a case like this the exposure-time is a matter of much importance, as it may constitute a considerable fraction of the star's whole period. Of the first forty-six spectrograms obtained here, all but three were taken with a dispersion of three prisms. The exposure varied from 30 minutes, on an especially good night, to 140 minutes on a night when the plate was taken "through thick clouds." A normal exposure for this star under average atmospheric conditions, with three



Provisional Velocity-Curve for β *Cephei*

prisms and the 24-inch (608 mm) camera, would be about 60 minutes. This is nearly one-quarter of the period, so that the early plates cannot give precise epochs for determining the period. For this reason I do not at this time attempt to give the period definitively; but I think it will prove to be very close to 0^d1904 or $4^h\ 34^m\ 11^s$.

The effect of the change of velocity during exposure will generally be to diminish the range of velocity and flatten the velocity-curve.

Provisional measures indicate a range of velocity (on the two-prism plates taken with short exposures) of about 34 kilometers, from about +12 km to about −22 km. This implies a velocity of −5 km for the system, and an orbital velocity of 17 km per second.

The accompanying diagram shows a provisional velocity-curve from the observations of July 6 and August 27. The plates of the latter date have been only roughly measured and approximately reduced. The sides of the squares represent 20 minutes of time and 2.5 km of velocity. Inasmuch as the velocity derived from a single plate might, under the circumstances, depart by 5 km from the true value, the agreement of the observations will probably be regarded as satisfactory; although the curve is of course inadequate for a proper determination of the orbit.

Assuming the period to be 274.2 minutes, and the circular orbital velocity to be 17 km per second, the radius of the orbital motion of the bright star would be only about 45,000 kilometers (28,000 miles), as projected upon the line of sight. It is therefore natural to infer that the plane of the orbit is greatly inclined to the line of sight. For instance, if the radius of the orbit of the brighter star is assumed for the moment to be the same as that found by Vogel for *Algol* (1.6 million km), then the inclination of the plane would lack only about $1\frac{1}{2}°$ of $90°$; and the observed projected velocity would have to be increased nearly forty-fold, yielding an actual velocity of over 600 km per second. Such speculations will be more appropriate after our plates have been fully measured and the orbit has been determined. They tend, however, to imply that the distance of the brighter body from the center of gravity of the system is very small, and they raise the question whether the bodies must not be nearly in contact. Fortunately the period is not a subject of speculation.

On some of the plates, certain lines have the appearance of complexity, and indicate the presence of a second component spectrum. This suggests that the second star may be not more than one or two magnitudes fainter than the brighter star.

The spectrum is an excellent example of the *Orion* type, with fairly sharp silicon lines in addition to those of helium, hydrogen, oxygen, and magnesium, and it may be regarded as well measurable,

in so far as not complicated by the superposition of lines from the second star.

It is certainly astonishing in working on this star to find that plates taken in immediate succession, with the epoch of mid-exposure separated by less than half an hour, show marked differences of radial velocity, at times reaching 10 kilometers.

So far as known to the writer, the shortest period hitherto determined for a spectroscopic binary is 1.45 days for *μ Scorpii*, as found by Professor S. I. Bailey;[1] and for *V Puppis*, as found by Professor E. C. Pickering[2] (the latter star an *Algol*-type variable[3]). We may compare with the period of *β Cephei* that of the variable *W Ursae Majoris*, $4^h\ 0^m$; and that of 14. 1904 *Cygni*, $3^h\ 14^m$, which is the shortest period known for a variable star.

YERKES OBSERVATORY,
October 1906.

[1] *Astrophysical Journal*, **4**, 253, 1896.

[2] *Ibid.*, **4**, 373, 1896; **7**, 139, 1898.

[3] The period of the light-variation of *R Canis Majoris*, found last year to be a spectroscopic binary, is $27^h\ 16^m$; but we have not yet obtained spectrograms enough to establish its period spectroscopically.

ON THE THEORY OF CEMENTED DOUBLETS

By HENRY C. LOMB

The theory of the aberrations of optical systems as developed by the lamented Abbe leaves little to be desired in point of perspicuity and elegance. The simplification accomplished by Abbe is due in great measure to the introduction of a certain function of the geometrical and physical constants of a refraction at a given surface, which function, known as Abbe's Invariant, may be defined as follows.

In Fig. 1, let ρ be the reciprocal of the radius OC of a spherical surface separating two media of refractive indices n and n', and let OA be the "optical axis" about which the portion of the surface considered is symmetrical; further, let σ and σ', respectively, be the reciprocals of the distances OS and OS' at which the incident ray PS (produced) and the refracted ray PS' meet the axis, all these distances being measured from the vertex O of the surface as origin, the direction of the incident light regarded as positive. (In Fig. 1, for example, ρ, σ and σ' are all positive.) Then, for rays near the axis, Abbe's Invariant Q is defined by the relation

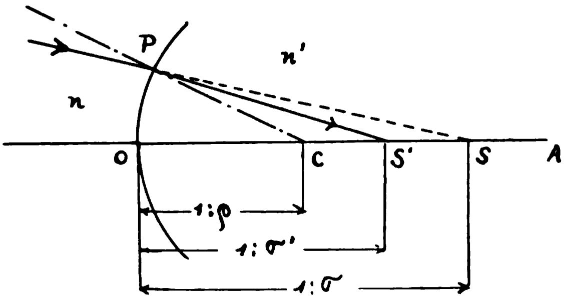


FIG. 1

$$n(\rho-\sigma)=Q=n'(\rho-\sigma') .$$

The name "invariant" refers, of course, to the property of the function remaining invariable or of the same form before and after the refraction. Its physical significance becomes apparent if we remark that Q is another form of the law of refraction

$$n \sin i = n' \sin i' ,$$

where i and i' are small angles of incidence and refraction.

In the following, we shall apply the formulae of Abbe to the case of a thin cemented doublet, i. e., of two thin lenses in contact, the two adjoining surfaces of the lenses having the same curvature. Considering, for the time being, monochromatic light only, the qualities which such an objective ought primarily to possess are:

1. It must have a prescribed *focal length*.

2. It must be free from *spherical aberration;* that is, all rays proceeding from a point S *on* the axis (Fig. 2) must be accurately refracted to a single point S' on the axis.

3. It must fulfil the so-called *sine condition;* that is, rays proceeding from a point P (Fig. 2) at a small distance from the axis and on a perpendicular SP to it, must be brought to a focus at a point P' also on a perpendicular $S'P'$ to the axis, the points S and S' being the same points considered under 2.

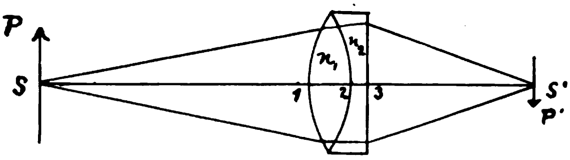


FIG. 2

The fulfilment of 2 is a prerequisite for the fulfilment of 3, and only when both conditions are simultaneously met can a correct image of an object of even moderate dimensions be formed by the lens-system.

We proceed to formulate these conditions analytically. Let ϕ be the reciprocal of the equivalent focal length, that is, the power, of the complete system, ϕ_1, ϕ_2 the powers respectively of its two component lenses. Then

$$\phi=\phi_1+\phi_2\,. \tag{1}$$

Employing the notation of Fig. 1, we have, for the spherical aberration of a system of k surfaces,[1]

$$S_{\mathrm{I}}=\sum_{k=1}^{k}Q_k^2\Delta_k\left(\frac{\sigma}{n}\right)=0 \tag{2}$$

and for the sine condition

$$S_{\mathrm{II}}=\sum_{k=1}^{k}Q_k\Delta_k\left(\frac{\sigma}{n}\right)=0\,. \tag{3}$$

[1] S. Czapski, *Grundzüge der Theorie der optischen Instrumente nach Abbe*, pp. 115, 148. Leipzig, 1904.

Here Δ_k denotes the variation of the quantity following it, before and after refraction at the kth surface; thus

$$\Delta\left(\frac{\sigma}{n}\right)=\frac{\sigma'}{n'}-\frac{\sigma}{n},$$

the primes indicating the values after refraction.

The sums (2) and (3) are particularly valuable in studying an already existing or an arbitrarily assumed optical system. They are readily computed in connection with the trace of an axial ray, and the influence of each surface on the final image is given by the size and sign of the term which it contributes to the total sum. But in order to discover all possible solutions which simultaneously fulfil some or all of the conditions noted above, it is essential that the expressions which determine them contain not, as here, several unknown quantities, but *one* unknown quantity only, with reference to which the equation can then be solved. For this unknown variable we take the optical invariant of one of the surfaces, namely that of the surface which is common to the two lenses. The particular advantages accruing to this selection will presently appear.

Consider the case of a single thin lens in air, of power ϕ and of glass having a refractive index n. Applying (2) to a system of two surfaces, we have for its spherical aberration,

$$S=Q_1^2\Delta_1\left(\frac{\sigma}{n}\right)+Q_2^2\Delta_2\left(\frac{\sigma}{n}\right). \tag{4}$$

Now, at the first surface of the lens

$$Q_1=\rho_1-\sigma_1\,,\qquad \frac{Q_1}{n}=\rho_1-\sigma_1'\,,$$

whence

$$Q_1\left(\frac{1}{n}-1\right)=\sigma_1-\sigma_1'\,.$$

That is,[1]

$$\Delta_1\left(\frac{\sigma}{n}\right)=\frac{\sigma_1'}{n}-\frac{\sigma_1}{1}=\left(\frac{1}{n}-1\right)\left(\sigma_1-\frac{Q_1}{n}\right). \tag{5}$$

Similarly, at the second surface of the lens

$$\Delta_2\left(\frac{\sigma}{n}\right)=\left(\frac{1}{n}-1\right)\left(\frac{Q_2}{n}-\sigma_2'\right). \tag{6}$$

[1] Cf. *l. c.*, p. 117.

Substituting (5) and (6) in (4),

$$S=\left(\frac{1}{n}-1\right)\left\{\sigma_1 Q_1^2-\sigma_2' Q_2^2-\frac{1}{n}(Q_1^3-Q_2^3)\right\} . \tag{7}$$

Further,

$$\sigma_2'-\sigma_1=\phi\ , \quad \text{and } Q_1-Q_2=\frac{n\phi}{n-1} .$$

Accordingly, as we eliminate Q_2 and σ_1, or Q_1 and σ_2' from (7), we find, after several reductions, for the spherical aberration of a single lens of power ϕ, the two equivalent forms

$$S=\phi\left\{\left(1+\frac{2}{n}\right)Q_1^2-\left(2\sigma_2'+\frac{3\phi}{n-1}\right)Q_1+\frac{n\phi}{n-1}\left(\sigma_2'+\frac{\phi}{n-1}\right)\right\} , \tag{8}$$

$$S=\phi\left\{\left(1+\frac{2}{n}\right)Q_2^2-\left(2\sigma_1-\frac{3\phi}{n-1}\right)Q_2-\frac{n\phi}{n-1}\left(\sigma_1-\frac{\phi}{n-1}\right)\right\} . \tag{9}$$ [1]

Now apply (9) to the first lens of the doublet (Fig. 2) and (8) to the second lens. Then, since the adjacent surfaces have the same curvature, the variable Q will be identical for both lenses, and employing the notation of the figure, we finally derive for the total ***spherical aberration of the cemented doublet,***

$$S_{\mathrm{I}}=\left\{\phi+2\left(\frac{\phi_1}{n_1}+\frac{\phi_2}{n_2}\right)\right\}Q_2^2+\left\{3\left(\frac{\phi_1^2}{n_1-1}-\frac{\phi_2^2}{n_2-1}\right)-2(\phi_1\sigma_1+\phi_2\sigma_3')\right\}Q_2$$
$$+\frac{n_1\phi_1^2}{n_1-1}\left(\frac{\phi_1}{n_1-1}-\sigma_1\right)+\frac{n_2\phi_2^2}{n_2-1}\left(\frac{\phi_2}{n_2-1}+\sigma_3'\right) . \tag{10}$$

In much the same manner we find for the *sine condition of the* ***doublet,***

$$S_{\mathrm{II}}=\left\{\phi+\left(\frac{\phi_1}{n_1}+\frac{\phi_2}{n_2}\right)\right\}Q_2+\left(\frac{\phi_1^2}{n_1-1}-\frac{\phi_2^2}{n_2-1}\right)-(\phi_1\sigma_1+\phi_2\sigma_3') . \tag{11}$$

Here

$$\sigma_3'-\sigma_1=\phi=\phi_1+\phi_2 . \tag{12}$$

Equations (10) and (11) are the relations sought; their vanishing carries with it the annihilation of the spherical aberrations in and without the axis, of a cemented doublet having a power of ϕ and having $1:\sigma_1$ and $1:\sigma_3'$ for conjugate focal distances. In the case of the telescope objective for which $\sigma_1=0$ and $\sigma_3'=\phi$, the equations reduce to a still simpler form.

We note that for any one particular value of the parameter ϕ_1 (or ϕ_2) there exist but two doublets which are spherically corrected

[1] (9) may also be obtained from (8) by reversing the direction of the light.

in the axis and but one form which fulfils the sine condition, but the root of (11) need not necessarily be a root of (10). In practice, we are restricted by the limited number of glasses available, so that it is not always possible to completely correct both spherical errors and, simultaneously, other errors (for example those of color) also. In such cases these unavoidable deviations from perfection must be apportioned to the best advantage.

The value of the equations (10) and (11), aside from the favorable form of the coefficients, lies in the fact that each of the spherical aberrations is expressed directly in terms of the powers of the component lenses and the prescribed conjugate focal distances. The symmetry and simplicity of the coefficients are evidently due to the peculiar choice of the unknown quantity, namely the invariant Q_2 of the cemented surface, about which surface the doublet is, in a measure, symmetrical.

The above methods are applicable to other aberrations for which algebraic expressions are known, and they may also be applied to more complicated optical systems by dividing the latter into doublets. But a more detailed discussion of this subject would exceed the limits of this paper.

ROCHESTER, N. Y.,
August 6, 1906.

COLOR-FILTERS FOR ASTRONOMICAL PHOTOGRAPHY WITH REFLECTING TELESCOPES

By ROBERT JAMES WALLACE

The great advantage now taken of photography in recording astronomical data, and the ease with which a visual refractor may be converted to a photographic by means of a color-filter, have gradually changed and enlarged the methods in telescopic work until there is now practically no branch which can not or could not be better performed by its aid.

The lens of the refractor is corrected for a certain limited spectral region—generally, the yellow-green near λ 5550, because in this region the eye is most sensitive to slight differences, the remaining hues coming to an approximate focus at varying distances from this point of correction. The function of the color-filter consists simply in absorbing from the incident light all other hues but those for which the lens is corrected.

Strictly speaking, it is not possible to construct a filter which will accomplish this end by itself without very greatly lowering the luminosity of the hue transmitted, nor is it necessary to do so. Because of the selective sensitiveness of the photographic plate we are able to divide the work between the filter and plate. If, for example, we select such a plate as Cramer's Instantaneous Isochromatic, we find a secondary maximum of photographic action which corresponds to the yellow-green of the spectrum; the insensitiveness of the plate (at normal exposure) to red or orange renders it unnecessary to absorb either of these hues by the filter, but only to absorb the ultra-violet, violet, blue, and to dim down the blue-green, the plate itself being but slightly sensitive to this hue.

In the case of the reflecting telescope the necessity for employing a color-filter has not been recognized to the same extent, because of the fact that all the component rays in white light are brought to a focus at the same plane, and it requires but a limited experience to obtain with this instrument a photographic record of telescopic objects which show structural detail far beyond the ability of the eye to perceive.

There is, however, one very serious defect in the photographs thus obtained; that is, an almost exact reversal of color-luminosity. If we compare the luminosity-curve of the spectrum with the intensity-curve of the ordinary photographic plate, we see in a moment how utterly false and unreliable is the result in so far as color is concerned. Considered even as a representation of *form*, it does not conform to the requirements of a "record of fact," because it is quite within the bounds of possibility to conceive of structural detail visible with a hue to which the plate is altogether insensitive.

This discrepancy between the visual luminosity and photographic intensity is very evident in the case of direct stellar photography, which becomes considerably worse when "extra-focal" images are made use of, as in the later methods of photographic photometry. The use of "isochromatic" plates tends but slightly to a betterment, for, when we consider that such a plate still retains its maximum sensitiveness to the violet end of the spectrum, then it can be seen that conditions are not greatly improved.

In consideration of the needs of this branch of the work, the writer undertook the preparation of a color-filter for use with the 24-inch reflector, to be used initially in obtaining negatives for photographic stellar photometry. Briefly stated, the method employed in adjusting such a filter consisted in (1) isolating a few dyes and making of them a special spectroscopic examination, singly and in combination; (2) coating trial filters with carefully measured amounts of gelatin containing known amounts of dye and estimating the approximate density of the spectrum as photographed through this filter against the normal luminosity-curve; (3) coating the optical glass with the amount of dyed gelatin as thus determined and measuring the density of the spectrum negatives taken through this screen; (4) photographic determination of the absorption at various exposures, and of the exposure increase.

From knowledge gained by a somewhat extended experience in making a large number of filters for various purposes, the following dye-stuffs were selected, estimated as being fairly near to the absorption required, viz.:

Tartrazine
Auramine O.
Metanil yellow S.
Nitrosodimethylanilin

From these dyes color-wedges were now prepared. A solution was made of

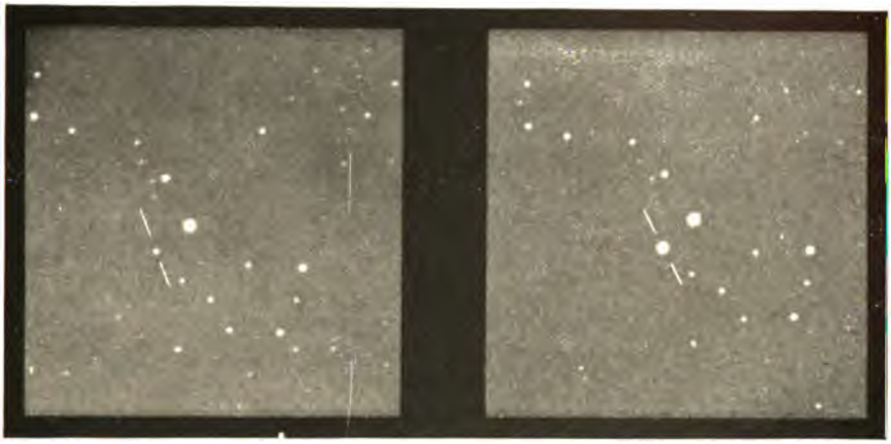
Gelatin (Fischer and Schmidt—extra hard) . .	2.5 grams
Distilled water	100.0 cc
Dye	0.25 gram

Seven cc of this solution at a temperature of 55° C. was flowed upon a plane glass strip 50×250 mm, and then laid aside to set in a drying-chamber, with one end raised to a height of 5 mm from the horizontal plane of the support, thus causing the gelatin to flow slowly toward the lower end. When dry a plane cover-glass was cemented on with Canada balsam and the edges bound. A cm scale was then ruled upon the glass with a writing diamond and a series of spectrograms made showing the absorption at every cm for constant exposure.

At this point it may be well to note the entire unsuitability of the prismatic spectrum for work of this nature. What we are concerned with principally is the rendition of the relative spectral *luminosity*—the dispersion is of no moment, provided it be sufficient to allow of the spectrum being easily read. The difficulty comes in the *interpretation* of prismatic results. We have of course the various dispersion formulae by Helmholtz, Ketteler, Cauchy, Hartmann, and others, but for work of this nature they are of no value whatever, because they do not take into account the absorptive effect due to density and composition of the glass composing the prism itself as it influences the luminosity. For example, supposing that a particular prism is of such a density and absorption that with normal exposure it gives a negative in which the ultra-violet is only impressed to λ 3900, then there is no formula which can supply the photographic intensities of the shorter wave-lengths to which the plate is normally sensitive, while the absorption even throughout the visible portion is still an unknown quantity and varies with every change in the refractive index. As an illustration of this want of reliability, Fig. 1 of Plate VIII shows the influence of this absorption in the comparison between the prismatic and diffraction spectrum in which the lack of concordance may be readily seen.[1]

[1] For the prism used $\mu_D = 1.6004$.

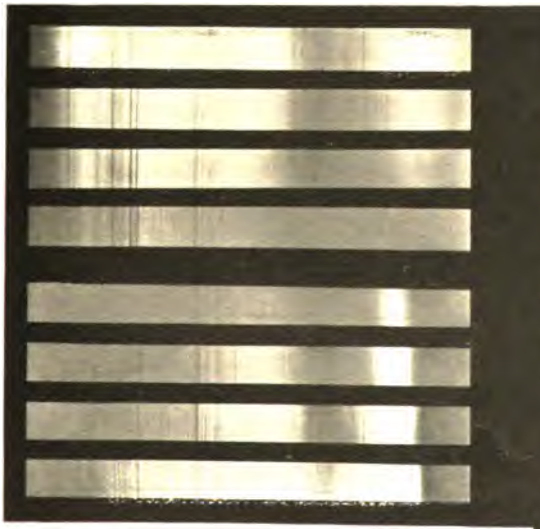
PLATE VIII



U CYGNI

4 *a*. On Seed "27" plate, without filter. 4 *b*. On Cramer "Trichromatic," with filter.

KH G F bE D KH G F bE D C B

1  2

 3

KH G F bE D

1. Comparison of prismatic and diffraction spectrum of diffused daylight.

2 and 3. Daylight diffraction spectrum, with varying exposures, approximating the visual luminosity-curve.

2. Pinacyanol plate and filter. 30^s to 30^m.

3. Trichromatic plate and filter 15^s to 8^m.

In all of the spectrographic work, therefore, use has been made of a replica grating of 15,150 lines to the inch, with the collimator directed at a constant angle to the northern sky, and illuminated by diffused light.

The function of the color-filter is to reduce the preponderance of action in the blue and violet region of the spectrum and absorb entirely the ultra-violet. The transmission throughout the remainder of the spectrum should be undimmed by any absorptive action due to the dye.

Examination of the negatives from the wedges, as above outlined, shows that in the dye Tartrazine (Badische Anilin- and Soda-Fabrik) we may obtain the first component of the filter sought for. Fig. 1 of Plate IX shows the record of this color-wedge, in which the ultra-violet transmission will be noted as extending down even into the dense end of the wedge. This transmission, which is masked by the absorption in prismatic spectra, becomes painfully apparent when the exposure is increased or the light rendered more intense, as is shown in Fig. 1*a*. Such a record serves the very useful purpose of showing the danger in the use of this much-vaunted dye for tri-chromatic and orthochromatic color-filters, even when used in extremely concentrated form. It will be observed that in scale-number 17 of this record the excessive density in the blue of the spectrum from F to G is well corrected for, but that the violet is still too strong, and the ultra-violet is of course transmitted. Another color-wedge, made of

Gelatin	2.5 grams
Water	100.0 cc.
Aesculin	0.2 gram

shows that at scale-number 23 or 24 we have an absorption corresponding to the requirements necessary for the second component of the filter, viz., absorption of the ultra-violet with a gradual absorption in the visible violet. (Plate IX, Fig. 2.)

The two color-wedges are now superposed upon one another with the selected scale-numbers in agreement and an exposure then made through the combination, which gave a result closely approximating the effect sought for. All preliminary exposures and

records are made on Cramer "Instantaneous isochromatic" plates, which have a comparatively low sensitiveness to the blue-green about λ 5050, for which due allowance must be made in the interpretation of the spectroscopic records. It will be evident that any absorption of the dye in the blue-green region would be instantly detected in the photographed spectrum.

Arrived now at a satisfactory point in the trial exposures, the next consideration is the production of a filter which will possess the same absorptive action as do the combined color-wedges at the points selected. Monpillard[1] has suggested a method for obtaining this result, which consists in measuring the thickness of the color-wedge at the selected point (e) and using this as one term of a simple proportion. Two other terms consisting of thickness (e_1) and weight of dye-stuff (d) are obtained by coating a separate plane glass of known area with a measured amount of dyed gelatin; the amount of dye necessary for the finished filter (x) is then to be found from the simple calculation,

$$\frac{e}{e_1}=\frac{d}{x}, \quad x=\frac{e_1 d}{e}.$$

This method commends itself by its simplicity, but in the hands of the writer it has not proven suitable. Presumably, if the thickness of the film on the color-wedge at the selected point were measured more delicately than in the method adopted, say, in the interferometer, then a much closer approximation might be arrived at than has been possible when using a. Brown & Sharp micrometer caliper. As it was, the *critical* adjustment of a filter is so delicate that a minute quantity of dye, either in deficiency or excess is fatal to correct performance. The making-up of a large quantity would also tend to reduce error, but for the manufacture of a single filter of special absorption such a course is not practical.

By a method of trial and error we may, however, very speedily arrive at an extremely satisfactory duplication. A plate of ordinary glass of exactly the same size as the desired filter is taken, and upon this is flowed a measured amount of the same solution as was used in making the color-wedge. This amount is approximately deter-

[1] *Comptes Rendus*, **141**, 31-33, 1905.

PLATE IX

FIG. 1

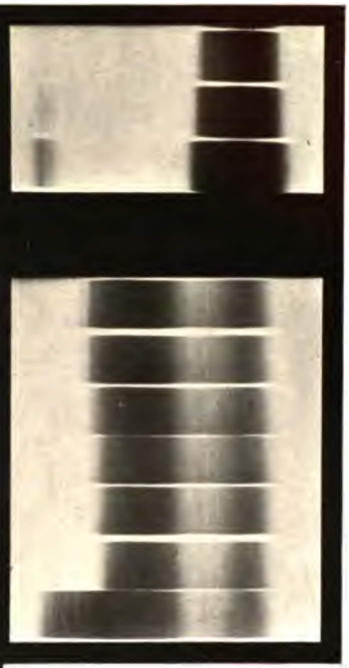


FIG. 1a

FIG. 2

FIG. 1. Spectral Record of Tartrazine Dye-Wedge. (Constant Exposure.)

FIG. 1a. Increased exposure through No. 26 of Tartrazine wedge, showing transmission of ultra-violet.

FIG. 2. Portion of spectral record of Aesculin wedge.

mined by visual observation, while in contact with a white surface on which also rests the wedge.[1]

As all dyed filters generally dry with a slight shift in absorption toward the red, allowance must be made for this; therefore two other glasses are coated with slightly smaller amounts of solution, and then dried rapidly by fan. Exposures to the spectrum are then made through each and any further correction noted.

In coating the optical glass for the finished filter the drying cabinet (which should be large and roomy) is carefully dusted with a damp cloth, and the supporting plate carefully leveled. The glass being coated with the determined amount, and laid upon the leveled plate, the door is closed, and left so until dry.

In filters of exact adjustment, where there are two components, it will be found advisable to flow each plate separately, as a much closer result can be thus arrived at than by combining the dyes and flowing once.

The solution actually used for coating the glass plates was made up as follows:

$$\left.\begin{array}{ll}\text{Gelatin,} & 5.0 \text{ grams}\\ \text{Water,} & 200.0 \text{ cc}\end{array}\right\}\text{Stock.}$$

Dye solution A:	Stock gelatin solution,	100.0 cc
	Tartrazine, 0.15 gram	
Dye solution B:	Stock gelatin solution,	100.0 cc
	Asculin, 0.2 gram.	

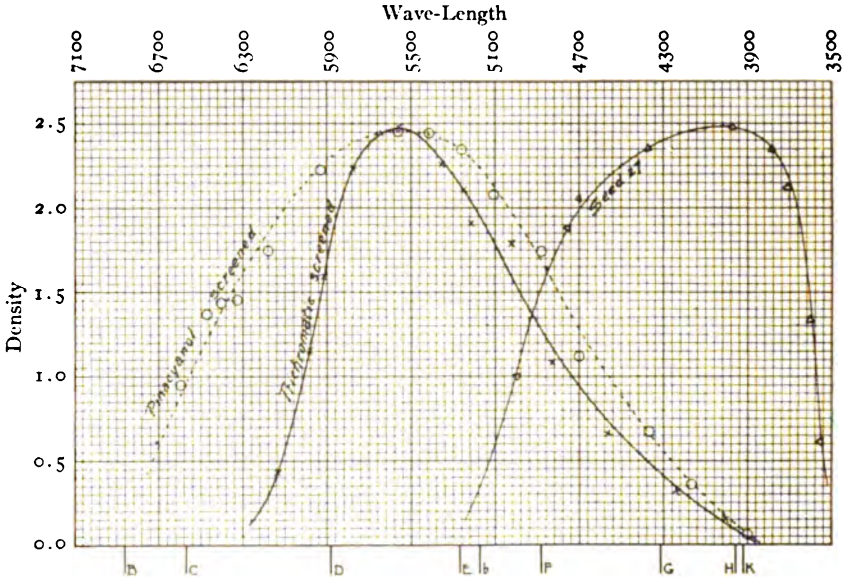
Of solution A 2.5 cc was flowed upon the glass plate of 58 sq. cm area, while 3.6 cc of solution B was flowed upon the cover-plate of similar size. The actual amounts of dye on each surface would then be A = 0.00375 gram, B = 0.0072 gram.

The process seems lengthy in the repetition, but is in reality quite rapid in performance. Especially is this the case where, as in the laboratory of the writer, the spectrograph, visual spectroscopes (prismatic and diffraction), and spectrophotometer are permanently set up in position for immediate use. The collection of dyes in color-wedge form, with their accompanying photographic records

[1] This method has been adopted after various trials with instrumental methods (colorimeter, tintometer, etc.) for, taking into account the shade change in drying, no greater reliability could be obtained.

ready for consultation, is obviously of inestimable value in many ways.[1]

In the testing of such a filter the first consideration is that of its influence upon the correct representation of the spectrum luminosity. A series of exposures was therefore made upon two plates, a Cramer "Trichromatic," and a Seed "27" bathed in pinacyanol. These negatives were then measured in the spectrophotometer and the curves



plotted; in each case that spectrum selected for measurement gave as its maximum a density[2] of 2.5 (Hurter and Driffield).

The best result, as will be seen from the curves, is obtained by the use of the pinacyanol-bathed plate, which was prepared in a bath of $\frac{1}{300,000}$ for a period of $2\frac{1}{2}$ minutes, and followed by a washing of about 3 minutes; the plates bathed were Seed "27 Gilt Edge." The spectra obtained upon this plate leave little to be desired (Fig. 2 of Plate VIII), but it would be obviously advantageous if use could

[1] All carefully measured filters made by the writer, such as those for the 40-inch Yerkes refractor, the Lowell Observatory, etc., have been derived in a similar manner.

[2] The development of all spectrum exposures is kept constant in constitution of developer, and time and temperature of development.

be made of a commercial plate already prepared and easily obtained. The "Trichromatic" was found to give very favorable results (Fig. 3, Plate VIII), where the extreme red was not required, although the lowering of sensitiveness in the blue-green at λ 5100 is still apparent, as will be seen from the measurement points on the mean curves.

In the series of varying time exposures upon this plate, it will be noted that the point of maximum intensity—i. e., highest luminosity—remains as a stationary point about λ 5580, midway between the Fraunhofer lines D and E. In the "Purkinje phenomenon" the maximum (visual) luminosity, in strong illumination, lies close to this point, but shifts toward the more refrangible end of the spectrum as the illumination decreases, and finally, at near the point of extinction, lies in the blue-violet. This, however, is a purely physiological phenomenon and in no wise affects the real maximum as recorded upon the photographic plate. The point of strongest action therefore remains constant.

The increase in the exposure time consequent upon the selective filtration, as compared with that of the Seed "27," was determined by equal exposures made with the Hurter and Driffield revolving sector disk. The exposures were made to diffused daylight and immediately followed one another; the entire time for both exposures being less than five minutes, the light meanwhile appearing constant. These plates were then developed together. The difference in exposure time necessary to obtain similar densities was then readily calculated from the distance apart of the plotted densities of the plates when measured, and was found to be for the Trichromatic and filter 9.2 times, and for the pinacyanol plate and filter 24.5 times. A number of subsequent exposures made in the camera confirmed these figures.

The absorption of the color-filter was next measured by the spectrophotometer in the yellow-green (λ 5500) and found to amount to 3.8 per cent.

The actual performance of the plate and filter in the telescope was determined by exposures upon objects of the class for which it was primarily constructed, viz., colored stars. Fig. 4, *a*, *b*, of Plate VIII shows the long-period variable *U Cygni* ($\alpha = 20^h\ 15^m\ 7^s$; $\delta = +47^\circ\ 26'$), *B. D.* $+47^\circ 3077$, which is classified by Chandler[1] in his color-scale

[1] Third Catalogue of Variable Stars, *Astronomical Journal*, **16**, 145, 1896.

as 9.3, "where 0 corresponds to white," and so on, "through increasing shades of intensity up to the deepest red of which we have cognizance in the heavens." In the eyepiece of the 24-inch reflector this star presents an extremely beautiful appearance, almost spectacular in effect. Mean visual estimations by Messrs. Parkhurst, Jordan, and the writer, on the evening of October 3, rate it as being *possibly* a trifle brighter than its neighboring white star (*B. D.* $+47^\circ 3078$), which is rated as of magnitude 8.3, and separated from it 1′ 35″. The photographic record of this intensely red star, as obtained upon the ordinary Seed "27" plate, shows it as far below its actual value; while, on the other hand, the beneficial effect of the filter and plate needs no comment.

The actual measurements of the disk diameters show that on the pinacyanol-bathed plate the red star is slightly larger compared with the white star, while on the "Trichromatic," owing to its less sensitiveness to the least refrangible end of the spectrum, the disk is of practically the same size.[1]

A further word may be said relative to the exposure through such a color-filter as has been described, and which applies generally to all others. For definitely comparable results it is essential that the temperature and time of development, and chemical constitution of the developer, be kept as constants. If this is so, then the only variable which enters into consideration is length of exposure. That this quantity must always be variable is unfortunately true, but to the worker of even limited experience the variance cannot be great, the trained observer being able to detect any decided "thickening" during the course of the exposure; this element of uncertainty obviously becomes greater as the exposure time is increased.

The influence of this variation upon the color-correction is indicated by the graduated exposures in Plate VIII, Figs. 2 and 3, and is self-explanatory. With increasing exposure up to that point which represents the true filter multiple, the remainder of the photographic opacity increases about proportionally to that at the point of maximum sensitiveness at λ 5600; beyond this point the spectrum shows a tendency to spread out at either end. This is to be expected from

[1] Further and complete information relative to this work is shortly to be published by Messrs. Parkhurst and Jordan.

the character of the filter where the absorption must be gradual and not in any way abrupt. This spreading (at the violet end) is a point, however, which need not be taken seriously, as it would require an exposure of about double the correct length of time to show any decided difference in the spectrum.

The drop in the reflectivity of the telescope mirrors as the silver films age[1] is a matter of no moment as affecting the relative exposure, as, for the work immediately under consideration, the screened exposure is always the same multiple of the unscreened plate, viz., 9.2 and 24.5 respectively. The tarnishing of the silver films ought, however, to be guarded against, as it exercises a more or less strong absorptive action upon the violet end of the spectrum according to the amount, and would thereby in critical work disturb the balance of action between the two sets of plates when taken at different periods.

It is a matter of some importance that the exposure multiple of the filter and plate over the Seed "27" be kept as nearly constant as possible, because, owing to the variance of the density-exposure curve of the plate with change in wave-length, a direct comparison of the faintest stars shown constitutes a very unreliable guide to exposure.

Although this filter has been made up primarily for use with a reflecting telescope in the work of photographic photometry, yet it will be plainly seen that its use does not end there. With the refractor it would be entirely unsuited, but with the ordinary high-grade doublet camera lens where all rays come to approximately identical focus, and on objects bright enough to allow for sufficient exposure, the gain in truthfulness of representation would be marked.

In conclusion the writer begs to acknowledge his indebtedness to Messrs. Parkhurst and Jordan for exposures at the telescope and general interest in the work.

YERKES OBSERVATORY,
October 8, 1906.

[1] C. A. Chant, *Astrophysical Journal*, 21, 211, 1905.

ON THE BRIGHTNESS OF THE INNER EDGE OF THE PENUMBRA IN SUN-SPOTS

By S. CHEVALIER

It is commonly admitted, I think, by astronomers that in a sun-spot the inner edge of the penumbra is brighter than the outer. Father Secchi, speaking of the currents which seem to form the penumbra, says in his work on the Sun (2d ed., p. 82):

> Ces courants sont moins condensés, moins lumineux, moins nettement tranchés à l'extérieur de la pénombre, tandis que près du noyau ils se pressent, se condensent et deviennent plus brilliants. Il arrive ainsi quelquefois que le bord de la pénombre contigue au noyau acquiert un éclat plus vif, presque égal à celui de la photosphère. La tache paraît alors composée de deux anneaux brillants concentriques. Ce n'est pas une illusion due à un effet de contraste; c'est un accroissement réel de lumière, du à une condensation de matière lumineuse dans le voisinage du noyau.

Recently, Young, in his *Text-book of General Astronomy*, writes (p. 138):

> The penumbra is usually composed of "thatchstraws," or long drawn-out granules of photospheric matter, which, as has been said, converge in a general way toward the center of the spot. At the inner edge, the penumbra, from the convergence of these filaments, is usually brighter than the outer.

The contention of Faye, that this appearance is a mere illusion, could be looked on as obsolete, were it not that a most prominent authority in solar physics, Sir Norman Lockyer, revived it in his *Chemistry of the Sun*. He says (p. 408):

> To the eye the outer edge, where the half-tone is in contrast with the photosphere, seems darker than the one which is in contrast with the black nucleus. This is a subjective appearance merely; as shown in photographs, the inner edge is *not*[1] brightened.

The high authority of the writer, as well as the force presented by his argument, will very probably influence many readers. On coming across these lines, I was much surprised, and determined to try to throw a little light on the question.

But, first of all, supposing it to be true that the inner edge as shown on photographs is *not* brightened, this would not be a peremptory

[1] The italics are the author's own.

argument for calling the eye observation a subjective appearance merely. With all respect to the authority of Sir Norman Lockyer, I cannot agree to this assumption. The argument to be complete must suppose the photographs to be perfect, or at least perfect enough to show so faint a detail as a slight difference of brightness between the outer and the inner edges. Now, everyone who is acquainted with photographs of the Sun knows how many details, unmistakably perceptible to the eye in a sun-spot, do not come out on the photographic film. The small size of the picture, the unsteadiness of the image caused by the agitation of the air, the brightness of the photosphere, are some of the causes to account for this. Therefore, from the absence on photographs of a detail observed at the eye-piece, it is not fair to conclude that the eye observation was an illusion. It is first necessary to prove that this detail, if true, should have come out on the photographs.

The first remark tends to weaken the argument of Sir Norman Lockyer, but does not solve the question. To do this I have examined attentively the photographs made during the last three months at this observatory. The inner edge of the penumbra, which, as shown on the photographs at the disposal of Sir Norman, is not brightened, as shown on the photographs made at the Zô-sè Observatory, is very generally more or less brightened. This is a mere question of fact; there is no room for argument and discussion. Sir Norman has assuredly examined with attention the best photographs he had at hand, and found the inner edge *not* brightened. Regarding his assertion nobody will entertain any doubt. But I hope that the annexed page of photographs will show that on other photographs the inner edge is very generally more or less brightened. And this is enough to solve the question. Almost all the nuclear sun-spots photographed here from January 1 to April 15 are represented on this plate. Only one or two, which had been photographed in bad conditions, are not included. They are enlarged to three times the original plates.[1]

On preparing these prints we have of course paid particular attention to this phenomenon. But, besides that, the enlargements are subject by themselves to lessen the differences of brightness. The slight differences, which are undoubtedly perceptible on a negative,

[1] The effect is better shown on a set of positives sent with the manuscript.—EDS.

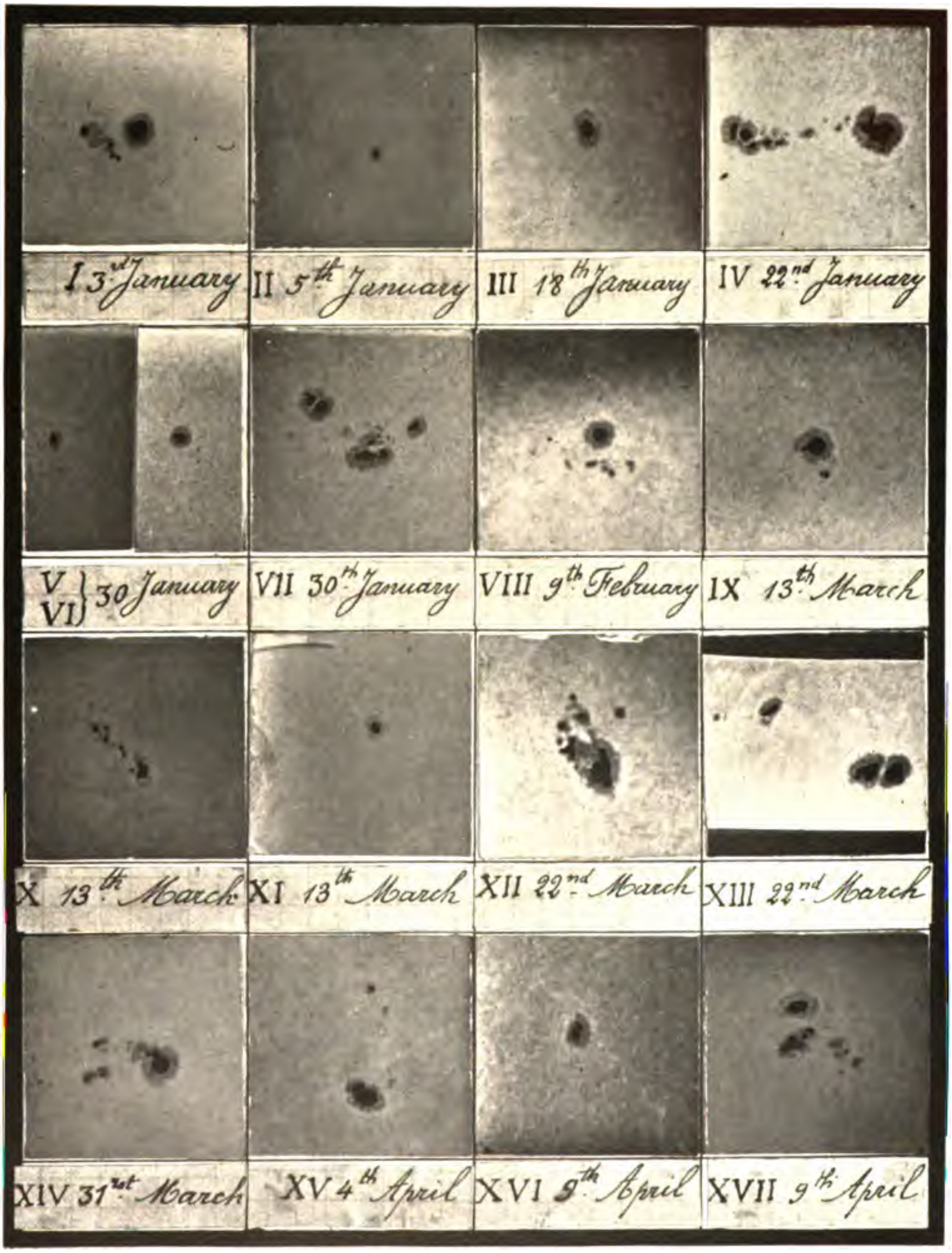
are not easily printed on paper. It is often necessary for that to sacrifice the general aspect of the picture. As in the present question the value of our photographs of the Sun will be of no slight consequence to bear conviction in the mind of the reader, I will ask the editor of the *Astrophysical Journal* to reproduce, enlarged to 3.25 times, the original negative of April 4, which I will select of course from our best photographs of this year.[1]

Now I think it will be worth while to show how far this phenomenon is perceptible on photographs, the more so that this study will lead to some interesting conclusions. For each of the seventeen sun-spots reproduced on the plate, I will tabulate the photographs obtained at this observatory, giving for each photograph, with the date, the clearness of the plate, the distance from the spot to the central meridian, and the apparent brightness of the inner edge over the outer. The Roman numerals refer to the numbers of the plate.

Date	Clearness of Plate	Dist 'nce to Central Meridian	Brightness of Inner Edge
No. I. Latitude 8°5, Longitude 282°, Surface 328 Millionths on January 3			
January 1....	Clear	1°	Very faint, almost imperceptible
" 2....	Clear	15	Well marked all round the nucleus
" 3....	Clear	29	Well marked; photographed through clouds
" 5....	Clear	54	Rather faint; well perceptible on NNE and S of the nucleus
" 6....	Clear	67	Well perceptible NE and SE of the nucleus
No. II. Latitude 10°0, Longitude 178°, Surface 63 Millionths. January 5			
January 5....	Clear	−49°	Strongly marked
" 6....	Clear	−37	Strongly marked
" 7....	Clear	−23	The penumbra is fading away
No. III. Latitude 8°5, Longitude 104°, Surface 320 Millionths. January 9			
January 9....	Very clear	−71°	Very strong except on W side
" 12....	Foggy	−32	Very faint, almost imperceptible
" 13....	Very clear	−18	Strongly marked
" 14....	Clear	− 5	Strongly marked
" 15....	Clear	8	Well marked, but not so strong
" 16....	Clear	21	Well marked
" 17....	Clear	34	Well marked
" 18....	Very clear	47	Very strongly marked
" 20....	Foggy	75	Strongly marked on the N side

[1] Although unable to comply with the author's request, we are prepared to vouch for the excellence of this photograph.—EDS.

PLATE X



Brightness of Inner Edge of Penumbra in Sun-Spots

Date	Clearness of Plate	Dist'nce to Central Meridian	Brightness of the Inner Edge
No. IV. Latitude $-11°.0$, Longitude 336°, Surface 377 Millionths. (Largest nucleus)			
January 25....	Very clear	10°	Faint, except on few places
" 26....	Very clear	25	Easily perceptible
" 27....	Rather clear	40	Faint
" 28....	Rather clear	53	Faint
" 29....	Clear	65	Well marked round the first nucleus
" 30....	Very clear	78	Clearly visible round the three nuclei
No. V. Latitude 15°.9, Longitude 302°, Surface 118 Millionths. January 29			
January 27....	Rather clear	6°	Faint, the penumbra is just making its first appearance
" 28....	Rather clear	18	Well marked
" 29....	Clear	31	Well marked on the W side
" 30....	Very clear	44	Well marked
No. VI. Latitude $-7°.4$, Longitude 286°, Surface 114 Millionths. January 28			
January 25....	Very clear	−40°	Well marked
" 26....	Clear	−26	Strongly marked
" 27....	Rather clear	−10	Faint
" 28....	Rather clear	1	Well marked
" 29....	Clear	14	Rather faint
" 30....	Very clear	28	Strongly marked
February 2....	Faint	70	Not clear
No. VII. Latitude 17°.0, Longitude 234°, Surface 297 Millionths. (Largest Spot of the Group)			
January 28 ...	Rather clear	−51°	Well marked
" 29....	Clear	−38	Visible only W of the second nucleus
" 30....	Very clear	−25	Faint round the first nucleus, well marked round the second
February 2....	Faint	18	Very faint
" 3....	Foggy	30	Very faint
" 4....	Foggy	42	Very faint
" 5....	Clear, faint	55	Well marked
No. VIII. Latitude 9°.5, Longitude 105°, Surface 175 Millionths. February 9			
February 5....	Clear, faint	−75°	Very faint
" 6....	Foggy	−59	Very faint
" 7....	Rather clear	−48	Well marked
" 8....	Tolerably clear	−31	Faint, well visible
" 9....	Clear	−21	Well marked
" 11....	Clear	8	Well marked

Date	Clearness of Plate	Dist'nce to Central Merid-ian	Brightness of Inner Edge
No. IX. Latitude 5°5, Longitude 74°, Surface 227 Millionths. March 12			
March 8....	Clear, faint	−60°	Well marked
" 9....	Clear	−47	Strongly marked
" 12....	Clear	− 5	Strongly marked
" 13....	Very clear	8	Very strongly marked
" 14....	Faint, soft	22	Very faint
" 15....	Very clear	36	Strongly marked
" 16....	Clear	50	Strongly marked
" 17....	Clear	67	Strongly marked on the N, and slightly on the W
No. X. Latitude 19°5, Longitude 107, Surface 411 Millionths. March 15			
March 12....	Clear	27°	Strongly marked S and W
" 13....	Very clear	40	Well marked
" 14....	Tolerably clear	55	Faint
" 15....	Clear	70	Well marked on N and S
" 16....	Clear	84	Too near the limb
No. XI. Latitude 6°0, Longitude 88°, Surface 100 Millionths. March 12			
March 12....	Clear	8°	Rather well marked
" 13....	Very clear	21	Well marked
" 14....	Tolerably clear	37	Faint
No. XII. Latitude 20°0, Longitude 313°, Surface 350 Millionths. March 21			
March 18....	Foggy	−45°	Invisible
" 19....	Clear	−34	Easily visible
" 21....	Clear	− 2	Irregularly marked round the nucleus
" 22....	Very clear	11	
No. XIII. Latitude 14°4, Longitude 273°, Surface 414 Millionths. March 19			
March 19....	Clear	−75°	Very faint
" 21....	Clear	−43	Strongly marked, photographed through clouds
" 22....	Very clear	−30	Strongly marked
" 30....	Clear	74	Well marked N and S of the nucleus
No. XIV. Latitude 8°3, Longitude 243°, Surface 109 Millionths. March 21			
March 21....	Clear	−74°	Spot too faint
" 22....	Very clear		Well marked
" 30....	Clear	43	Well marked
" 31....	Clear	57	Well marked

Date	Clearness of Plate	Dist.'nce to Central Meridian	Brightness of Inner Edge
No. XV. Latitude $-12°.8$, Longitude 174°, Surface 427 Millionths. March 31			
March 30....	Clear	$-32°$	Well marked on the S
" 31....	Clear	-17	Faint
April 1....	Foggy	-3	Faint
" 2....	Clear	11	Well marked
" 3....	Rather clear	24	Well marked
" 4....	Very clear	37	Strongly marked
" 6....	Clear	65	Strongly marked N, S and W
" 7....	Clear	78	Too near the limb
No. XVI. Latitude 20°.0, Longitude 106°, Surface 312 Millionths. April 1			
April 1....	Foggy	$-69°$	Well marked
" 2....	Clear	-55	Strongly marked N, S and E of the nucleus
" 3....	Rather clear	-42	Very strongly marked
" 4....	Very clear	-30	Very strongly marked
" 5....	Clear	-4	Strongly marked
" 7....	Clear	9	Strongly marked
" 9....	Clear	38	Strongly marked
" 10....	Clear	49	Well marked
No. XVII. Latitude 26°.0, Longitude 43°, Surface 314 Millionths. April 7			
April 6....	Clear	$-66°$	Too near the limb
" 7....	Clear	-53	Well marked on N, S and E
" 9....	Clear	-25	Faint
" 10....	Clear	-15	Rather well marked
" 11....	Clear	-2	Well marked
" 12....	Foggy	12	Well marked

From these tables it seems possible to derive the following conclusions:

1. The brightness of the inner edge of the penumbra in a sun-spot is a very common phenomenon, undoubtedly true.

2. There is a great difference with regard to this point, from one sun-spot to another.

3. For each sun-spot there is not any apparent variation in this phenomenon, resulting from its place over the disk. It must, however, be noted that the first appearance of it comes out on the north and south of the nucleus, when the spot is entering on the visible hemisphere, then on the east, and finally on the west side. When the spot is approaching the west limb, the disappearance begins with the

east side, then the west, and finally the north and south. It is just the same as for the penumbra itself. This remark, though of a negative character, is of no small importance, as it shows a wide difference between the brightness of the inner edge of the penumbra and the brightness of the faculæ.

4. This brightness of the inner edge seems to be one of the characteristic notes of a sun-spot, subject, very probably, to some real but only slight changes.

5. There is a strong probability that the sun-spots of a regular shape present this phenomenon with a more marked brilliancy.

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THE SPECTROCOMPARATOR[1]

By J. HARTMANN

In the *Publicationen des astrophysikalischen Observatoriums zu Potsdam*, Bd. **18**, Stück 53, 1906, which has recently appeared, I have communicated a procedure for the measurement of lines in spectrograms which is based on the application of a special measuring apparatus. In the following I will give the most important points concerning the purpose of this instrument, its construction, and its application.

Hitherto the method of measurement of the Doppler displacement of lines in stellar spectra consisted in setting the cross-wires of a measuring microscope upon one after another of the lines of the stellar spectrum and the adjacent terrestrial comparison spectrum, generally that of iron. In order to compute the displacements of the lines from the screw readings and from these the value of the radial velocity of the star, the wave-lengths of the lines must be assumed as known. In the case of the first-type stars, where there is a paucity of lines, this method has so fully sufficed that a better substitute is scarcely to be made. For the few lines of these stellar spectra can be measured in their entirety without great expenditure of time and their wave-lengths, belonging as they do to a few gases, principally hydrogen and helium, are known with great precision. The complete measurement and reduction of such a spectrum, when the convenient method is employed which I described in the *Astronomische Nachrichten*, **155**, 81, 1901, requires about two hours.

The conditions are quite different in spectra of the later star types, where there are many lines. In the first place, the lines in these spectra are so numerous that their complete measurement and reduction would require many days, and in the second place a rigorous reduction of such a material has hitherto not been at all possible because the wave-lengths of the lines are not known with sufficient accuracy. On this account, observers have until now limited them-

[1] Translated by Philip Fox from the *Zeitschrift für Instrumentenkunde*, **21**, 205–7, July 1906. The electrotypes have been kindly loaned by Director Vogel of the Astrophysical Observatory at Potsdam.

selves to a partial treatment of such spectra, measuring only a small number of lines, whereby the major part of the rich material present in the plate remains unutilized. In addition it must be noted that the value of the radial velocity derived from these measurements must undergo a slight correction when definitive values of the wave-lengths used become known. The difficulty of ascertaining the correct wave-lengths for these star spectra lies in part in the fact that the stellar spectrographs used for the exposures are not sufficiently powerful to separate fully the lines lying near together, so that these very often merge together. For the truly isolated lines exact wave-lengths are also lacking. The observer must take them from Rowland's table of the solar spectrum, the wave-lengths of which, up to the present, have not been adjusted to the laboratory iron spectrum in a manner free from objection, so that the systematic differences in the two systems of wave-lengths must enter into and falsify all spectroscopic measurements of velocity. In addition the truly isolated lines are frequently so fine as to be hard to set upon with the cross-wire; and further it is never possible until the end of the whole computation to make certain, in a way free from arbitrariness, from the accordance of the different values found for the velocity, whether or not decidedly erroneous wave-lengths have been assumed for numerous lines.

All of these difficulties are obviated by the new method of measurement which makes possible the complete measurement and reduction of a spectrum, however rich in lines, and the derivation of the definitive velocity in from one to two hours. This great advantage is reached not by setting as formerly the cross-wire upon single lines, but by simultaneously bringing into coincidence numerous lines of one spectrum and the corresponding lines of another.

The exposures of the spectrograms with the spectrograph are made as formerly, the terrestrial comparison spectrum, generally iron, appearing on both sides of the star spectrum. In the plates exposed with the Potsdam Spectrograph III, the stellar spectrum is 0.25 mm wide and on both sides lie the iron spectra 0.60 mm wide, the intervals between the stellar- and iron-spectra being 0.25 mm. With the same spectrograph a plate of the solar spectrum is made which should have a width of 0.55 mm, thus leaving intervals 0.1 mm

wide. The exact widths may be easily secured with the help of my occulting device.

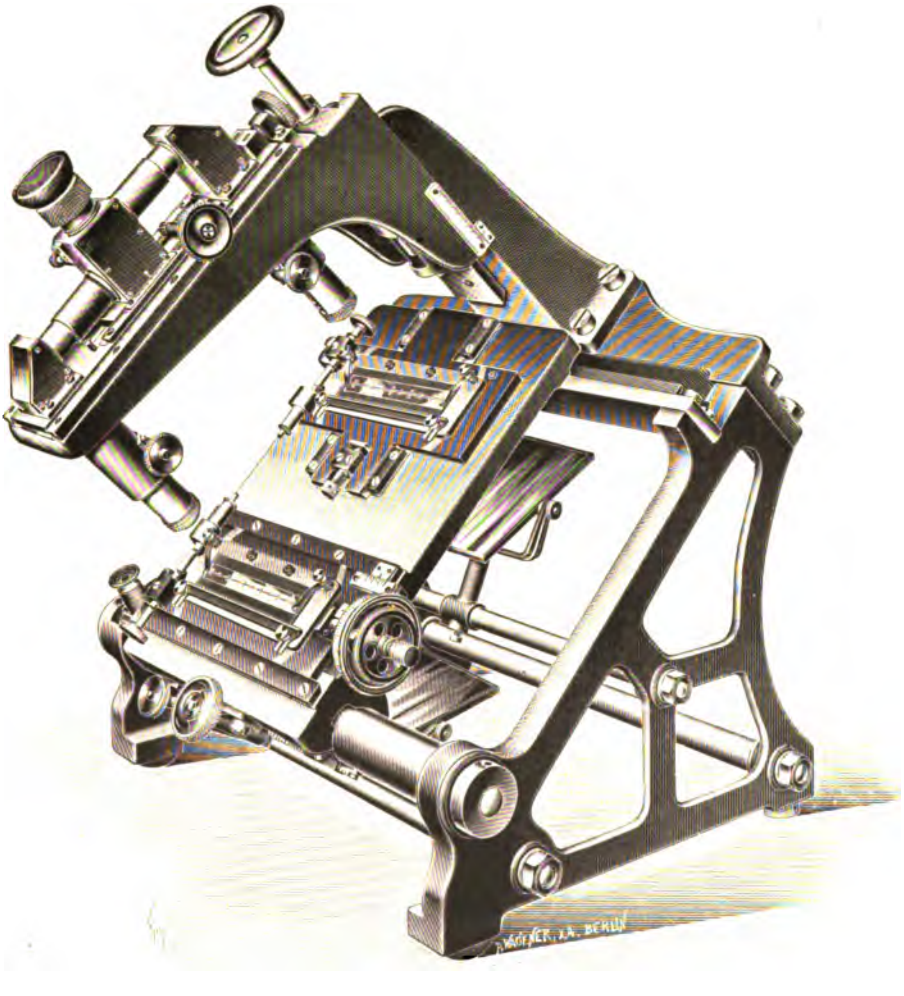

FIG. 1

The measuring apparatus was constructed according to my designs by the firm of Carl Zeiss in Jena, under the especial direction of Dr. Pulfrich. Fig. 1 shows the general appearance of the instrument, which I now proceed to describe.

The stage of the measuring apparatus, on which are clamped the two spectra to be compared, is shown in Fig. 2 in one-fifth its natural size. It holds the device for the orientation of the two plates. The solar spectrum mentioned above is attached by means of two microscope clamps to plate A_1, which has an aperture 1 cm wide and 12 cm long for the illumination of the spectrum. The plate A_1 can be turned about a short central pivot E_1, by means of the

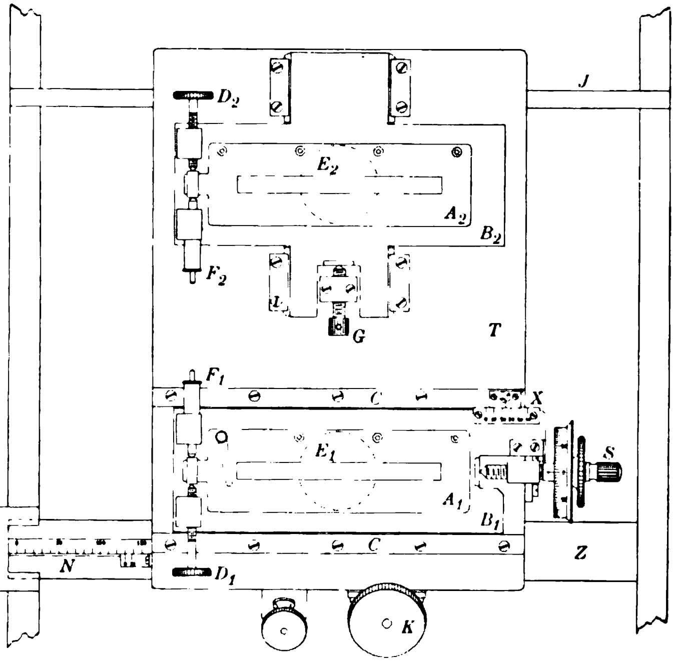

FIG. 2

screw D_1 and the opposing spring F_1. The pivot E_1 is mounted on the plate B_1 lying under A_1. B_1 runs in dovetail guides C, and can be moved from right to left on the Table T by means of the micrometric screw S. The screw has a pitch of 0.5 mm, and its head has 100 divisions, so that the displacement of the spectrum from right to left can be read to 0.0005 mm. The screw runs 45 turns in its nut and is usable for somewhat over 2 cm. Two long spiral springs lying underneath the table press the plate B_1 with its agate buffer against the rounded end of the screw. The number of complete

turns of the screw is read on the scale X which has half-millimeter divisions.

The stellar spectrum to be measured is mounted in similar fashion on the plate A_2, which may be turned about the pivot E_2. Plate

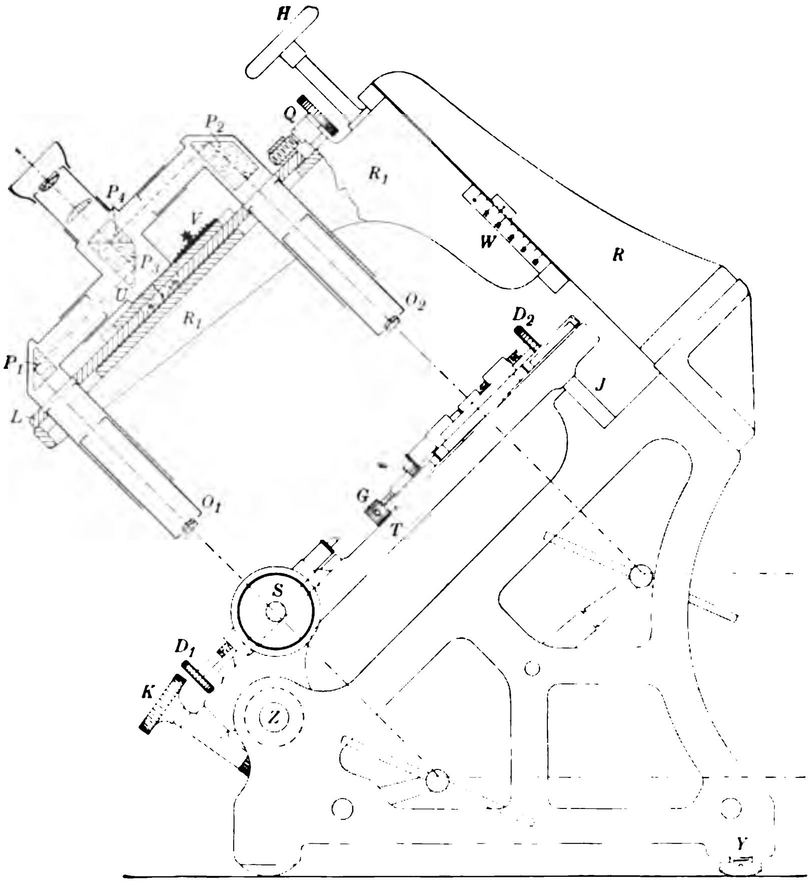

FIG. 3

B_2, carrying A_2, may be moved up and down on the table T by means of the screw G, thus enabling the distance between the two spectra to be regulated.

The whole table T is movable from left to right on the steel cylinder Z, 35 mm in diameter, and the steel guide J by means of the pinion K meshing in a rack lying underneath. Near this is

seen a clamp for holding the table in an invariable position. The setting can be read on the scale N, graduated in half-millimeters, by means of a vernier and magnifying lens.

The stage T, as is apparent[1] in Fig. 3, is carried by its guides Z and J in a plane inclined 45°. Above it stands the carrier of the double microscope RR_1, the construction of which is manifest in Fig. 3. The optical axes of the two objectives O_1 and O_2 are perpendicular to the two spectrograms clamped to the table below. The two objective tubes, each provided with draw tubes 41 mm long and adjustable with rack and pinion, are fastened on the plate L at a fixed distance from one another. The plate L may be moved about 1 cm in its dovetail guides on the surface of the carrier R_1, by means of the screw Q. At the upper ends of the two objective tubes the beams of light are reflected toward the prism-system P_3P_4 by the right-angled prisms P_1 and P_2. At the surface of contact of the two prisms P_3 and P_4 the union of the two beams takes place in the following manner.

On the hypothenuse surface of the prism P_4 the portion shown shaded in Fig. 4 is silvered, and then the prism P_4 is cemented to the prism P_3. In the eyepiece the field of view has the form shown within the circle. If the two spectra are laid under the microscopes O_1 and O_2 and properly adjusted, the observer perceives, as shown in Figs. 5 and 6, part of the stellar spectrum or its comparison spectra on the shaded portion of the surface, while the remainder of the field is filled with the image of the solar spectrum and its comparison spectra lying under O_1.

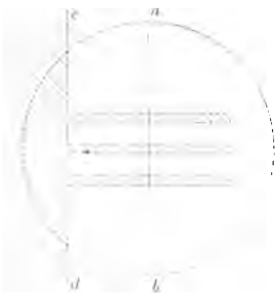

FIG. 4

Through the middle of the hypothenuse surface of the prism P_4 a fine black line ab is drawn perpendicular to the strips of mirror. It serves not for real measurement but merely to roughly mark the middle of the field of view. I will call it simply the cross-wire, in analogy to the measuring cross-wire of other microscopes. Since this cross-wire is inclined 45° to the optical axis of the eyepiece, only a short stretch of it will appear

[1] In the schematic Fig. 3 all screws are omitted for the sake of simplicity.

sharp in the field of view at any time. In like manner the edges of the two outer strips of the mirror will appear somewhat out of focus if the eyepiece is set carefully upon the middle strip. In the measurements this is entirely immaterial, for the observer does not see the mirror at all when the plates are properly illuminated, but sees only the images reflected from them, and these lie accurately in the focal plane.

The prism-system P_3P_4 is mounted in a small box, from which it may be easily removed in order to substitute another. By this means it is possible to use different forms of mirror as desired. The apparatus has a prism-system additional to that described above, which serves for bisecting the field of view, so that the entire lower half of the field is filled by the image of the plate lying under O_1, and the upper half by the image of the plate under O_2.

The small box holding the prism-system and also carrying the eyepiece is mounted on a slide which may be moved by means of the rack at V, the positions being read on the millimeter scale U. If the box approaches the microscope O_1, this microscope is shortened and the magnification diminished, while at the same time the magnification of O_2 is increased. By this gradual variation of the relative magnification of the two microscopes the observer can make the two images of the spectrograms in the field of view so exactly of a size that they can be brought simultaneously into sharp coincidence throughout their entire extent. This arrangement for the establishment of images of exactly equal size, applied here so far as I know for the first time, is of fundamental importance in the entire process of measuring.

Finally it is to be mentioned that the upper part of the microscope carrier R_1 may be moved about 5 cm up and down in the dovetail guides on the lower part R, by the use of the hand wheel H. This motion allows the magnification of the microscopes to be changed simultaneously, so that the width of the images of the spectra may be made to agree with the given width of the mirror in the prism-system; that is, the two outer strips are made to lie in the centers of the comparison spectra. The height of the microscopes is read on the millimeter scale W. If this motion, which allows a variation of 30 per cent. in the size of the image, does not suffice for adapt-

ing the width of a given spectrum to the distance of the mirror-strips, the observer can substitute another pair of objectives of suitable focal length.

The entire frame of the apparatus is sufficiently stable despite its light and pleasing form. It rests upon four feet, of which the one at Y is adjustable by a screw, and in the usual way, by testing opposite corners, any unevenness of the bed may be accommodated so that the instrument stands on its four feet free of strain.

Since the measuring makes no high demands on the mechanical construction, the testing is very simple. It is never assumed in the measurement that the motions are exactly in a straight line or perpendicular to one another, and consequently this investigation can be omitted. Only the optical parts and the measuring screw must undergo a close test. Since deterioration of the images may result from the passage of light through the prisms and from the two reflections, the observer assures himself against this by the perfect sharpness of the images when observing a fine test object. Further, the three strips of the mirror in the prism-system should be sharp and the borders approximately parallel to each other and to the cylindrical guide Z. This can be tested by bringing a point of the image of a spectrum, clamped in position, upon the edge of a mirror-strip, when upon moving the table the point should follow along the edge of the strip.

I will pass over the investigation of the screw, which was carried out in the usual way, simply remarking that the screw of the Potsdam instrument is especially good. In the stretch of 35 revolutions which I investigated neither the periodic nor the progressive errors ever amounted to 0.001 rev. = 0.0005 mm. For the 8 middle revolutions which alone are used in the measurements the periodic error is

$$\overset{\text{Rev.}}{-0.0001285}\cos A + \overset{\text{Rev.}}{0.0003484}\sin A\ .$$

Though the maximum of this is only 0.00037 rev. = 0.000185 mm, it may not be neglected in the measurement of stellar spectra, for in unfavorable cases it may cause an error of 0.27 km in the determination of the velocity, while a precision of 0.1 km is now attainable in these measurements. I shall show farther on how the slight influence of the periodic errors can be made harmless by proper arrangement

of the measurements. The progressive errors need not be considered, for in the measurement only fractions of a revolution are used

In order to simplify the measurements, the observer determines a few constants of the instrument, viz., the middle points of the scales N, X, and U, and constructs a table for the elevation W.

I call the central setting, N_0, of the table, that when the image of the center of rotation of the pivot E_2 is on the cross-wire. Clamp the table in this position and move the slide B_2 by means of the screw until the center of rotation of E_1 lies on the cross-wire. This gives the reading X_0.

The scale U is an index to the setting of the relative magnification of the two microscopes, or, as I call it briefly, the image-extension (*Bilddehnung*). To find the position U_0 in which the two magnifications are equal, lay under the microscopes any two plates which have on them a sharp mark and bring the images into coincidence by turning the measuring screw. If the table T is moved back and forth on the cylindrical guide, the marks remain in coincidence in the entire field of view only when the magnifications of the two microscopes are equal. If the mark lying under O_1 moves less than the other, then the image from O_1 is less magnified, the microscope O_1 is too short, and the prism-system is therefore to be moved upward in the positive direction of the scale U, by means of the rack V. By this motion the sharp setting on the images is somewhat disturbed, and the observer must correct by refocusing the two objectives. To simplify this, which frequently recurs in the measurement of spectra, I have so arranged the racks of the draw-tubes that to attain a sharp setting, the pinions must be turned in the same direction as the pinion V was previously turned. The observer should mark once for all the direction in which V must be moved: when the image of the spectrum lying under O_1 (later the solar spectrum) is too *small*, the motion of the prism-system is then *upwards*.

As a final simplification for the coming measurements, the observer can form a small table which gives the setting of the elevation-scale W for various widths of spectra. The microscope-carrier R_1 must always be set so that the two outer strips of mirror lie in the centers of the iron spectra. In order to find this setting quickly by means of the scale W one needs only to know the size of object this mirror

distance represents at various readings of W. Measure then the distance between the centers of the comparison spectra and set W accordingly.

To construct this table lay a scale of 0.1 mm divisions, having about the thickness of the spectrograms, on the plate A_1 so that the divisions lie horizontally. Keeping the "image-extension" constant at U_0, make a series of settings on the scale W, focus for each setting with the objective draw-tube and estimate on the scale the distance between the centers of the two outer strips.

Two pairs of objectives belong to the Potsdam instrument. They are of 55 mm and 45 mm focal length and I designate them I and II. Further, since the objective draw-tubes could not be made long enough to use the whole of the scale W, I have had a pair of extension tubes, 20 mm long, made which may be screwed into the draw-tubes. With the combinations now possible the following values $2D$ were obtained from measurements of the mirror distance on the scale:

Objective	W	$2D$	p_1	p
I without extension tube.........	6	1.57 mm	2.54	21.2
	10	1.53	2.60	21.7
	15	1.47	2.71	22.6
	20	1.42	2.81	23.4
	25	1.37	2.91	24.3
I with extension tube............	30	1.33	3.01	25.1
	35	1.28	3.11	25.9
	40	1.24	3.21	26.8
	45	1.21	3.30	27.5
	50	1.17	3.40	28.4
	55	1.14	3.50	29.2
II without extension tube..........	0	1.10	3.62	30.2
	5	1.06	3.76	31.4
	10	1.03	3.87	32.3
	15	1.00	3.98	33.2
	20	0.97	4.11	34.3
	25	0.94	4.24	35.4
II with extension tube............	30	0.91	4.38	36.5
	35	0.89	4.48	37.4
	40	0.87	4.58	38.2
	45	0.85	4.69	39.1
	50	0.83	4.80	40.0
	54	0.81	4.92	41.0

It is seen that the mirror can be set sharply in the middle of the comparison spectra when the distance between them lies between 0.81 mm and 1.57 mm. All the exposures with the Potsdam Spectrograph III lie within this range.

The value $2D$ offers each time an easy means of computing the magnification. Since a knowledge of this is also of interest I will include a determination of it. In order to find the objective magnification p_1, divide the interval between the outer mirror strips, as projected in the focal plane, by $2D$. I have measured this distance microscopically and found it to be 3.982 mm. From this is obtained the objective magnification given in the table under p_1.

The magnification of the eyepiece cannot be readily computed directly from its focal length, for the beam of light passes not through air, but almost entirely in the glass of the prism-system behind the eyepiece. I have therefore measured directly outside the eyepiece, the angle of divergence 2ϕ between the two rays coming from the two outer mirror-strips. This gives $2\phi = 7\overset{\circ}{.}6$. From this follows the eyepiece magnification referred to a distance of vision of 25 cm

$$p_2 = \frac{50 \tan \phi}{3.982} = 8.34 ,$$

and the combined magnification given in the table

$$p = p_1 p_2 .$$

If for cases in which the enlargement (which may be brought up to 41 diameters) appears too great, there is included a second eyepiece for the apparatus which gives half the above magnifications.

I shall now give an illustration of the use of the apparatus, treating it briefly and referring for details to the more exhaustive communication mentioned in the beginning.

The solar spectrum placed under the microscope in the measurement of stellar spectra I shall call the fundamental spectrum because all velocities measured depend upon it. With the comparator the difference of the displacements of the lines is directly measured; and therefore the difference of velocities as given by the stellar spectrum and fundamental spectrum. Since the observer can compute the radial velocity of the Sun, the velocity of the star is at once found. I will only mention here further that the spectrum of any star can be used in place of that of the Sun as the fundamental spectrum, but for the sake of simplicity I shall speak only of the solar spectrum,

On the solar spectrum plate very near the iron spectrum the observer marks a number of places by means of dots, and numbers

them as shown in Figs. 5 and 6. The places are so chosen that when the dot is upon the cross-wire, a number of good lines of the iron spectrum are on the mirror-strips on both sides of the cross-wire. For these regions the observer computes the velocity-factors, s, that is, the velocity which 1 rev. of the screw represents for each region. These values are conveniently written upon the right edge of a card upon which appears also the value of W and of V_0, and a small table of the values of f, of which I am about to speak. The card for the fundamental spectrum III 758 has the following form:

FUNDAMENTAL SPECTRUM III 758
SUN, 1905 MAY 16.
$W=30.0$; $V_0=+0.31$ km

	12	13	14	15	16
23............	1.23569	1.26718	1.30207	1.34125	1.38584
24............	1.19342	1.22188	1.25319	1.28803	1.32724
25............	1.15366	1.17956	1.20787	1.23912	1.27399
26............	1.11600	1.13969	1.16544	1.19369	1.22496
27............	1.08024	1.10201	1.12556	1.15126	1.17952
28............	1.04614	1.06623	1.08786	1.11136	1.13707
29............	1.01354	1.03214	1.05211	1.07369	1.09721
30............	0.98216	0.99944	1.01793	1.03785	1.05945
31............	0.95191	0.96801	0.98518	1.00362	1.02355
32............	0.92292	0.93796	0.95396	0.97111	0.98956
33............	0.89496	0.90904	0.92399	0.93997	0.95713
34............	0.86790	0.88112	0.89513	0.91006	0.92605

12	492
13	479
14	465
15	450
16	437
17	422
18	407
19	396
20	383
21	373
22	361
23	349
24	337
25	326
26	315
27	304
28	294
29	285
30	275
31	266
32	259
33	252
34	244

The measuring of the stellar spectrum is done as follows: The observer sets for the value of W given on the card and for the image-extension, U_0. If X is the accidental reading of the measuring screw, set the table at the reading $N=N_0-X_0+X$; at this point the axis of the pivot E_1 is on the cross-wire. The fundamental spectrum is then clamped on the plate A_1 in position I, by which I denotes that with the longer wave-lengths to the right, or in which they appear to the left in the microscope. The observer sets the eyepiece sharply on the middle mirror-strip and brings the image of the plate in sharp coincidence with it by means of the draw-tube O_1. Using the screw Q the middle strip is set exactly in the middle of the

solar spectrum, the table is moved to the end of the spectrum by means of the pinion K, and by the screw D_1 the spectrum is reset until it is again symmetrical to the mirror. The fundamental spectrum is then parallel to the cylindrical guide and its adjustment is complete, a process requiring less than a minute. When the instrument is in continuous use the observer will generally allow the fundamental spectrum to lie undisturbed for a long time and the adjustment described above is eliminated.

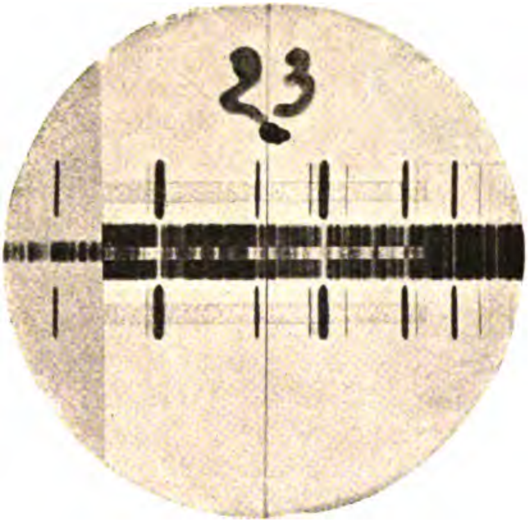

FIG. 5

The adjustment of the star spectrum is almost equally simple. It is clamped on the plate A_2 with the same region of the spectrum in the field of view as with the fundamental spectrum. The objective O_2 is set carefully, the table is brought to its central position N_0, and the stellar spectrum is moved by the screw G until it appears at an equal height in the field of view with

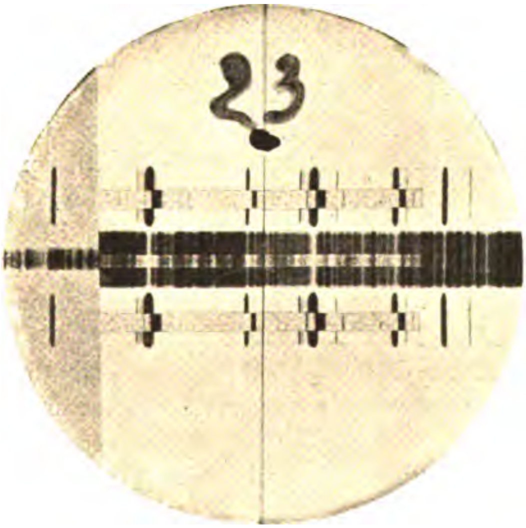

FIG. 6

the fundamental spectrum. This adjustment is made with the iron lines of the two plates alone and they are compared near the line of separation *cd* in Fig. 4. The table is moved again sidewise to the end of the spectrum and the stellar spectrum is again set at an equal height by means of the screw D_2. The stellar spectrum also is now parallel to the cylindrical guide.

Then the "image-extension" is to be so adjusted by the rack *V* that the two spectra in the field of view appear of equal size. By observing the rule given above, concerning the direction in which *V* is to be turned, this, too, is easily accomplished.

If the stellar spectrum, as is frequently the case, is not uniformly strong throughout its width, the observer moves the microscopes by means of the screw *Q* until the middle strip of mirror falls upon the best part of the stellar-spectrum, and with this, all the preparations are ended.

The real measurement consists essentially, while one of the various numbered points is on the cross-wire, in bringing first, the stellar spectrum into coincidence with the adjacent solar spectrum, and then the two iron spectra into coincidence, as may be seen from Figs. 5 and 6. Since the spectra exactly coincide throughout the length of the field of view, the observer recognizes at a glance which lines are common to both spectra and can be used in establishing the coincidence. For this the lines need not be sharply defined nor symmetrical, but wide groups, edges of bands, and even the dark intervals of the negative between lines are equally applicable. On account of these continual comparisons of spectra, the observation is very interesting, and moreover is so rapid that the complete measurement of a spectrum rich in lines is accomplished in about a half-hour. The number of the coincidences of lines actually observed in this time is very great, since each setting of the screw depends on very numerous simultaneously observed coincidences.

When the spectrum is measured in the first position, both the stellar spectrum and solar spectrum are reversed and the measurement is repeated, thus eliminating the physiological errors dependent upon the position. At the same time the small effect on the result of the periodic screw errors mentioned above is eliminated. For this purpose the observer has only to heed that the measurement in

the second position is made at a place in the screw 180° from that in the first.

The reduction is as simple as the measurement, as I will illustrate with an example. The following scheme contains all the computations which can present themselves in any case, but under circumstances about half of it may be omitted. I remark further that mark No. 13 of the fundamental spectrum is at λ 4606, mark No. 34 at λ 4070.

The four columns designated *Fe* and * give the screw-readings taken in the measurement. The amount of the line-displacement is deduced from them, viz.:

$$\text{in position I, } d_1 = Fe - * ,$$
$$\text{in position II, } d_2 = * - Fe .$$

The column $d_2 - d_1$ is computed only as a check. Negative values of $d_2 - d_1$ are seen to predominate. This is common to all series and arises from the above-mentioned physiological difference of perception dependent on the position of the spectrum. Its influence on the result is eliminated by forming the mean d from the d_1 and d_2. The computer now places the card described on p. 296 so that the values of s belonging to the various regions stand opposite the corresponding values of d; then with the help of a multiplication table he forms the product $sd = V_* - V_0$. These are the values of the difference in velocity between the star and the fundamental spectrum, of which the mean M_1 in our case is $+11.60$ km.

The reduction here described is so simple that it can be made in a few minutes, and it is so easy to look over that large blunders are scarcely possible. However I use this computation only as a control and deduce the definitive value of the velocity in another still more simple and more accurate way.

In obtaining the mean value M_1, the individual differences of velocity $V_* - V_0$ from the various parts of the plate are given equal weight. This is not rigorously correct on account of the strongly increased dispersion toward the violet end of the spectrum. Since one can suppose that on the average the linear amount d of the line-displacement can be measured throughout with the same precision, the values $V_* - V_0 = sd$ should receive, in the formation of the mean,

PLATE III 775. *α Boötis* 1905 MAY 26, $9^h 44^m$ C. E. T.
FUNDAMENTAL SPECTRUM III 758.

MARK No.	FIRST POSITION			SECOND POSITION			$d_2 - d_1$	d	$V_* - V_o$	F
	Fe	*	d_1	*Fe*	*	d_2				
	Rev.	Rev.	Rev.	Rev.	Rev.	Rev.	Rev.	Rev.	km	km
13	27.403	0.382	+0.021	27.876	0.901	+0.025	+0.004	+0.023	+11.0	−0.6
14	.406	.380	26	.875	.899	24	− 2	25	11.6	0.0
15	.404	.377	27	.876	.900	24	− 3	25_5	11.5	−0.1
16	.408	.376	32	.869	.898	29	− 3	30_5	13.3	+1.7
17	.410	.380	30	.873	.902	29	− 1	29_5	12.4	+0.8
18	.413	.385	28	.870	.900	30	+ 2	29	11.8	+0.2
19	.423	.395	28	.862	.891	29	+ 1	28_5	11.3	−0.3
20	.424	.392	32	.860	.886	26	− 6	29	11.1	−0.5
21	.427	.389	38	.856	.888	32	− 6	35	13.1	+1.5
22	.427	.398	29	.856	.889	33	+ 4	31	11.2	−0.4
23	.426	.393	33	.853	.882	29	− 4	31	10.8	−0.8
24	.427	.392	35	.852	.885	33	− 2	34	11.5	−0.1
25	.427	.388	39	.857	.889	32	− 7	35_5	11.6	0.0
26	.423	.390	33	.857	.888	31	− 2	32	10.1	−1.5
27	.426	.385	41	.855	.893	38	− 3	39_5	12.0	+0.4
28	.423	.380	43	.858	.893	35	− 8	39	11.5	−0.1
29	.424	.386	38	.857	.892	35	− 3	36_5	10.4	−1.2
30	.426	.380	46	.856	.900	44	− 2	45	12.4	+0.8
31	.421	.373	48	.856	.905	49	+ 1	48_5	12.9	+1.3
32	.421	.378	43	.859	.900	41	− 2	42	10.9	−0.7
33	.424	.380	44	.855	.898	43	− 1	43_5	11.0	−0.6
34	.430	.377	53	.852	.895	43	− 10	48	11.7	+0.1
		Σd_1	+0.787		Σd_2	+0.734		M_1	+11.60	
			+0.734							
			+1.521							

log $=0.18213$ $\qquad M_2 = +11.57$ km
log$f$ $=0.88112$ $\qquad V_o = + \;0.31$

log$M_2 = 1.06325$ $\qquad V = +11.88$ km

weights which are proportional to $\frac{1}{s}$. Designate the mean so calculated with M_2, then

$$M_2 = \frac{\Sigma \frac{1}{s}(V_* - V_o)}{\Sigma \frac{1}{s}} = \frac{\Sigma d}{\Sigma \frac{1}{s}}.$$

This correct manner of reckoning thus leads to an extraordinarily simple scheme of computation. It is unnecessary to form the individual products, sd, but only to multiply the sum of all the d's with a

factor which is constant so long as the same regions are used. Since

$$\Sigma d = \frac{1}{2}(\Sigma d_1 + \Sigma d_2),$$

I set

$$f = \frac{1}{2\Sigma\frac{1}{s}},$$

and from this comes

$$M_2 = f(\Sigma d_1 + \Sigma d_2).$$

The values of log f are given on the card described above for the first and last regions employed as arguments. In our illustration we measured from mark No. 13 to No. 34, whence we have log $f=$ 0.88112 and from this, as given under the scheme of reduction, $M_2 = +11.57$ km, which agrees closely with M_1. After adding to M_2 the value of V_0, also given on the card, the velocity sought, of the star relative to the Earth, is obtained,

$$V = +11.88 \text{ km}.$$

This can be referred to the Sun in the usual way.

As seen in the example the entire measurement and computation is extraordinarily simple, so that the complete treatment of a stellar spectrum, which formerly would have required many days, now needs at most two hours. In addition to this, the new method is perfectly rigorous, since no inadmissible suppositions were made and nothing was omitted.

On account of the extraordinarily great number of coincidences used, the accuracy of the new method is greater than in the earlier method; but more especially on account of the elimination of all errors which were formerly introduced into the result from the adoption of incorrect wave-lengths of the lines observed the value of the velocity is considerably more certain and is definitive. In the paper cited in the beginning of this article I have made a more thorough investigation of the accuracy of the method from the standpoint of a greater quantity of observational material, of which I shall here give only the result. While hitherto in the best series of observations the probable error of a velocity derived from a plate amounted to not less than 0.25 km, the new method with the comparator succeeds in determining the motion within 0.1 km.

In conclusion I will remark that the comparator, beside its use in the measurement of line displacement, as described here, is also suitable for all comparative studies of various spectra, and for the investigation of the errors of small scales.

Astrophysikalisches Observatorium,
Potsdam, June 1906

REVIEWS

Vorlesungen über theoretische Spektroskopie Von A. GARBASSO. Leipzig: Barth, 1906. Pp. viii + 256, with 65 figures. Marks, 8.

This series of twenty lectures by Professor Garbasso represents one of the courses which he offers in the University of Genoa. The title alone raises a rather large question as to whether the science of spectroscopy is yet in a stage of advancement sufficient to permit of a general theory. In the opinion of many, including, we believe, Professor Garbasso himself, this question must be answered in the negative. The attempt, therefore, to see what can be done in the way of reaching a general view along different lines, such as the mechanical, electromagnetic, or electrostatic, is for this very reason exceedingly interesting.

It is important, however, for the reader to bear in mind what the author distinctly states at the beginning, namely that he has omitted from the volume everything which may not be considered as the description of a physically possible model.

The work is divided into four sections, of which the first is devoted to the experimental data of the science and to the mathematical tools employed in their investigation. The second, third, and fourth parts are given to the mechanical, electromagnetic, and electrostatic explanations, respectively.

The description of phenomena to be explained includes normal and anomalous dispersion, the Kayser-and-Runge series, variation of spectrum from one part of source to another, dependence of spectrum (of any one element) upon various physical conditions, phenomena of absorption, and the Zeeman effect.

The second lecture deals with an interesting attempt, on the part of Garbasso and Aschkinass, to build up a reflecting, refracting, and dispersing body from a large number of Hertzian oscillators. A short chapter on optical resonance, a clear statement concerning the use of the Lagrangian equations, and a few general mathematical theorems complete the first section of the volume.

The second section applies the equations of Lagrange, in a rather elegant manner, to the problem of normal dispersion according to the views of Cauchy, to the problem of anomalous dispersion according to

the mathematical theory of Helmholtz; and to the problem of the complex pendulum giving two frequencies and furnishing a mechanical analogue to illustrate Kundt's rule for the displacement of absorption bands in chemical compounds.

The third section is given over to various systems of Hertzian oscillators. Helmholtz's electromagnetic theory of dispersion is clearly set forth, Cauchy's formula coming out, of course, as a special case. The periods of various conductors and systems of conductors are determined with exceeding neatness by the Lagrangian method. Many of these conductors yield radiation of more than one frequency, thus giving groups of lines, which the author calls "doublets," "triplets," etc. But in no case is the approximation to Kayser and Runge's formula (i. e., all the frequencies in the series determined by three constants) sufficiently close to give one confidence of being on the right track. Thus, on p. 180, where a series of four lines is derived and discussed, it will be observed that three of these are used in the determination of the constants, so that only one line remains by which to test the formula. And while the agreement in this one case is fairly good, it does not follow that, in a series consisting of a larger number of lines, the agreement would be at all satisfactory.

In Part IV the electrodynamics of moving bodies is taken up, the Zeeman phenomenon is predicted and discussed very clearly—all in terms of the equations of Lagrange and Maxwell. Stoney's planetary (or kinematic) theory of the atom is set forth in a simple and clear manner.

The last two chapters contain an altogether excellent exposition of the atom proposed by Kelvin and profoundly modified by Lorentz, J. J. Thomson, and others.

The history of spectroscopy is filled with chapters which teach us to be very careful in the interpretation of experimental facts. Various interpretations have been given by different men at different times, to such a phenomenon as the long and short lines of Lockyer, or to the curvature of the lines in the Schuster-Hemsalech experiment, or to Lenard's separation of the electric arc into various regions by means of the slitless spectroscope. This variety of interpretation given to fundamental phenomena shows us at once how much we are in need of, and yet how far we are from, a general theory of spectroscopy.

The volume under review represents a laudable attempt to try out several different hypotheses. It will be of especial interest to the student of dynamics, containing as it does a series of illustrations of good dynamical method.

H. C.

A Compendium of Spherical Astronomy, with its Applications to the Determination and Reduction of Positions of the Fixed Stars. By SIMON NEWCOMB. New York: The Macmillan Co.; London: Macmillan & Co., Ltd., 1906. Pp. xviii+444, with 36 figures. $3, net.

Newcomb's *Compendium of Spherical Astronomy* is the most important addition to the literature of the subject since the appearance of the works of Chauvenet and Oppolzer. The volume is invaluable both to the advanced student and to the professional astronomer. It contains much already familiar, though presented with a freshness and directness that is new; a considerable amount of material hitherto inaccessible; and, most important of all, the results of the author's large experience with all that concerns the positions and proper motions of the stars. But it is evident from almost the first page that the discussion of the fundamental question of stellar positions and motions has been in the mind of the author as the real purpose of the book, for in the earlier chapters everything is prepared for the attack upon this problem, while time determination is barely touched upon, and the theory of the determination of latitude, eclipses, occultations, and transits is not even mentioned.

The space allotted to this review admits of only the briefest mention of the contents. The volume is divided into three parts. The first deals with preliminary subjects, including an introductory chapter of interest to the computer, a chapter on interpolation and mechanical quadratures, and one on the method of least squares. The last, of 44 pages, accepts the Gaussian formula of probability without development; and gives a concise statement of the different classes of errors and their determination, of the combination of observations by weight, and of the derivation and solution of a series of normal equations. Applications to cases of three and two unknowns are also made, that for the latter being arranged with special reference to the determination of proper motions.

Part II deals with the fundamental principles of spherical astronomy, which are developed with considerable detail. Spherical co-ordinates, the measure of time, parallax, aberration, astronomical refraction, and precession and nutation are the subjects treated. The last two, contained in chapters viii and ix, will be of the greatest interest to the professional astronomer. The chapter on refraction summarizes the results of recent meteorological investigations in their bearing upon the relation connecting atmospheric density and altitude, states the hypotheses of Newton, Bouguer, Bessel, and Ivory, and adds a fifth, which assumes a decrease in temperature in constant geometrical progression with the altitude. Two

forms are derived for the general equation for refraction. The first of these is the more useful when the zenith distance is not near 90°. It has the form

$$R = am \tan z\,,$$

where a is a constant, and

$$m = 1 - m_1 \sec^2 z + m_2 \sec^4 z - \ldots\ldots$$

The coefficient m_1 is shown to be independent of the relation connecting density with altitude. Whatever hypothesis is made, therefore, concerning the relation between these two quantities affects only the remaining terms of m, and since $m_2 < 0.000005$, and the coefficients of the higher terms are even smaller, it appears that the particular hypothesis used, excepting Bessel's, which deviates in another particular, is of importance only for large values of the zenith distance. This fact is of course well known, but the author's method of development exhibits the cause with especial clearness. The integrations necessary for the evaluation of m_2, m_3, etc., are carried through for the hypotheses of Newton and Ivory. The chapter ends with an account of the method employed in the construction of tables of refraction.

Chapter ix derives the numerical values of the various precessional motions and of the obliquity from the fundamental data for the motions of the poles of the equator and ecliptic, gives a clear exposition of the relation of nutation to precession, and contains a list of the usually included nutation terms.

Part III deals with the reduction and determination of the positions of the fixed stars. The first two chapters of this part form an important contribution to astronomical science. They contain improved methods for the reduction of star positions from one epoch to another and to apparent place. Rigorous trigonometrical developments are given for circumpolar stars as well as approximate formulæ for the more frequently occurring cases. The application of both methods of reduction is facilitated by the introduction of tables which are given in the Appendix. The processes here presented are designed to replace the older methods which, in certain cases, are unsatisfactory on account of the long interval through which precessional and proper motion reductions must now be made.

Chapter xii gives a clear account of the principles underlying the meridian determination of right ascensions and declinations, both fundamental and differential. The last chapter is devoted to an exposition of what the author has found to be the best method of deriving positions and proper motions from published results of observation. It is prefaced with a brief historical account of the development of meridian circle obser-

vation, and contains besides a description of the procedure involved in the formation of a fundamental system, with special references to the systems of Auwers, Boss, and Newcomb.

The notes and references appended to each chapter contain much valuable bibliographical material and many comments, suggestive and critical. The most important formulæ and the tables are collected in an appendix of 60 pages.

The usual number of misprints, apparently inevitable in a first edition, have made their appearance, but none of those noted are likely to cause the reader any great difficulty.

F. H. SEARES.

Elementi di Astronomia ad uso delle Scuole e per Istruzione privata. Compilati dal P. ADOLFO MÜLLER d. C. d. G. Roma: Desclée, Lefebvre E. C. Vol. I, Astrometria, Astromeccanica, 1904. Vol. II, Astrofisica, Astrochronaca, 1906. Pp. 600 each, with a total of 450 figures.

These two excellent volumes will meet with a hearty welcome from all who wish to be informed not only on the essentials of astronomy, but also on its connection with other sciences, its history, development, and present status.

Astrometry, in Vol. I, embraces the doctrine of the sphere, the construction of ancient and modern astronomical instruments, and some elementary mathematical formulæ in their application to the problems of practical astronomy. Astromechanics presents ancient and modern ideas regarding the mechanics of the solar system, eclipses, the calendar in its various transformations, the Galileo question, and the like.

Astrophysics, in Vol. II, contains short treatises on photography, spectroscopy, geology, meteorology, and terrestrial magnetism, and then investigates the physical constitution of the heavenly bodies. There is also a very condensed history of astronomy for handy reference.

Literary men will be pleased with the author's profuse references and quotations from all the great masters, from Aristotle, Plato, Cicero, Seneca, Ptolemy, Copernicus, Galileo, Newton, Herschel, and a host of others. The progress of the science is shown not only by the dicta of its great pioneers, but also by engravings of their instruments and the results of their observations. As the author is Father Secchi's legitimate successor, it was to be expected that, especially in astrophysics, Secchi's views should be prominently presented. The fact that the work was written in the Italian tongue and printed in Rome, will also appeal to some readers.

WILLIAM F. RIGGE.

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

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A SPECTROSCOPIC INVESTIGATION OF DR. G. URBAIN'S PREPARATIONS OF TERBIUM[1]

By G. EBERHARD

Although terbium was discovered as early as 1843, no one of the many chemists who have occupied themselves with its production have succeeded until recent years in obtaining a preparation that is even tolerably pure, from which the chemical properties and the spectrum of terbium could be studied. As we know today, Mosander, the discoverer of this element, had at most only from 1 to 2 per cent. of terbium in the substance which led him to the discovery; and the only thing that he could establish was that terbium formed a dark "superoxide" which strongly colored the other earths, which could be reduced on heating in a current of hydrogen.

The reasons for the failure of these attempts at isolation lie in the fact that terbium belongs to those rare elements which never occur except in excessively small quantities, and that its separation is even today one of the most difficult problems in the chemistry of the rare earths.

Under these circumstances it is not surprising that many chemists, and among them Bunsen, doubted the existence of terbium, and the contention lasted for many years, until finally its existence was conclusively demonstrated.

[1] Translated from advance proofs of a paper appearing in the *Sitzungzberichte der Kgl. Akad. der Wiss.*, Berlin, 1906, p. 384.

Undoubtedly the greatest advance was made by Lecoq de Boisbaudran and Demarcay. Lecoq had a few centigrams of the preparation, at least fairly rich in terbium, since he succeeded in finding the faint absorption band at λ 488 $\mu\mu$, but he was not in a position to adequately separate his preparation from the related earths. He further did not recognize that he was dealing with terbium, but thought he had found a new element ($Z\delta$). Later Demarçay[1] had a terbium of similar properties in which, independently of Lecoq, he found the band at λ 488, and was even able to give several lines of the spark spectrum. He also believed that he had found a new element (Γ). These two chemists are indeed the only ones who attained a fairly strong concentration of terbium in their preparations.

The Parisian chemist, Dr. G. Urbain,[2] was the first to succeed in actually isolating this interesting substance, after he had carried on for several years continuous fractionations by new methods which he invented, and obtained 7 grams of the costly substance in very great purity. He was able at the same time to furnish the chemical proof that his preparation represented a single element, that this was identical with Mosander's terbium and had an atomic weight of 159.2. The only thing lacking for a complete description of this element was the accurate investigation of its spectrum, by which alone an element is characterized in a wholly unambiguous way, along with its chemical properties. The investigation of the spectrum is of the highest importance, particularly in the case of the rare earths which possess such an uncommon similarity in their chemical characteristics. Frequently observers have thought they had found new elements when they were actually dealing only with mixtures of several of these so closely related substances. Furthermore, the spectroscopic investigation alone can settle what related elements of this group are contributing impurities, and whether the amount of the impurity is so low that the preparation can be regarded as chemically pure. For this reason spectrum analysis has since its foundation stood in the closest relation to the chemistry of the rare earths, and exact results have been obtained only by the co-operation of the chemist and the spectro-

[1] *Comptes Rendus*, **131**, 387, 1900.

[2] *Ibid.*, **139**, 736, 1904; **141**, 521, 1905.

scopist, as Kayser[1] has so aptly shown in the introduction to his paper on the spectrum of yttrium.

Dr. Urbain then requested me to complete the researches in this direction, and, thanks to the numerous and exceedingly valuable preparations which he put at my disposal, the present research could be carried out and the proof furnished, with all the precision desired, that we were concerned with an actual element. In spite of the very great purity of Urbain's preparations, the work could be executed with the prospect of success only by the spectroscopic investigation of a large number of its fractionations, in addition to terbium itself, as well as of its related elements, inasmuch as no previous spectroscopic investigations were available (with exception of the eight spark lines given by Demarçay), and indeed under the circumstances such investigations could not exist. As a matter of fact, Messrs. Exner and Haschek, after the investigation of a whole series of fractionations of the yttrium earths, expressed the opinion that terbium does not exist.[2] This investigation was greatly simplified by the fact that Urbain succeeded in producing, in an adequate degree of purity, dysprosium, which follows terbium and which hitherto had never been produced, and was only known by a few absorption lines. Thus, it has now been possible to follow the behavior of the terbium lines from their appearance (in gadolinium) to their disappearance (in dysprosium).[3]

For photographing the spectra I employed a concave grating spectrograph[4] of the Astrophysical Observatory at Potsdam, which made it possible to obtain with a single exposure on a film (mounted on a stiff celluloid foundation) the second order spectrum from λ 2200 to

[1] "Die Bogenspektren von Yttrium und Ytterbium," *Anhang zu den Abhandlungen d. K. Preuss. Akad. d. Wiss.*, 1903.

[2] *Bogenspektren der Elemente*, I. p. 19. No criticism should be made of these gentlemen on that account, for they had at their service inadequately differentiated fractionations, and too impure preparations. For instance, their "holmium" consisted of gadolinium, terbium, dysprosium, neoholmium, yttrium, and erbium.

[3] Before this investigation was concluded, Dr. Urbain sent me a series of the purest preparations of dysprosium, designated as *Dy* 31, 35, 38, 41, of which *Dy* 41, which I have already partially investigated, contains very faintly only a few of the very strongest terbium lines.

[4] A description of this apparatus may be found in the *Zeitschrift für Instrumentenkunde*, **25**, 371, 1905.

λ 4700. On account of the excessive cost of the preparation, only very small quantities were available, for which reason it would be undesirable to make several exposures for the different regions of the spectrum. The substances in the form of oxides were vaporized on carbon rods which had previously been thoroughly heated, with a current-strength of about 15 amperes at 120 volts. Iron served as comparison spectrum, but in order to eliminate any changes in the apparatus between the two exposures, a very small quantity of the substance under investigation was vaporized on the carbon crater simultaneously with the iron. Then, if there were any small displacements of the spectrum with reference to the comparison spectrum, such as might be due to slight changes of temperature, this could be determined by the measurement of the lines common in the two strips of spectrum, and hence they could be made harmless. The films were laid between sheets of optically plane glass, and measured on machine No. 4 of this Observatory, the screw of which has no periodic errors, as has been shown by an investigation by myself and Dr. Münch. The progressive errors, which were also determined, run very smoothly and are small.

I have employed the values given by Exner and Haschek for the wave-lengths of the iron lines, for the following two reasons. Too small a number of the decidedly more accurate wave-lengths of the iron lines by Kayser are available for an investigation in which the lines are needed at small intervals, so that, unfortunately, I could not employ them. Furthermore, the wave-lengths of the terbium lines ought to be referred as closely as possible to the system of Exner and Haschek, since the tables of wave-lengths by these gentlemen are at present the only complete ones, and they deservedly have found a very wide circulation. It is true that the accidental errors in the values of Exner and Haschek often reach several hundredths of an Ångström unit, but I hope that I have made myself somewhat independent of these by the use of a great number of lines, and by a graphical correction of their wave-lengths. The precision of the wave-lengths of the terbium lines as given below is the same as that of the measurements of Exner and Haschek.

Dr. Urbain placed at my disposal the following preparations:

A. *Gadolinium.*—(1) Entirely pure gadolinium from the minerals monazit and xenotin.

(2) A number of gadolinium fractionations from europium to terbium. The oxide of the last of these (*Gd* 37) is of faint yellow color and gives the strongest terbium lines quite conspicuously.

B. *Terbium.*—(1) "Nitrates doubles de *Ni. Gd-Tb* (14)." The oxide which was strongly yellow already shows all of the strong terbium lines and those of medium strength.

(2) "$Z\delta_7$." The dirty dark yellow oxide already gives all the terbium lines along with those of gadolinium.

(3) "$Z\delta_5$." The oxide is almost black with a touch of reddish-brown. It is an almost pure terbium, but it still yields the strong lines of gadolinium in the arc.

(4) "$Z\delta_3$" and (5) "$Z\delta_1$." The oxides are almost black with a faint touch of the reddish-brown. The spectrum is essentially that of terbium.

(6) "*Tb-Dy* 7, 8, 9." The oxides are of the dirty dark yellow. All the lines of terbium and almost all those of dysprosium are present. The three fractionations show only very small differences.

C. *Dysprosium and neoholmium.*—(1) "Queues du fractionnement des ethylsulfates de *Tb*; traces *Ho*, tres-riches *Dy*" (Q). The oxide is of a dirty bright yellow color. The lines of dysprosium here appear in full number, and faintly those of terbium.

(2) "Terres riches en *Ho*, pauvres en *Dy*, riches en Y. Fraction (4–5)" (*Ho*). The oxide is white with a faint touch of yellowish. The spectrum contains essential lines of yttrium, the strong lines of neoholmium and the stronger lines of dysprosium.

(3) "*Dy* 31, 35, 38, 41." The oxide is of a faint dirty yellow. *Dy* 41 is a pure dysprosium.

Complete measurements were made of the following photographs of these preparations: $Z\delta_7$, $Z\delta_5$, $Z\delta_1$, *Tb-Dy* 8, Q, with a total of 10,300 lines beside 2000 iron lines. Subsequently I also worked upon the greater part of *Dy* 41. Some single lines were measured in all fractionations. All of the plates, measurements and reductions were made exclusively by myself.

The material yielded by these measures I have critically discussed

in order to obtain certainty as to the impurities. First, with the aid of the tables of Exner and Haschek, those lines were eliminated which were due to elements not belonging to the group of rare earths. This included the lines coming from the carbon rods, particularly *Ba*, *Ca*, *Al*, and *Si*. It is probable that the lines of the different band spectra of carbon were not fully eliminated, but I believe that only the fainter ones are left in the tables, since during the measurement all the stronger ones could be recognized by comparison with the control spectrum in which they also occurred, and they could be left out. The ultra-violet cyanogen bands confused the image of the spectrum considerably, and they certainly suppressed or concealed some of the fainter lines of terbium in these regions. Of the other elements we should particularly have expected *Pt*, *Ni*, *Bi*, and *Mg*, which might have crept in during the fractionation; but the preparations proved to be entirely free from these elements. The same was true of the rare earths *Ce*, *La*, *Pr*, *Nd*, *Sa*, *Eu*. Gadolinium was the first element whose lines appeared. Since Messrs. Exner and Haschek dealt in their investigations with gadolinium from Demarçay, I sought out the impurities on the basis of those tables, and employed my own photographs of Urbain's preparations only in isolated cases.[1] I have collected in Table I all the stronger lines of gadolinium which did not coincide with terbium lines, and I give the intensities (I) which they had on the spectra I measured.

It appears from this table that Q is wholly free from gadolinium, and *Tb-Dy* 8 practically so, while $Z\delta_1$ only contained very slight traces.

For eliminating the lines of yttrium I have employed the tables of Kayser and those of Exner and Haschek, and give the results in Table II. I would particularly point out that, according to this, yttrium is certainly lacking in $Z\delta_7$, $Z\delta_5$, and $Z\delta_1$. All the yttrium lines are also absent from *Dy* 41.

For hunting down the lines of the other yttrium earths I have been completely dependent on my own researches, since all the preparations used by Exner and Haschek were very complicated mixtures; indeed,

[1] I have not been able to find the strong gadolinium line λ 4063.62 (20) of Exner and Haschek's tables. On my plates there are at this point the two lines λ 4063.54 (4) and λ 4063.76 (2).

in the case of some of them (erbium, thulium, and ytterbium) the question of their elementary character is still doubtful. Fortunately Urbain had succeeded in isolating dysprosium, so that by the measurement and provisional tabulation of the dysprosium spectrum I was able to select at least those lines which most particularly come into consideration as impurities. Provisional values of the wave-lengths and intensities of the stronger dysprosium lines are contained in Table III, in so far as they did not coincide with terbium lines, and the intensities of these lines in the terbium preparations are again given with them.[1]

TABLE I

Exner and Haschek		I (Eberhard)				Exner and Haschek		I (Eberhard)			
λ	I	$Z\delta_5$	$Z\delta_1$	*Tb-Dy*8	Q	λ	I	$Z\delta_5$	$Z\delta_1$	*Tb-Dy*8	Q
3027.74	8	6				3814.18	10	3			
3100.66	10	6	0	0		50.85	8	1			
3350.63	8	6	2	0		51.15	8	1			
3418.87	8	3				52.65	10	6			
22.62	10	6	2			94.88	8	3			
39.37	8	3				3916.70	10	5	1		
40.13	8	5				4037.49	10	3			
81.49	8	4				38.03	8	3			
3545.94	10	3	0			49.59	8	4			
85.12	10	3				50.05	10	5			
3646.36	15	4				63.62	20				
54.78	8	2				70.51	10				
56.31	8	3	0			85.73	10	4			
64.78	8	5	0			98.80	10	3	1	0	
71.39	10	3				4212.16	8	4			
3712.88	8	5				26.02	8	4			
43.68	10	3				62.24	10	7	0		
68.60	20	7				4327.29	10	6			
96.62	10	2				42.35	10	3			

TABLE II

Exner and Haschek		I (Eberhard)			Exner and Haschek		I (Eberhard)		
λ	I	$Z\delta_5$	$Z\delta_1$	*Db-Dy*8	λ	I	$Z\delta_5$	$Z\delta_1$	*Db-Dy*8
3600.01	10			0?	4177.74	30			
3710.48	20			1	4309.81	20			
3774.52	20				4375.12	50			
3788.88	30			2	4398.25	20			
4077.54	20				4422.81	10			
4102.57	20								

[1] For the purpose of comparison I have added Exner and Haschek's wave-lengths and intensities in the spectrum of "holmium," which appear to belong to dysprosium.

TABLE III

Eberhard						Exner and Haschek		Eberhard						Exner and Haschek	
λ	Q	*Tb-Dy8*	*Zδ*$_1$	*Zδ*$_5$	*Zδ*$_7$	λ	I	λ	Q	*Tb-Dy8*	*Zδ*$_1$	*Zδ*$_3$	*Zδ*$_7$	λ	I
2906.53	6	4				.53	2	3266.32	5	2				.33	2
14.09	6	0	0			.08	3	69.24	6	3				.22	2
34.62	6	5				.60	3	72.88	6	3				.85	1
44.70	5	2				.67	3	80.25	5	4	1			.22	3
53.88	5	1				.82	2	88.06	5	2				.05	2
77.57	6	3	0			.53	1	3308.97	9	6	2			.99	10
86.06	7	2				.04	dp. 2,2	16.42	7	4	2			.42	4
91.73	5					.73	2	17.22	6	3	1			.23	2
3002.51	5	2				.51	2	20.02	7	6	3			.02	5
03.89	6	2				.86	2	41.12	6	4	1			.12	4
15.22	6	2				.16	2	42.00	5	3	2			.00	2
15.82	6	4				.77	2	58.38	7	2				.39	2
17.10	6	2				.09	dp. 1, 2	68.24	6	5	1			.21	5
25.73	6	2				.71	2	84.23	8	1				.21	2
26.30	6	5	0			.26	3	85.15	9	7	4	0		.19	10
29.95	5	2				.91	2	88.83	6	0					
36.83	5	2				.80	2	96.31	7	6	0			.31	6
47.68	7	4				.66	3	3407.94	8	5	3			.94	6
51.60	5	3				.57	2	14.95	6	4				.95	3
52.44	5	2				.45	2	19.73	8	4	2	1		.72	4
60.17	5	2				.14	2	29.15	6	2				.23	1
62.76	5	4	0			.72	3	29.57	7	4	2	0		.56	2
72.03	6	5	0			.03	3	45.73	8	6	3			.72	8
73.68	6	4				.66	3	50.05	5	3				.02	2
3102.04	7	3				.02	3	54.49	6	3	2			.50	dp. 4, 2
03.36	5	2				.36	3	63.51	6	2	0			.46	1
03.97	5	3				.94	3	71.27	6	3				.27	2
05.12	5	0				.10	2	71.73	7	3				.72	3
20.28	5	3				.30	2	77.23	6	4	3			.21	4
28.51	7	4	0			54	3	97.97	5	2				.93	2
40.74	6	4	2	0		.75	4	3501.59	5	2				.56	2
41.25	6	4	0			.23	4	04.68	7	3				.64	3
43.92	5	2				.93	2	17.41	6	3	1			.38	3
46.25	6	3				.24	2	24.15	8	5	3			.19	10
52.00	5	3	1			.00	2	31.89	10	10	3	2		.90	20
87.80	6	3	0			.80	2	35.12	5	4	3	0		.12	8
93.41	6	3				.42	3	36.18	6	4	3	0		.15	8
3205.61	6					.57	1	38.67	7	5	3			.65	8
08.96	6	3	2	0		.94	2	42.48	6	4	1			.50	6
12.20	5	0				.16	1	45.13	5	3	1	0		.12	2
15.31	6	3	0			.30	3	50.39	7	6	3			.37	5
16.75	8	7	3			.77	6	51.77	5	3	2			.79	4
23.30	6	3				.41	3	63.31	5	3	2			.30	4
25.23	6	2				.23	2	73.99	5	4				.99	3
36.77	7	3				.75	2	74.30	5	3	1	0		.31	4
48.47	5	3				.45	2	76.42	5	3	2			.40	5
56.38	5	3	0			.40	2	77.05	5	4	2			.01	5
60.82	5	3	2	1		.84	2	96.21	5	2				.18	2
66.12	5	3	2			.12	1	3602.97	5	1				.96	2

TABLE III—*Continued*

Eberhard						Exner and Haschek		Eberhard						Exner and Haschek	
λ	Q	*Tb-Dy8*	$Z\delta_1$	$Z\delta_5$	$Z\delta_7$	λ	I	λ	Q	*Tb-Dy8*	$Z\delta_1$	$Z\delta_5$	$Z\delta_7$	λ	I
3614.24	4	2				.21	2	3983.78	7	6				.82	10
24.43	5	3				.40	2	84.40	6	4				.38	4
35.41	6	3				.39	3	96.84	6	6				.90	10
36.38	5	2						4000.59	10	9	4	1		.67	20
37.45	7	3				.40	3	11.44	5	5	0			.42	3
44.05	6	2				.04	3	24.57	6	4	0	0		.56	3
45.54	10	7	4	2		.52	10	27.94	6	4	0			.91	1
46.01	5	4	3			.96	3	42.12	6	4				.11	3
72.45	6	4	3			.43	4	46.13	10	10	4	2		.11	20
74.20	6	5	4			.21	4	48.51	5	3				.46	2
85.96	7	4				.92	3	50.73	7	7	3			.72	10
94.98	7	6	1	0		.98	8	73.31	8	6	4	2		.30	10
97.65	6	3				.65	3	78.10	10	9	4	0		.11	15
98.34	6	4	1			.32	4	4111.50	7	5	3			.51	10
3701.79	6	3				.76	3	19.45	6	3	0			.47	3
24.63	5	4	1			.59	4	24.79	6	4	0			.80	3
57.51	6					.52	6	29.58	6	6	2	0		.60	5
84.05	5	3				.08	2	43.26	6	6	2			.25	6
86.31	6	4	3			.34	8	68.14	8	7	3			.15	8
88.56	5	3	2			.57	5	4206.74	7	7	4			.72	4
3812.41	6					.43	2	11.92	10	10	5	1		.89	20
25.82	5	4				.77	3	18.28	9	8	2			.26	5
72.30	7	5	3			.28	10	21.32	8	7	3			.29	10
92.08	5	2				.05	1	25.34	7	6	3			.31	5
98.68	7	6	3	0		.69	10	47.52	5	4				.47	2
3914.14	5	2	0			.13	2	95.18	9	6				.18	8
15.02	6	3	0			.02	4	4325.30	6	2	0			.20	3
15.75	6	3	1			.74	4	39.83	6	5	1	0		.80	3
31.67	5	5	3			.72	5	58.65	7	6	1			.60	3
42.66	6	4	0			.66	4	74.43	6	6	0			.42	3
44.85	9	9	4	1		.87	20	74.94	6	5	0				
54.69	6	4	1			.68	3	95.15	9	7	4	0		.12	3
78.70	8	8	4			.70	10	4449.92	9	9	4			.88	10
79.61	6	5				.57	3	68.34	6	5	0			.31	3

$Z\delta_7$ is accordingly entirely free from dysprosium, and $Z\delta_5$ has only faint traces of the strongest lines.

I have not yet had available a pure neoholmium, and it is therefore evident that the preparations at my disposal supplement each other in the most fortunate fashion.

An accurate separation of the lines of the components of erbium, thulium, and ytterbium[4] is not possible at present, for the reasons

[4] Auer von Welsbach has recently separated erbium and ytterbium into two components each, but nothing more precise is yet known in the matter.

given above, and there would therefore have been great danger that lines of these elements might be present in my tables for terbium. The fact that this is not the case is due to the circumstance that Urbain discovered and employed a method of fractionation in which the very considerable quantities of yttrium in the raw material separated between neoholmium and erbium. Therefore the neoholmium, and so much the more the dysprosium which separates before this, and still again the terbium separating still before this, must be free from erbium, thulium, and ytterbium. The entire absence of the very strong yttrium lines in the preparation $Z\delta_7$, $Z\delta_5$, and $Z\delta$ is, under the circumstances, an absolute proof of the absence of erbium, thulium and ytterbium from the terbium preparations. I have not failed to search among the terbium lines for the strong lines which Exner and Haschek ascribe to these elements, but with negative results, as was also ultimately the case for scandium. From this discussion it is clear that Dr. Urbain in fact has succeeded in producing terbium in a remarkable degree of purity, after it had been sought in vain for sixty years by numerous investigators of the first rank.

In the table of terbium lines which now follows, each value of the wave-length and of the intensity is the mean of three, and many times of four, separate values which were obtained in the measures of $Z\delta_7$, $Z\delta_5$, $Z\delta_1$, *Tb-Dy*8. Only in very isolated cases (chiefly for faint lines) are the figures in the table the mean from two separate values, and this only when the lines in question were present in $Z\delta_7$ as well as in $Z\delta_5$, so that it is almost certain that they belong to terbium. I measured all the lines present on the spectrograms, in so far as they could not be recognized at once as lines of carbon. The intensity 1 was assigned to the faintest lines which were still well measurable, the intenity 10 to the strongest lines. The very faintest lines which were just visible, but no longer measurable with certainty (intensity = 0), were omitted from the following table; but there were not many of them. In estimating the intensity, both the degree of blackening and the width or the whole impression of the lines were the criteria, and only in the case of diffuse lines was the intensity made to depend principally on the degree of blackening. The symbols br, dp, and r denote broad, double, and diffuse toward the red. If the assignment of a line

of terbium is not wholly free from doubt, this fact is indicated by the symbol ? following it.

In the tables I have given, along with the values of the wave-lengths and intensities found by myself, also those of Exner and Haschek for the holmium spectrum which appear to belong to terbium; and, further, those lines of gadolinium, yttrium, and dysprosium which are close to terbium lines. The values for dysprosium are also here to be regarded as only provisional.

TABLE IV

Eberhard						Exner and Haschek	
λ	Q	*Tb-Dy*8	$Z\delta_1$	$Z\delta_5$	$Z\delta_7$	λ	I
3399.06	2	0	1			3399.09	10
3416.55	1					3416.57	8
3485.02	2					3484.97	10
3515.74	1					3515.72	10
3891.21	3					3891.18	20
4045.64	1					4045.59	10
4054.05	2					4054.08	10

The study of the behavior of the separate lines gave me the following general results:

1. There are no indications of a resolution of gadolinium.

2. From Urbain's preparations there would appear to be no other element between gadolinium and terbium.

3. The terbium production by Urbain appears to be a simple body. well defined by a characteristic spectrum—an element—since there are no indications that it could be resolved.

4. The preparations made by Dr. Urbain (particularly $Z\delta_3$ and $Z\delta_1$) are pure to such an extent that a determination of the atomic weight with them must give a very nearly correct value for this constant.

5. Those lines which could be followed farthest toward gadolinium and also toward dysprosium are $\lambda\lambda$ 3523.82, 3676.52, 3703.01, 3704.05, 4005.62, 4278.71. They can serve for proving the presence of terbium in minerals and raw materials. In fact, with the aid of these I was able to find terbium in samarskite oxide and in monazite oxides free from the cerium and thorium, and in gadolinite-yttrium earths.

6. The lines of terbium are not present in the solar spectrum, or at least not with appreciable intensity.

TABLE V

Eberhard		Exner and Haschek		Remarks	Eberhard		Exner and Haschek		Remarks
λ	I	λ	I		λ	I	λ	I	
2759.65	1				2996.12	1	.12	1	
69.69	2	.65	1		99.18	3			*Gd* .20 (5)
84.63	1				3004.71	2	.70	1	
2800.66	1	.65	2	*Dy* .65(2)	05.65	4	.66	1	
09.46	3	.40	1		09.44	4	.42	1	*Gd* .47 (1)
12.79	1	.75	1		10.71	4	.70	1	
27.56	1				12.18	1			
44.08	1				13.07	1			
55.10	1				13.77	2	.81	1	
55.80	1				16.36	5	.36	dp. 1, 1	
57.84	3	.79	1		19.32	3	.27	1	
61.47	1				20.47	4	.44	1	
84.82	1	.77	1		22.10	1			
86.03	1				23.57	3			
86.42	3	.38	1		23.88	1			
87.54	1				27.46	4	.44	1	
94.58	2	.58	1		29.38	1			
97.58	5	.59	1		31.72	5	.73	1	
98.96	4	.95	1		34.22	2			*Gd* .20 (8)
2901.67	1	.70	1		35.00	1			
10.44	2	.47	2		37.14	1	.13	1	
11.96	2	.93	1		38.37	1	.40	3	*Dy* .43 (6)
14.87	4	.88	1		38.81	2	.80	1	
15.70	3	.70	1		43.74	1br			
16.37	3	.38	1		45.08	5	.06	1	
19.06	2				47.11	1			
24.28	1				51.24	5			
24.64	1				52.34	1			
26.07	1				53.41	2			
31.53	1				53.71	6	.71	2	*Gd* .70 (3)
33.02	3	.99	1		61.96	1			
33.94	1				62.90	1			
34.90	1				64.20	4	.15	1	
36.40	1				64.69	1	.67	1	
40.17	3	.13	1		65.32	1	.28	1	
41.81	1				65.82	1			
45.04	2				66.02	1			
45.84	2				67.31	5	.30	1	
46.93	1			*Dy* .95 (3)	69.14	6	.11	dp. 1, 1	
49.20	1				70.19	7	.17	2	
50.18	1	.18	1		72.79	5	.82	1	*Gd* .74 (3)
56.35	4	.32	1		76.18	2			
60.72	1				79.01	6	.99	2	
63.01	1	.00	1		80.27	1			
64.12	1	.12	1		81.68	2			
64.87	2	.85	1	*Dy* .90 (2)	82.17	1	.20	1	?*Gd* .15 (10)
68.96	3	.97	1	*Dy* .02 (2)	82.53	5	.52	1	
77.89	2	.91	1		83.04	1			
87.17	1				86.14	1			
88.81	2	.80	1		86.91	2			
92.00	1				87.66	1			*Dy* .67 (2)

TABLE V—*Continued*

Eberhard		Exner and Haschek		Remarks	Eberhard		Exner and Haschek		Remarks
λ	I	λ	I		λ	I	λ	I	
3088.55	6br	.52	1		3158.20	1			
89.25	1	.19	1	*Gd* .31 (1)	58.80	2			*Gd* .75 (1)
89.70	5	.66	2		59.35	2			
91.80	1				59.52	2			
92.13	1			*Gd* .15 (2)	62.55	2			
93.06	1				63.02	4			
93.32	1			*Dy* .25 (3)	63.95	3			
93.55	1	.56	1	*Dy* .63 (1)	64.21	3	.22	1	
93.96	1	.95	2		64.88	3			
96.97	2	.00	1	*Gd* .92 (1)	65.86	5	.87	1	
97.53	1	.52	1		67.62	4	.60	2	
3102.66	5				68.42	5			
03.10	5	.10	1		68.71	1	.74	1	*Dy* .76 (2)
09.27	2	.28	1		69.93	5			
09.93	2	.90	4	? *Dy* .89 (6)	71.30	2			
12.53	1				72.04	1			
12.68	1				73.84	4			
13.72	4				73.94	1	.90	2	
14.56	1				74.76	5	.80	1	
17.37	2				75.55	1			
18.01	2	.02	2	*Gd* .07(1),	76.90	1			
				Dy .04 (4)	77.15	1			
19.73	4	.71	1		77.61	1br	.66	1	*Dy* .63 (2)
21.55	1	.55	1		79.94	1			
22.06	3	.07	1		80.66	6	.72	1	
22.93	3				81.30	1			
23.16	4	.17	1		82.91	1			
24.12	2			*Gd* .16 (3)	83.39	3	.34	1	*Dy* .32 (3)
24.65	1				83.76	3			
29.00	1				83.98	3	.01	1	
30.78	1br				85.67	2			
31.48	3				86.33	2			
34.37	4	.39	1		87.37	6	.39	2	
35.41	2			*Dy* .51 (6)	88.15	6	.18	1	
37.40	2				88.68	2			
38.75	1			*Gd* .81 (1)	89.87	1	.92	1	*Dy* .90 (2)
39.77	6	.74	1		90.10	2			
40.18	4	.15	1		90.80	1br	.80	1	*Dy* .76 (2)
43.56	1				92.58	1	.09	1	
44.56	1	.60	1		93.13	1			
45.33	4	.31	2	*Dy* .32 (3)	94.31	2			
46.35	2				94.83	4			
46.83	2				95.74	5			*Y* .75 (3)
47.07	4	.02	1		98.15	1			
47.27	4	.25	1		99.67	7	.65	1	*Gd* .67 (1)
48.84	4	.85	1		3200.84	3			
54.85	1				02.06	1			
55.22	1				02.82	2			?
55.74	3				03.15	2			
57.28	1	.30	1	*Dy* .31 (2)	07.19	3	.23	2	*Dy* .25 (6)
57.62	1	.64	1	*Dy* .66 (2)	07.63	4			

TABLE V—*Continued*

Eberhard		Exner and Haschek		Remarks	Eberhard		Exner and Haschek		Remarks
λ	I	λ	I		λ	I	λ	I	
3208.09	3				3262.83	3			
09.66	3				63.09	5	.10	1	
10.15	1				63.87	1			
10.32	3				64.05	3	.03	1	
15.15	3				64.19	1			
15.63	1				65.05	3			
19.04	8	.05	2		66.09	2	.12	1	
20.09	6	.10	2		66.55	6	.52	1	
20.34	1				68.26	4	.20	1	
23.10	3				68.65	3	.66	1	
24.84	1				70.70	1br			
25.63	1			*Gd* .57 (2)	70.83	1			
27.10	1				72.53	4	.48	1	
27.61	1				73.28	2			
29.30	2				74.31	3			*Gd* .30 (1)
29.81	1br				74.45	2			
30.13	4				75.84	1	.87	1	*Dy* .92 (2)
30.85	1				77.45	3	.45	1	
31.18	3	.17	1		77.85	1br			
31.59	1				79.22	1			
32.16	2				80.44	6	.40	2	
33.69	1				81.55	6	.55	1	
34.62	2				83.26	6	.27	1	
35.88	3			*Dy* .97 (6)	83.96	1			
36.29	2			*Gd* .26 (1)	85.16	5	.16	1	
39.39	1				85.33	3			
39.74	2br	.69	1		86.29	1	.30	1	
40.11	5	.14	1		87.08	1br			
40.78	4				87.70	3	.67	1	
42.06	1				91.69	4	.68	1	
43.32	3				93.20	4r	.18	2	
44.74	1				94.16	4			*Gd* .21 (2)
45.31	3			*Dy* .25 (5)	95.44	2	.50	1	
45.52	1				98.34	2	.34	1	
46.64	1				98.79	4	.76	1	
47.32	1				3304.22	2	.19	1	
47.98	1				04.39	2	.45	1	*Dy* .42 (2)
49.75	2				05.07	3	.09	2	
50.63	1				05.51	2			*Dy* .55 (2)
50.88	1				07.53	5br			
51.06	1				07.92	1			
51.37	4	.42	5	*Dy* .41 (7)	08.66	2			*Gd* .66 (1)
52.44	6	.45	1		09.28	2			
53.68	1				10.24	1			
55.34	2	.36	1		10.49	1			
58.53	1				10.92	2			
59.54	4	.51	1		12.70	1			
60.23	4			*Dy* .12 (6)	12.90	3	.86	3	
60.80	1	.84	2	*Dy* .83 (4)	14.45	3			
60.97	2				15.17	1			
61.86	5				17.71	2			

TABLE V—*Continued*

Eberhard		Exner and Haschek		Remarks	Eberhard		Exner and Haschek		Remarks
λ	I	λ	I		λ	I	λ	I	
3318.17	1			*Gd* .19 (1)	3375.18	4	.16	1	
19.31	1				76.51	2	.47	1	*Dy* .50 (3)
19.93	1br			*Dy* .02 (5)	76.79	3	.74	1	
21.02	1				77.80	4			
21.25	4br	.23	1		78.86	4			
22.38	5	.40	1		79.02	4	.00	2	
23.52	3				79.32	1			
24.03	3	.06	1		80.78	4	.79	1	
24.54	8	.56	4		81.01	2			
25.64	3	.64	1		81.74	1	.70	1	
27.24	3	.20	2		82.91	4	.91	1	
29.20	5	.20	1		86.61	4			*Dy* .70 (3)
33.33	3	.30	1		87.77	1			
34.07	2				88.51	1	.48	1	
34.47	1				89.72	2			
34.62	3	.56	1	*Dy* .56 (1)	90.15	3			
35.56	2				90.78	1			
36.87	5	.90	1		91.05	1			*Gd* .04 (1)
38.18	3	.14	1		91.39	3			*Gd* .45 (1)
39.15	5	.14	1		91.80	1			
39.75	2				92.20	2			
43.10	1				93.12	1	.10	1	
43.71	1	.69	2	*Dy* .65 (1)	93.68	2			*Gd* .76 (2), *Dy* .71 (7)
43.90	1								
45.00	1				94.94	3	.97	1	
46.49	1	.45	1		95.35	3			
47.41	3	.36	1		97.36	1			*Gd* .35 (1)
48.25	3	.20	1		97.76	2			
48.43	1	[illegible]			98.49	4	.45	1	
48.70	3	.70	1		98.69	1			
49.57	6	.54	1		99.23	3			
50.64	2			*Gd* .63 (8)	3400.10	2			*Gd* .12 (3)
51.58	1				00.64	3	.62	1	
53.05	3				00.99	4	.01	1	
57.50	3			*Dy* .46 (1)	02.45	4			
59.47	1				04.91	3			
59.99	2				06.16	3			
60.39	2				07.25	1	.30	2	*Dy* .31 (2)
62.32	5			*Gd* .40 (8)	09.00	1			
64.33	2			*Gd* .36 (2)	10.04	1			
64.48	1				10.56	3			
65.01	7	.01	1		10.83	3	.85	2	
65.42	3				13.92	5	.92	5	*Dy* .94 (7)
66.31	1	.33	1		15.59	1			
67.31	3	.30	1	*Gd* .22 (1)	16.40	4			
68.67	1				16.71	1			
70.73	3	.70	1		17.87	2			
71.65	3	.65	1		18.11	1			
72.50	4	.50	1		20.47	5	.46	1	
72.85	1	.90	5	*Dy* .88 (3)	23.21	1			
74.56	3				23.56	1			

TABLE V—*Continued*

Eberhard λ	Eberhard I	Exner and Haschek λ	Exner and Haschek I	Remarks
3424.14	2			*Gd* .07 (5)
24.46	1			
24.83	1			
25.57	1			
26.06	2			*Gd* .08 (3)
26.48	2			*Gd* .46 (2)
28.29	1	.26	2	
28.87	1			
29.24	1	.26	1	
29.92	1			
30.31	1			
30.49	1			
30.76	4			
32.03	1			
32.51	3			
33.38	5	.40	1	
34.57	2	.51	5	*Dy* .50 (6)
35.08	1			
35.70	1			
36.27	1			
37.12	3	.06	1	*Dy* .07 (1)
38.73	3			
39.22	1			
39.87	2	.86	1	*Gd* .93 (3)
40.55	4	.55	1	
41.84	1			*Gd* .92 (2)
44.06	1			
44.74	3	.70	1	
44.91	1			
46.52	5	.46	1	
49.02	1			
49.61	2			
52.56	2			
53.01	1			
53.60	1			
54.24	3			*Gd* .30 (3)
55.55	1			
56.15	2	.13	10	*Dy* .13 (3, very broad)
56.71	1	.68	4	*Dy* .67 (6)
57.18	3			*Gd* .17 (2)
58.77	3	.73	1	
60.57	3	.54	2	*Dy* .54 (2)
61.14	4	.10	5	*Dy* .10 (5)
62.68	1			
62.98	1			
63.12	2			*Gd* .14 (3)
64.76	1br			
66.13	2			
67.03	2			*Gd* .09 (3)
68.17	5	.15	1	

Eberhard λ	Eberhard I	Exner and Haschek λ	Exner and Haschek I	Remarks
3468.55	1	.55	2	*Dy* .58 (2)
69.84	1			
70.01	1	.00	1	*Dy* .06 (1)
70.52	3	.46	1	
71.87	3			
72.52	3	.49	1	
72.95	6	.95	1	
73.18	2			
73.94	2			*Dy* .84 (3)
75.46	1br			
76.28	2			
76.45	2	.47	1	*Gd* .45 (2)
79.40	1			
80.33	4	.32	1	
80.63	1dp	.57	1	
81.69	2			
82.98	2	.91	1	*Dy* .93 (3)
83.20	3	.16	1	
83.85	3	.83	1	
84.86	1br			*Dy* .84 (2)
87.43	3			*Dy* .38 (2)
87.78	2			*Dy* .72 (3)
88.96	1			
89.65	4	.63	1	
89.91	2			
90.42	1			
91.41	2	.39	1	
91.94	1			
92.13	2br	.13	1	*Gd* .10 (6)
92.69	3	.66	1	
93.14	2			
94.39	2			
95.53	2			
96.43	1	.46	3	
98.90	1	.82	3	*Dy* .86 (3)
99.47	1dp			
3500.20	1			
00.42	3			
00.99	5	.96	1	
04.18	1			
04.91	2			
05.26	1	.24	dp1,1	
06.05	3			
07.58	3	.60	1	
09.32	10	.30	8	Perhaps double
10.25	2	.22	1	*Gd* .26 (1), *Dy* .28 (2)
11.21	1	.15	1	
12.73	2	.70	2	*Gd* .66 (4), *Dy* .70 (3)

TABLE V—*Continued*

Eberhard		Exner and Haschek		Remarks	Eberhard		Exner and Haschek		Remarks
λ	I	λ	I		λ	I	λ		
3513.24	4	.20	1		3571.49	1			
14.01	1				72.25	2	.28	1	
14.31	1	.27	1		75.77	1dp			
15.16	3	.16	1		76.03	2			
15.64	2				76.78	1	.74	1	
16.31	2				77.21	3			
16.79	1				79.36	5	.33	1	*Dy* .27 (2)
19.91	4				85.24	2	.21	5	*Dy* .22 (3)
20.95	2	.96	1		87.60	3	.56	1	
23.10	1	.02	2	*Dy* .02 (2)	87.86	1	.90	1	*Y* .86 (1)
23.33	1				89.74	1			
23.82	7	.81	2		91.55	2	.58	4	? *Gd* .56 (2), *Dy* .57 (6)
25.27	3	.27	1	*Gd* .28 (1)					
25.79	3	.81	1		91.78	2			
26.02	1				93.23	3br	.29	1	
26.91	1				93.88	2	.85	1	
29.93	2				94.41	2	.42	1	
30.57	1	.57	1	? *Dy* .57 (1)	94.73	1	.75	1	
32.00	2				95.12	3			*Dy* .20 (5)
32.84	2				96.52	5	.52	1	
33.79	2				96.97	1			*Gd* .99 (1)
33.99	2				97.96	1			
36.51	3				98.21	3			
37.32	1				3600.19	2			
37.88	1			*Dy* .83 (2)	00.55	6	.50	4	*Dy* .56 (4)
38.03	3				00.94	1			
39.05	2				01.68	1	.65	1	
39.97	1				01.86	2			
40.41	5	.41	1		02.64	1			
43.38	3				05.04	4	.04	1	*Gd* .00 (4), *Dy* .04 (1)
44.03	4								
44.54	1			? *Dy* .54 (3)	06.14	2			
46.64	2				06.31	3	.26	5	*Dy* .28 (6)
51.14	1dp				07.66	1			
52.12	3	.17	1	*Dy* .19 (2)	08.00	1			*Gd* .01 (1)
55.42	3				08.35	1			
55.87	1	.83	1	?	09.20	1			
56.24	1				09.70	1dp			
56.39	1				10.00	1			
58.91	1br				11.42	3			
59.30	1br	.42	2	?	11.61	3			
59.54	2				13.17	3	.21	2	*Dy* .22 (3)
59.88	3				13.47	1			
61.88	6br	.83	3		13.81	3	.80	1	
63.05	3	.09	1		14.79	3	.77	1	*Dy* .86 (2)
65.87	3	.84	2		15.58	1			
66.25	1	.23	1	*Dy* .21 (1)	15.77	4br	.76	1	
67.50	5	.50	2		16.71	4	.70	2	
67.99	1				18.01	5	.98	1	*Dy* .95 (3)
68.64	8	.63	4		18.31	2	.25	2	*Dy* .23 (4)
69.11	5	.14	2		19.86	4	.85	1	

TABLE V—*Continued*

Eberhard		Exner and Haschek		Remarks	Eberhard		Exner and Haschek		Remarks
λ	I	λ	I		λ	I	λ	I	
3622.27	2	.25	1		3668.14	1	.11	1	
24.02	4				68.64	1	.66	1	*Dy* .69 (1)
24.89	3				69.79	1	.79	1	
25.66	6	.64	2		70.80	1			
26.27	1				71.52	2			
26.63	5	.61	1		72.51	2			*Dy* .45 (6)
27.01	2			*Gd* .02 (1)	75.91	2			
28.31	3br	.35	1		76.52	8	.48	3	*Dy* .58 (1)
28.87	1			*Y* .85 (4)	78.04	3			
29.59	4br			*Gd* .66 (2),	78.94	2			
				Dy .57 (5)	81.62	1			
30.40	3	.35	5	*Gd* .39 (1),	82.45	7	.45	2	
				Dy .39 (6)	83.44	1			
31.20	2			*Dy* .20 (1)	84.99	2	.00	1	
31.64	4				87.34	1			
33.43	6			*Dy* .41 (2)	88.32	5	.30	1	
35.58	3				89.28	4	.24	1	*Dy* .18 (1)
38.59	4	.58	1	*Dy* .64 (3)	89.88	2			
39.06	1	.03	1	*Dy* .05 (1)	91.32	7br	.32	2	
39.95	4	.99	1	*Dy* .00 (2)	92.15	1			
40.91	1	.95	1	*Dy* 97 (3)	93.00	4			
41.80	6	.77	1		93.74	2	.73	1	*Gd* .76 (1)
42.19	1				94.87	1			
42.50	1				96.45	3	.40	1	
42.82	3				97.03	4	.05	2	
43.43	2				97.91	1			? *Gd* .89 (5)
43.90	1				99.50	4			
44.28	2				3700.28	1			
45.96	3	.96	1	*Dy* .00 (5)	00.47	1			
46.28	1			*Gd* .36 (15)	01.47	1			
46.58	1				03.01	8	.99	4	
47.19	4				03.64	1			
47.87	4				04.05	8	.05	3	
49.51	1				05.25	2			
50.55	7	.51	3	*Dy* .57 (2br)	06.54	4	.53	1	
51.08	3	.10	1	*Gd* .09 (2)	08.03	3			
52.03	3	.00	1		09.51	4			
52.43	2	.41	1		11.92	8br	.90	2	*Dy* .81 (5)
53.11	3				14.56	1br			
54.02	2	.00	1	? *Dy* .02 (2)	16.24	2			
55.01	5	.99	2	*Dy* .02 (3)	16.58	2	.54	1	*Gd* .52 (5)
56.63	1			*Dy* .58 (1)	17.07	1	.05	2	*Dy* .10 (3)
56.90	1				17.64	3			*Gd* .60 (4)
58.42	2			*Gd* .35 (1)	18.06	1			
59.01	6dp	.00	3		18.65	2			
59.58	3				19.62	3			*Gd* .63 (10)
60.62	2				20.46	1			
60.89	3	.85	1		20.59	1			
63.27	5	.25	1		22.79	2	.79	1	
64.45	1				23.26	3			
65.75	1				25.11	2			

TABLE V—*Continued*

Eberhard		Exner and Haschek		Remarks
λ	I	λ	I	
3725.59	2br	.59	1	*Gd* .63 (3), *Dy* .62 (1)
28.83	2	.88	1	
29.12	3			
30.07	5	.05	1	
30.57	1			
32.00	1			
32.54	5	.53	1	
34.88	2			
35.17	2			?
38.57	1br			
39.06	2			
40.51	3			
41.35	4	.32	2	
41.76	3			
42.04	3	.02	1	*Gd* .05 (1), *Dy* .02 (2)
43.27	3	.21	1	*Dy* .24 (2)
45.20	3	.17	2	
46.72	3			
47.33	4	.33	1	
47.52	6	.48	1	
48.97	1			
49.47	1			
49.88	1			
51.81	1br			
52.37	1br			*Gd* .42 (1)
53.74	2			
55.38	6	.35	2	*Gd* .38 (1)
57.58	3			*Dy* .52 (6)
58.06	4			
58.46	1			? *Gd* .46 (5)
59.53	3br	.50	1	
60.34	1			
61.25	1			
62.88	1	.89	1	
65.26	6	.28	2	
67.65	3			*Dy* .74 (2)
75.41	2			
76.62	7			
77.63	2	.57	2	*Dy* .59 (3)
79.36	2			*Dy* .32 (2)
80.14	1			
81.80	1			
83.62	3			*Dy* .67 (5)
85.52	1	.54	3	? *Dy* .54 (3)
87.35	4			*Y* .31 (2)
89.20	1			
89.86	1			
90.06	2			
92.35	1			

Eberhard		Exner and Haschek		Remarks
λ	I	λ	I	
3793.69	3	.70	2	
94.50	1br	.51	1	
98.01	1			
98.66	3	.70	1	
99.09	1			
99.72	3			
3801.96	2			
02.33	2			
06.53	1			*Dy* .47 (4)
07.00	3	.00	2	
11.82	2			
12.84	2	.83	1	
13.28	1			*Dy* .27 (2)
16.33	1br	.31	3	*Dy* .35 (3)
20.26	1			
20.55	1			
20.93	2			
23.23	1			
24.51	2	.50	2	
42.61	4			
43.16	3			
44.39	1			
48.91	4			
52.02	1			
74.30	6	.25	10	*Dy* .34 (4)
85.27	1	.23	1	
86.18	1			
87.00	3	.96	1	
87.85	2			*Gd* .87 (1)
88.40	2	.37	1	
90.03	1			
93.52	3			
94.73	1dp			
96.16	3			
96.74	4	.78	1	
97.45	1			
98.03	3	.01	1	
99.34	8	.32	3	
99.72	3	.77	1	
3900.92	1			
01.47	3	.47	2	
01.79	1			
02.14	1br			
03.29	1			*Dy* .23 (1)
04.33	1	.35	2	*Dy* .38 (3)
04.72	1br			
05.72	1	.73	3	? *Si* .69 (10), *Dy* .69 (2)
06.71	1			
08.23	3	.17	1	
08.82	2			

TABLE V—*Continued*

Eberhard λ	Eberhard I	Exner and Haschek λ	Exner and Haschek I	Remarks	Eberhard λ	Eberhard I	Exner and Haschek λ	Exner and Haschek I	Remarks
3909.30	4	.28	1		3956.29	3			
09.69	4	.71	1		57.49	2			
10.31	2	.28	1	*Dy* .24 (1)	58.11	4	.11	2	*Dy* .19 (4)
10.56	1				58.49	6	.52	1	
10.78	1			*Dy* .70 (1)	58.76	1			
11.01	2				60.28	1br			*Gd* .27 (1)
12.42	1				60.84	2			
12.93	2dp			*Dy* .00 (2)	62.12	1			
13.62	2				62.76	1	.75	2	*Dy* .74 (2)
13.93	1			*Gd* .92 (1)	63.06	1			
14.72	2				65.25	2	.25	1	*Dy* .26 (1)
15.54	5	.57	1		66.09	4			
17.11	1				67.36	3			
17.45	2	.48	3	*Dy* .49 (2)	67.80	2			
18.97	2				70.35	4	.36	1	
19.67	7r	.64	1		71.90	3	.90	1	*Gd* .91 (4)
20.11	2				72.22	2	.24	1	
20.88	4				73.07	1			*Dy* .18 (1)
21.16	1				74.41	4	.47	1	
21.91	1				74.84	2	.86	2	
22.23	5	.25	1		76.99	10	.00	5	
22.88	5	.92	1		80.41	1			
23.13	1				81.29	5	.32	1	
24.53	2			*Dy* .60 (2)	82.02	10	.09	8	*Dy* .10 (7)
24.95	2				82.47	1	.47	1	
25.57	7	.57	2		84.00	3			
29.94	2dp			*Dy* .86 (1)	84.18	3			
30.95	3				84.96	1			
32.50	2br				85.22	2			
35.38	7r	.35	2		86.48	3	.49	1	
37.30	1	.30	1	*Dy* .32 (1)	87.84	1	.78	1	
37.77	2	.74	1		89.63	1			
39.08	8	.08	2		90.75	3			
41.32	3	.34	1		91.73	1			
41.53	1				92.33	1			
42.34	3	.34	1		93.00	1			
43.07	1				93.67	4			*Dy* .71 (2)
43.81	2				95.27	1			
44.36	1				95.93	1	.94	1	*Dy* .91 (1)
45.07	2br				97.52	3			
47.02	6	.05	3	*Dy* .09 (5)	98.24	1	.20	2	*Dy* .20 (1)
48.45	4	.46	1		98.53	3			
49.67	1				99.03	2	.05	1	
50.05	1				99.50	4	.55	1	
50.27	2				4000.13	4	.13	1	
50.55	3			*Y* .52 (10),	01.40	1	.45	1	*Gd* .40 (3)
				Dy .52 (5)	02.30	5br	.31	1	*Dy* .35 (1)
50.91	1				02.70	7	.72	2	
52.02	1				03.91	2	.92	1	
54.18	4	.17	1		04.03	1	.00	1	
55.89	1br	.83	1		04.64	2	.59	1	

TABLE V—*Continued*

Eberhard λ	Eberhard I	Exner and Haschek λ	Exner and Haschek I	Remarks
4005.62	8	.64	3	
06.07	3			*Dy* .01 (1)
07.32	1	.28	1	*Dy* .30 (1)
07.86	1	.89	1	*Dy* .91 (3)
09.34	2	.30	1	*Gd* .35 (2)
09.65	3			
10.18	2	.21	2	*Dy* .23 (3)
12.94	5	.92	2	*Dy* .97 (1)
13.36	3	.40	1	
15.66	1br			
16.09	1dp			
16.44	1br			
17.01	1	.00	1	*Dy* .06 (1)
17.99	1br			*Dy* .92 (2)
18.51	1dp			
19.28	5	.25	1	
20.60	5	.62	3	
21.25	2			
23.02	3	.03	1	
23.87	2	.85	2	*Dy* .89 (3)
24.21	3			
24.88	3			
25.84	4			*Dy* .78 (2)
27.54	1br			
28.13	1			
28.42	5	.45	3	*Dy* .51 (5)
30.11	1			
31.77	6	.78	2	
32.45	5	.48	2	
33.18	7	.20	3	
36.35	1			
36.57	1			*Dy* .51 (4)
38.08	2			*Dy* .01 (2)
39.35	2			
39.64	2	.63	1	*Gd* .63 (1)
40.26	1			
40.56	3			
42.00	1			*Dy* 12. (5)
42.47	2			
43.81	2br			*Gd* .85 (1)
47.32	3			
48.07	1	.96	1	
51.65	1			
51.95	4	.95	2	*Dy* .95 (2)
52.56	1			
52.99	6	.96	1	
54.22	3			
57.18	1			
58.59	2			
60.54	4	.53	1	
61.01	5	.00	1	

Eberhard λ	Eberhard I	Exner and Haschek λ	Exner and Haschek I	Remarks
4061.74	5	.70	2	*Dy* .69 (1)
62.36	1			
62.94	1			
64.02	4	.97	1	
66.33	5	.33	1	
70.25	3	.25	1	*Dy* .24 (1)
70.69	2			
71.36	2			
72.44	2			*Dy* .46 (1)
72.84	4			*Dy* .76 (1)
73.88	3	.86	1	*Gd* .99 (10)
74.12	1	.13	2	*Dy* .14 (2)
74.22	1dp			
75.32	3	.32	1	
75.99	1			
78.59	2			*Gd* .60 (6)
80.94	1			
81.38	4	.38	2	
82.37	2			
82.95	2			
83.35	2			*Dy* .26 (1)
83.81	2			*Gd* .87 (2), *Dy* .75 (1)
84.40	2	.40	1	*Dy* .43 (1)
84.99	2			
86.77	3	.75	1	
87.85	2			*Gd* .85 (3), *Dy* .94 (2)
89.48	3	.50	1	
89.66	2	.64	1	
91.49	1			
92.34	3	.34	1	
94.20	1			
94.58	6			
96.09	1			*Dy* .25 (2)
97.56	2dp			
99.32	1			
99.62	2			
4101.08	5br			
01.82	4			
02.67	3			
03.60	4			*Dy* .48 (9)
04.09	6			*Dy* .00 (4)
05.54	3	.50	1	
06.17	1	.12	1	?
07.90	1br			
09.70	1			
12.66	3	.68	1	
13.05	3			*Dy* .18 (1)
14.27	3	.21	2	*Dy* .23 (2)
15.49	2br	.46	1	*Gd* .54 (1)

TABLE V—*Continued*

Eberhard λ	Eberhard I	Exner and Haschek λ	Exner and Haschek I	Remarks
4116.68	1			
17.40	1			
20.09	3	.12	1	
20.69	2			
21.19	1			
22.38	1			
22.66	2			
23.93	1			*Dy* .88 (1)
26.90	1			
27.44	3			
30.32	1			
31.28	1			*Dy* .18 (3)
31.63	3	.65	1	*Gd* .65 (3)
33.01	2	.03	2	*Dy* .02 (2)
35.55	3	.53	1	*Gd* .60 (1)
38.79	1			*Dy* .70 (1)
39.22	2			
39.94	3			
40.93	1	.95	1	
41.71	4	.67	4	*Dy* .67 (6)
42.61	1			
43.76	3			
44.55	8	.58	3	
47.13	1			
48.34	1			
49.30	3	.30	1	
50.71	2			
51.28	1	.26	3	
53.65	1			*Gd* .69 (2)
56.44	3			
58.46	1			
58.70	2			
61.64	1br			
66.69	1			
69.27	3			
69.50	1	.45	1	*Dy* .40 (2)
70.07	1			
70.65	2	.71	1	
71.21	3			
71.94	5			*Y* .01 (2), *Gd* .86 (2)
72.77	3			
73.00	1			
73.64	4			
76.04	1			
79.14	2			
79.97	1			
80.54	4			
81.03	3	.07	1	*Gd* .05 (2)
81.50	3			
84.49	2			*Gd* .48 (10)

Eberhard λ	Eberhard I	Exner and Haschek λ	Exner and Haschek I	Remarks
4185.10	2			
86.38	2			
87.33	3			
88.27	2			*Gd* .20 (2)
88.68	3			
90.28	1			
91.80	3	.83	5	*Gd* .75 (2), *Dy* .81 (3)
93.29	1			*Gd* .29 (1)
93.55	1			
93.94	1			
94.14	1			
96.91	3			
97.27	1			*Gd* .21 (3)
98.61	2			
99.87	1			
4200.83	2			
01.18	6	.15	1	
03.91	4	.88	1	
06.05	1			*Dy* .70 (6)
07.06	1			
07.67	2			
08.83	3			
13.62	4			*Dy* .76 (2)
14.56	4	.55	1	
15.25	4	.32	5	? *Gd* .13 (6) *Dy* .30 (5)
16.11	1	.06	1	
16.83	1			
17.66	3			
18.06	1			
19.01	1			
19.32	3			
20.24	3			
22.85	1			
23.46	3			
24.43	3			*Y* .43 (1)
26.63	9br	.58	1	
30.74	2			
31.54	2			
32.05	3			*Dy* .18 (5)
32.36	3			
32.98	4			
34.37	1			*Dy* .33 (1)
34.80	1			
35.52	2	.54	1	
39.44	3			
40.11	1			*Dy* .01 (5)
40.33	1br			
42.43	1			

TABLE V—*Continued*

Eberhard λ	Eberhard I	Exner and Haschek λ	Exner and Haschek I	Remarks
4242.73	3			
44.73	1			
45.36	1			
45.95	1			*Dy* .07 (4)
46.77	2			*Gd* .70 (3)
48.73	1			*Dy* .65 (2)
50.44	2	.50	1	*Dy* .55 (1)
51.50	2			
51.88	6	.85	1	*Gd* .90 (10), *Dy* .94 (1)
54.17	1			
55.39	7	.36	1	
56.30	1			
58.37	8	32	2	
60.94	1			
63.85	1			*Dy* .75 (1)
64.82	1			
65.15	1			
66.53	5	.49	1	
69.49	1			
69.87	6	.87	1	
71.73	1			
72.44	2			
73.38	2			*Dy* .33 (2)
75.39	4	.36	1	
76.91	8br	.86	3	*Dy* .88 (4)
77.93	3			
78.71	10	.67	4	
80.67	1			? *Gd* .69 (8)
81.48	1			
85.30	5	.30	1	
85.92	2	.98	1	*Gd* .96 (2)
86.29	1			*Gd* .26 (2)
87.06	3	.05	1	
89.89	4	.87	1	
94.53	1			
95.50	1			
96.49	3			*Gd* .46 (3)
98.55	4			*Gd* .60 (1)
4300.08	4	.05	1	
01.11	1			?
03.12	4			*Dy* .17 (3)
04.17	2			
07.38	3	.35	1	
08.85	7	.80	10	*Dy* .82 (7)
10.61	4	.56	1	
11.17	1	.16	1	*Gd* .14 (2), *Dy* 18 (1)
11.74	2	.72	1	
12.26	3			*Dy* .15 (1)
13.40	2	.40	1	

Eberhard λ	Eberhard I	Exner and Haschek λ	Exner and Haschek I	Remarks
4315.87	2			
19.05	9	.01	3	
20.45	1			
22.39	5	.40	2	*Gd* .35 (2), *Dy* .37 (1)
23.04	3			
23.83	3			*Dy* .95 (2)
26.00	9	.04	2	
26.64	6	.62	3	*Dy* .58 (2)
29.10	5	.08	2	*Gd* .06 (1), *Dy* .08 (4)
30.53	1			
32.30	5	.30	1	
34.84	2			
36.66	7dp	.64	2	
37.79	5	.82	1	
38.62	6	.60	3	
40.79	6	.79	2	
42.70	9br	.73	1	
48.49	2	.51	1	*Dy* .52 (1)
49.78	2			
50.92	2	.93	2	
51.80	1dp			
53.39	9	.35	1	
56.28	4	.25	1	
57.01	7	.99	2	
60.31	5	.35	1	*Dy* .36 (2)
66.18	1			*Y* .20 (1)
67.67	5			
72.22	2	.23	1	
76.59	3			
81.47	2			
82.60	5	.61	1	
84.24	1			
85.85	4			
86.24	5	.21	1	
88.42	4	.43	1	
89.20	1			
91.10	5			*Gd* .12 (3)
94.20	2			*Y* .23 (1)
96.75	5			
4400.97	1			*Gd* .93 (1)
01.72	3			
03.36	5			*Gd* .30 (4)
05.58	5			
09.68	4br			*Dy* .55 (8)
16.43	5			
20.38	2			
23.28	5	.27	1	
24.64	1			
27.85	2			

TABLE V—*Continued*

Eberhard		Exner and Haschek		Remarks	Eberhard		Exner and Haschek		Remarks
λ	I	λ	I		λ	I	λ	I	
4428.50	1				4503.75	1			*Dy* .79 (1)
30.27	1br				04.72	1			
30.88	1			*Gd* .82 (5)	09.20	5			
32.88	3				11.68	5			
34.63	5				13.12	3			
35.17	2			? *Ca*	14.47	3			
35.73	3				16.07	1			
36.27	5	.24	1		19.92	1dp			
39.12	4				24.52	1			
39.53	2				25.14	1			
41.42	1			*Dy* .35 (1)	26.08	1			
41.67	1				29.03	1			
48.20	3				29.91	2			
51.79	2				30.75	1			
52.96	2br				31.98	1			
53.31	1				34.33	2			
55.07	1			? *Ca*	37.10	4			*Gd* .11 (1)
58.65	1				37.38	3			
59.57	3				40.74	1			
61.43	1				46.63	1			
62.38	1br				47.64	1			
65.88	1				48.52	1			
67.87	2				49.26	3			
69.29	1				49.90	3			
69.56	1				50.61	3			
69.90	1br				51.10	1	.06	1	*Gd* .15 (1)
71.34	1	.31	1		56.65	3	.63	1	*Dy* .66 (2)
71.89	2				57.07	1			
73.00	1				57.45	1			
73.46	1			*Gd* .45 (1)	62.41	4			
4473.87	1				63.87	6			
85.85	1				65.04	1			
88.33	2				73.54	2			
89.93	2				78.88	6	.88	1	
90.84	1				80.55	2			
91.19	1				85.01	2			
93.25	7	.26	1		91.73	2			
4501.47	1			*Dy* .44 (1)					

By the settlement of the terbium question an important step has been made in the investigation of the yttrium earths, and it will now be possible for the first time to successfully undertake the discussion of the further earths of this group, dysprosium and neoholmium. I hope before long to be able to give complete tables of wave-lengths for the first of these elements, based again upon preparations from Dr. Urbain.

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ON THE RELATION BETWEEN STELLAR SPECTRAL TYPES AND THE INTENSITIES OF CERTAIN LINES IN THE SPECTRA

By SEBASTIAN ALBRECHT

During the past summer, in connection with the measurement of the spectrograms obtained at the Mills Observatory in Chile by Professor Wright, assisted by Dr. Palmer, an investigation of the individual spectrum lines was begun, with a view of determining whether there is a shift of any of the lines which is progressive from spectral type to type. Several lines were found which undergo such a progressive change as indicated by the radial velocities obtained from them, this being especially noticeable for the line λ 4468.663.[1] Since then, several lines which were formerly not measured have been included in the observing list, largely for the purpose of studying their behavior in the different types of spectra, rather than for use in the radial velocity determinations. An examination of Rowland's tables shows that in most, but not all, cases studied, lines apparently single are in reality blends of two or more close components. The nature of the variations found is such as to indicate varying intensities of the same components, rather than the presence or absence of different components in the different types. It was the intention, when sufficient data had been obtained, to make comparisons with the enhanced and weakened lines in the spark and arc spectra. When a list of sun-spot lines[2] in the region covered by the southern Mills plates became available, a comparison with these was made instead, and the preliminary results are given in this paper. The investigation was limited to stars of types F to Mb inclusive on the Harvard classification. In this classification the Sun is of type G.

The principal result of the comparison is the very strong indication that the physical conditions in the stars as we pass from the F to the Mb type vary roughly in the same direction as from the Sun

[1] In preparing this manuscript I note that Slipher has called attention to the fact that this line gives too large positive velocities in *α Boötis*.

[2] Walter S. Adams, "Sun-spot Lines in the Spectrum of *Arcturus*," *Astrophysical Journal*, **24**, 69, 1906.

to the sun-spots. It is not intended to convey the impression, however, that any one type has been found in which the conditions are exactly the same as in the sun-spots, though an early K type is probably nearest to it. In the Mb type the relative intensities of the lines, as shown both by their appearance and their residuals, have gone far beyond what they are in the Sun, whereas in the F type they much precede the condition in the solar spectrum. Adams (*loc. cit.*) has shown a striking similarity to exist in the intensities of sun-spot lines and of the corresponding lines in the spectrum of *Arcturus* (type K), while Hale and Adams[1] have made a similar comparison of intensities for *a Orionis* (type Ma) in the region λ 5393 to λ 5703.

A large number of lines might be mentioned which change greatly in intensity and appearance as we proceed from the F to the Mb type, this change being frequently very prominent even from the G to the K type, and which are not included in Adams' list of lines affected in sun-spots. Among the most striking of these are the *Cr* lines, λ 4254.5 and λ 4274.9, which become very strong, wide, and diffuse as we follow the scale of stellar types. A few cases of contradictory evidence have also been found, in which the residuals show a definite and decided shift in the opposite direction from that which would be expected from the intensities assigned to the components in sun-spots. Among these the line λ 4435.2 may be especially mentioned. For this blend both my value of the wavelength, as determined from the spectrograms, and the progressive trend of the residuals indicate a shift of its center toward the violet instead of toward the red. In one or two cases also the residuals tend to show an oscillation of the line, in one direction from the F to the G (or early K) type, and in the opposite direction from the G to the Mb type. Some of the observed differences between sun-spots and stellar K types, in which the physical conditions may be similar, may be due to the fact that in the stars this condition is the average condition in their entire atmospheres, while in the case of the sun-spots some effects may be altered by overlying layers of gases and vapors or by other local circumstances. Nevertheless the similarities are sufficiently striking to promise much for this line of study. The results here given depend not only upon the appearances

[1] *Astrophysical Journal*, 23, 400, 1906.

of the lines, but primarily upon quantitative measurements of their positions.

It is possible that for variable stars of large light-changes, some traces of velocity variations may be found, corresponding to small changes in spectral types from maximum to minimum. Spectrograms for a few cases are available and they will be examined.

Dr. Palmer made a simple tabulation of the residual velocities, without regard to spectral type, for a large number of lines in the spectra measured by himself, for the purpose of improving the wave-lengths of the lines used. The values in the old tables had been obtained by blending the components according to the intensities in the Sun assigned to them by Rowland. The wave-lengths credited to him in this paper are the average wave-lengths, deduced by him, for these lines in all the stars measured. In the case of three lines he found two values each. For one of the three lines, one value was for the types G, K, M, and the other for types A and F, while for the other two lines one value related to the types F, G, and K, and the other to the types K5M and Mb.

Acknowledgments are due Director Campbell for encouragement and assistance in the prosecution of the work.

In the tables following the residuals are given in kilometers. The capital letter at the head of each column denotes the stellar type. The intensities for the Sun are from Rowland's tables, while those for sun-spots and for *Arcturus* are from Adams' article (*loc. cit.*), where n.c. = not affected.

	Element	Intensities In Sun	In Sun-spots
λ 4258.219		1 N	
.477	*Fe*	2	3-4

In the F type this blend is strong, of nearly average width, and has sharp edges. In the G type it is broad and somewhat fuzzy, the widening having occurred toward the longer wave-lengths. In the Mb type the edges are usually sharp.

My residuals, based on Palmer's mean value λ 4258.43 are:

For F type, mean of 4 plates	= -3.6 km		
" F5G Pec., " " 3 "	= -5.7	Broad and fuzzy lines.	
" K type, " " 1 "	= $+4.1$		
" Mb " " " 4 "	= $+5.0$		

These figures will no doubt be changed somewhat by more extensive measures.

The shifting of the center of the blend indicates an increase in intensity and probably also a widening of λ 4258.477 as we go from the F to the M types, which is in the same direction as from the Sun to the sun-spots.

λ 4283.169 *Ca* Intensity in Sun, 4 In sun-spots, 5

The preliminary residuals seem to indicate a widening toward the violet in the G types, and toward the red in K5M and Mb types, but this cannot be considered as established until confirmed by a longer series of observations.

Residuals (in km)

F	F8G	G	G5K	K	K2M	K5M	Mb
+1.2	−2.3	−3.1	−1.7	+0.5	+1.5	+3.4	+4.3
+0.9	−0.8	−1.7		+1.6	−1.7	+3.0	+1.2
+1.9	−0.3	−2.0		−1.8		+5.3	
−0.2		−3.2					
+3.1							
+3.3							

λ 4299.989 Intensity in Sun, 1N
4300.211 *Ti* 3 In sun-spots, 2–3

In the F type this line is very strong and sharp. It is much weaker in the G type, and still weaker in the Mb type. The F, G and K types give small positive residuals, while two plates of type K5M give small negative residuals. This shift, if confirmed by a longer series, would indicate in the M types a weakening of the *Ti* line sufficiently to make λ 4299.989 1N effective.

λ 4337.72 *Cr* Intensity in Sun, 3 In sun-spots, 4
38.08 *Ti* Intensity in Sun, 4 In sun-spots, 3
.43 *Fe* Intensity in Sun, 1 In sun-spots, 1–2

In the F type this line is sharp, strong, and of slightly greater than average width, and the residuals show it to have practically the wave-length of the *Ti* component. In the G type it is broad and fuzzy, and occasionally the *Cr* and *Fe* lines are separated by a darker region. In the M types the *Ti* line is very much weakened, in fact almost absent, making the *Cr* line look sharp and narrow.

λ 4339.617 *Cr* Intensity in Sun, 4 In sun-spots, 5
.822 *Cr* Intensity in Sun, 3

This blend is very faint in the F type. In the G type it is fairly strong. In the M type it is very strong and sharp, and the components show a decided tendency toward resolution. Both components seem to increase in intensity, and the blend gives good velocities for types from G to M, there being no shift either way, as is shown by the following residuals:

G	G5K	K	K2M and K5M	Mb
+1.8	±0.0	−2.5	+0.4	−0.5 km
	−1.2	+0.3	−1.4	+0.7
	+1.2	+0.9	−1.1	−0.1
			+2.1	+0.8

		Intensity in Sun	In Sun-spots	
λ 4340.634	*Hγ*	20N	15	weakened on red edge mainly.

This line is stronger in the F type than in the G type. In the M type it is very weak, probably less than one-tenth of its intensity in the F type. There is a tendency to give positive residuals in the F and G types, and negative residuals in the M types, the residuals in each case being small, however.

A5F	F	F8G Pec	F8G	G	G5K	K	K5M	Mb
+3.9	+1.8	+5.8	+2.4	+1.5	+1.2	−0.8	−1.0	−1.3km
	+0.8		−0.8	+2.7	+2.3	+1.1	−3.3	−0 4
	+1.9		+3.9	+1.6	+2.2	+1.3	−0.1 (K2*M*)	−1.3
	+3.9			+2.0	+0.8	+1.5	−2.9	
	+1.6			+1.9	+0.8	+0.1	−0.8	
	+2.2			+1.8		+1.4	−1.5	
	+1.5					−2.2		
						−2.0		
						+0.6		
						+1.2		
						−2.1		
						+2.9		
						+1.0		
						+0.5		
						+1.6		
						+1.4		
						−0.6		

		In Sun	In Sun-spots	In *Arcturus*	
λ 4352.908	*Fe*	4	6-7	7-8	widened toward red.
53.044	*V*	0			

In the F type this line is faint and narrow. In the G type it is more than twice as strong, and it continues to increase in strength

through the K to the M types, where it is more than four times as strong as in the F type. The consistent trend and small range of the residuals indicate that this line can safely be used, if allowance is made for the type. The residuals show that the widening toward the red becomes quite marked in the K type, and increases through the K5M to the Mb type. There is little doubt that this blend will also give good results in the F type.

F 8 G and F 8 G Pec	G	G 5 K	K	K 2 M and K 5 M	Mb
−3.7	+0.8	−0.7	+1.6	+3.6	+3.4 km
+1.5	+1.0	−0.6	+3.1	+3.0	+4.9
−1.0	+1.7	+2.2	+0.8	+3.0	+4.6
+1.5	−1.1	+2.8	+1.4	+1.0	+4.7
−3.5		±0.0	+2.6	+3.9	+3.9
		+1.7	+3.1	+2.2	+6.5
			+1.5	+4.4	
			+2.2	+3.3	
			+0.8		
			+2.0		
			+0.9		
			+2.1		
			+3.4		
			+2.6		
			+1.4		
			+4.8		
			+2.2		
			−0.2		
			+0.8		
			+4.3		
			+1.9		
			+3.2		
			+3.6		
Means −1.0	+0.8		+2.2	+3.0	+4.7

λ 4355.257 *Ca?* Intensity in Sun, 2 In sun-spots, 2–3, In *Arcturus*, 3
.509 Intensity in Sun, 0

In the F type this line is vv faint. In the G and K types it is usually good, being stronger in the K type, however, than in the G type. In the M type it is broader and generally fuzzy.

λ 4371.14	*Zr*	Intensity in Sun, 1	In sun-spots, n.c.	In *Arcturus*, 6
.22		Intensity in Sun, 1	In sun-spots, n.c.	
.32		Intensity in Sun, 0	In sun-spots, n.c.	
.44	*Cr*	Intensity in Sun, 2	In sun-spots, 3–4	

This blend is frequently resolved into two components, the one toward the red being in all cases brighter than the one toward the

violet. They are faint in the F type, both components are stronger in the G type, and in going toward the M type the component toward the red alone seems to increase in strength. My residuals, which are principally for types K and K5M, give for the wave-length of this blend the value λ 4371.368, which is slightly greater than would be obtained from blending the component according to their intensities in the sun-spots (λ 4371.356).

λ 4387.007 *Ti?* Intensity in Sun, 1 In sun-spots, 0

This line is best in the F type, but it has a slightly fuzzy appearance. It diminishes in brightness in the G and K types, and in the M types it is nearly gone. The few residuals that I have are all plus, but small, which tends to show that the solar companion at λ 4387.220, 1N, has but a slight effect in determining the position of the center of the blend.

λ 4390.149 *V* Intensity in Sun, 2 In sun-spots, 3–4 In *Arcturus*, 3–4

This line is very faint in the F type and increases in strength continuously through the G and K types to the M type. For this

G	G5K	K	K5M	Mb
−0.8 (Sky)	−1.5	−2.7	+1.1	+1.8
−5.1	−2.9	−1.9	−0.9	+4.0
−3.7		−1.9	−0.5	+0.2 (Ma)
−3.4		−4.0	−3.2	+2.6
		−4.5	+0.4	+1.0
		−3.1	+1.0	±0.0
		−3.1	+0.4	−0.6
		+1.5	+0.2	+1.6
		−2.2	−0.7	+0.9
		−3.5	−0.3	
		−6.9	−0.1 (K2M)	
		−0.9	−0.4	
		+0.9	−2.0 (K2M)	
		−3.0		
		−1.9		
		−4.1		
		−2.1		
		−4.4		
		−2.8		
		−3.0		
		−0.7		
		−2.3		
Means −3.2		−2.7	−0.4	+1.3

line Dr. Palmer found the values λ 4390.118 for types F, G, and K, and 4390.178 for types K5M and M. My residuals, however, show

that the variation of the wave-length is progressive, the residuals changing from fairly large negative values in the G type to nearly zero values in the K5M type and positive residuals in the Mb type. The line is widened gradually toward the longer wave-lengths as we proceed from the G to the Mb types.

λ 4395.201 *Ti* Intensity in Sun, 3 In sun-spots, 3–4 } In *Arcturus*, 7
.413 *V*, *Zr* 2 3

This line increases continuously in width and intensity from the F to the Mb type. It is fairly sharp in all of these types, even in the Mb type where it is very broad and strong. Both components increase in intensity, but the residuals show a gradual and continuous shift of the center of the blend toward the red. It is an easy line to set on with the micrometer thread, and the residuals indicate that it gives excellent results.

F	F8G	G	G5K	K	K5M	Mb
−3.0	−2.2	−0.8	+0.1	±0.0	+1.7	+2.4 km
−2.7	−4.1	+0.2	−1.8	−0.5	+0.9	+2.7
−0.5			+0.3	+0.8		+2.9
−1.8				−0.3		
				+0.1		
				−1.2		
				+0.3		
				+0.1		
				+0.3		
				+0.6		
Means −2.2	−3.2	−0.4		±0.0	+1.3	+2.7

λ 4400.555 *Sc* Intensity in Sun, 3
.738 *V* 1 In sun-spots, 2–3 In *Arcturus*, 3

My value for the wave-length of the blend as obtained for types K and K2M is somewhat greater than the value obtained by blending the components with the intensities 3 and 3. The residuals show the gradual shifting of the center of the blend toward the red as we go from the F to the Mb type. In the F type the line is narrow and very good, the vanadium component being absent. In the G type the vanadium component becomes effective. In the Mb type it has the same intensity as the *Sc* component, and the resulting blend is strong and broad.

λ 4406.810 *V* Intensity in Sun, 2 In sun-spots, 4 In *Arcturus*, 5

This line is good in types G to Mb, and increases in strength in

the same direction. In the F and F8G types it is scarcely visible. The preliminary residuals indicate no shift of its center.

λ 4407.810	*V*	Intensity in Sun, 2	In sun-spots, 7–8,	In *Arcturus*, 7
.871	*Fe*	4		

This line is good in types F to Mb, and it is not much wider in the latter than in the former. The residuals seem to indicate a slight shift toward the violet, but this is still uncertain. A large shift cannot be expected from such close components, and it will require a longer series of measures to show which component is increased in intensity, if they are not both increased.

λ 4408.364	*V*	Intensity in Sun, 2	In sun-spots, 3	In *Arcturus*, 3
.582	*Fe*	3	6	6
.683	*V*	2		

This line is good in types F to Mb. It increases considerably in intensity and in width, and occasionally it is somewhat fuzzy in the Mb type. The residuals indicate that it widens symmetrically.

λ 4416.636	*V*	Intensity in Sun, 0	In sun-spots, 2–3	In *Arcturus*, 3
.985		2	1–2	1

In the F types the vanadium component is absent or at least not effective, and the line λ 4416.985 is very good. In the types G and K the vanadium companion begins to appear, and the blend is usually fuzzy. In the M types the vanadium component is strong, considerably stronger than the other component, which is also shown by the large negative residuals in these types. The value of the wave-length for the Mb type (λ 4416.746) as obtained from the residuals is in fair agreement with the value of the blend according to intensities in the sun-spots (λ 4416.723).

F	F8G	K	Mb
+1.1	+1.8	+1.3	−16.5
+0.5			−15.9
−1.0			−15.9
+1.8			−16.6
			−16.8
			−16.1

λ 4417.450	*Ti*	Intensity in Sun, 0	In sun-spots, 2	In *Arcturus*, 3
.577	*Co*	∞		
.740		∞		
.844	*Ti*	3		

In the F type the line is narrow and good, the *Ti* component λ 4417.844 alone being effective, as is shown by the residuals. In the G type the line is still fair, but in the K and M types the companions at λλ 4417.450 and .577 have become of about the same intensity as the line .844. In the K and M types the line is usually very fuzzy, and on good plates the two components are resolved.

λ 4425.608	*Ca*	Intensity in Sun, 4		
26.201	*Ti*	o Nd?	In sun-spots, 2	In *Arcturus* 2

The *Ti* component is absent in the F type. In the G type it shows but is quite faint. In the K type it is as strong as the *Ca* line, while in the Mb type it is about twice as strong as the *Ca* line, i. e., intensity 8. The two lines are usually completely resolved, but occasionally they are somewhat run together.

λ 4427.266	*Ti*	Intensity in Sun, 2	In sun-spots, 3	In *Arcturus*, 8
.482	*Fe*	5	n.c.	

This line broadens somewhat from the F to the Mb types, and increases considerably in strength. It is usually very good in types F8G to Mb, and the residuals show that it gives excellent results. A larger number of residuals will be required to establish definitely the apparent slight shift of the center of the blend toward the violet in the Mb type.

λ 4428.711 *V-Cr* Intensity in Sun, 1 d? In sun-spots 2 In *Arcturus*, 2

In the F type this line is absent. In the G type it is very faint but usually measurable. It increases in strength from the K to the M types and is a very good line. It is apparently shifted toward the violet in the M types, and this is also borne out by the trend of the few residuals that I have, and by my value of the wave-length (λ 4428.684) which was found for types K to M. If it is double, as indicated by Rowland, it is probably the component toward the violet that is intensified.

G 5 K	K	K 2 M	K 5 M	Mb
+3.6	+2.5	−0.8	−3.3	−2.5 km −2.6

λ 4433.948	*Fe*	Intensity in Sun, 1	In sun-spots, n.c.	In *Arcturus*, 2
34.168	*Ti*	o Nd?	1	

This line is very faint and fuzzy in the F type. In the types G to Mb it is fair. The wave-length found for it from the plates for types G5K to Mb is λ 4434.066, which is in good agreement with the value λ 4434.058 obtained by blending the components with the intensities given for them in the sun-spots.

λ 4435.129	*Ca*	Intensity in Sun, 5	In sun-spots, 6	In *Arcturus*, 8
.321	*Fe*	2	n.c.	

This line is good, especially in types F and G. It increases in strength continuously from the F to the Mb types. My value of the wave-length, 4435.201, is in good agreement with the value 4435.208 that would be obtained by blending with intensities 5 and 3 respectively. This value and the trend of the residuals indicate a shifting of the center of the blend toward the red as we go from the F to the Mb type. This is in the opposite direction from that indicated by the intensities of the components as given by Adams for the sun-spot spectra.

F	F 8 G	G	G 5 K	K	K 5 M	Mb
−3.4	−0.2	−1.0	+0.8	+1.9	+1.8 (K2M)	+1.3 km
	−3.2	−2.5		−0.4	+3.3	+3.7
				−1.4		+1.0
				+0.2		+1.2
				−0.6		
				+0.6		
				+1.1		

λ 4441.881 *V* Intensity in Sun, 3 Nd? In sun-spots, 5 In *Arcturus*, 4–5

This line is good in types F to Mb, increasing in strength in the same direction. The residuals do not show any decided evidence of shift from type to type.

λ 4443.976 *Ti* Intensity in Sun, 5 In sun-spots, 4–5 In *Arcturus*, 4–5

This line is strong and sharp in the F type, being several times as strong as in the G type. It continues to diminish in intensity from the G to the Mb type, where it is quite faint and fuzzy. It is not good for measurement in the Mb type with the dispersion of the southern Mills spectrograph.

λ 4468.663 *Ti* Intensity in Sun, 5 In sun-spots, 4–5 In *Arcturus*, n.c.

This is a good line in all types from F to Mb. It is quite strong in the F type. It seems to broaden somewhat toward the red in the

M type. The progressive trend of the residuals from negative values in the F type to approximately zero values in the G5K type and large positive residuals in the Mb type is very decided.

	F	F8G	G	G5K	K	K5M	Mb
	−1.3	−2.2	−0.3	+1.4	+2.8	+5.7	+6.8 km
	+1.7	−0.3	−1.5	+0.9	+2.6	+0.8	+7.0
	−4.1	−2.9	−0.9	−0.7	+3.9	+6.3	+3.5
	−3.0	−0.9		−0.9	+1.1	+2.5	+6.1
	−3.7	+1.4		+2.7	+1.0	+4.7	+4.8
				−2.1	+0.6	+4.7	+1.9
					+2.2	+4.7	+1.7
					+0.4	+5.2	+5.9
					−0.4		
					+1.2		
					+1.3		
					+2.7		
					+3.7		
					+4.3		
					+1.5		
					−0.3		
					+1.9		
					+0.6		
					+0.4		
					−1.5		
					−3.0		
					+3.3		
					−0.1		
					+0.7		
Means	−2.1	−1.0	−0.9	+0.2	+1.3	+4.3	+4.7

MOUNT HAMILTON,
October 19, 1906.

ORBIT OF THE SPECTROSCOPIC BINARY λ *ANDROMEDAE*

By KEIVIN BURNS

λ *Andromedae* (1900, $\alpha = 23^h\ 32^m.6$, $\delta = +45°\ 56'$) is a K-type star of photographic magnitude 5.0. The lines are very sharp, especially in the region of $H\gamma$. The binary character of this star was announced by Campbell[1] in 1899, and plates taken later in the same year showed the period to be $20^d.5$. The first eighteen plates were taken with the original Mills spectrograph,[2] $H\gamma$ central, and the others with the remounted Mills spectrograph (λ 4500 central; collimator, $f = 724$ mm (28.5 inches); camera $f = 521$ mm (20.5 inches); titanium comparisons). The star is relatively fainter in the region of λ 4500, $1^h\ 30^m$ exposure being required to give the same density as was obtained in $1^h\ 05^m$ with $H\gamma$ central. However, a part of the increase is due to the greater focal length of the second camera. Definitive measurements seemed to show a discrepancy between the series taken with the first Mills spectrograph and the plates taken with the second Mills, and a complete series was secured with the new instrument in order to determine the nature of the disagreement. Following is a list of the plates of this star, and of check-plates measured in connection with the two series.

No.	Plate	Gr. M. T.			Velocity
1	546 B	1897	November	16.712	+11.05 km
2	1015 B	1898	October	18.719	1.04
3	1041 D		October	26.828	8.50
4	1322 D	1899	July	5.935	12.28
5	1327 A		July	11.956	2.19
6	1331 D		July	12.929	1.65
7	1338 D		July	16.948	1.47
8	1349 D		July	24.995	13.02
9	1369 C		August	2.897	0.38
10	1378 D		August	8.976	4.67
11	1459 C		September	11.778	2.41
12	1486 B		September	21.744	12.12
13	1511 C		October	16.776	12.03
14	1518 B		October	17.769	9.68
15	1529 A		October	24.705	0.51
16	1539 A		November	1.680	10.53
17	1546 B		November	6.734	9.30

[1] *Astrophysical Journal*, 10, 178, 1899. [2] *Ibid.*, 8, 123, 1898.

No.	Plate	Gr. M. T.			Velocity
18	2544 A	1902	September	14.946	12.26
19	3366 F	1904	July	26.928	12.88
20	3398 B		August	16.791	13.70
21	3407 E		August	21.970	4.67
22	3425 A		September	4.786	13.02
23	3455 A		September	18.975	5.66
24	3458 B		September	19.906	8.78
25	3502 C		October	24.829	0.91
26	3527 A		November	7.633	11.15
27	3564 D		November	22.735	12.83
28	3603 A	1905	February	27.640	0.48
29	3820 A		June	7.985	2.29
30	3833 E		June	13.910	7.91
31	3837 E		June	14.928	10.61
32	3841 E		June	18.949	15.84
33	3844 D		June	19.935	14.40
34	3848 D		June	20.933	12.09
35	3853 E		June	21.963	8.97
36	3855 D		June	25.974	3.09
37	3858 D		June	27.981	1.10
38	3862 A		June	28.939	0.34
39	3865 D		July	1.985	2.84
40	3868 D		July	2.922	4.24
41	3895 B		July	7.949	13.37
42	3900 B		July	13.979	7.33
43	3914 B		July	19.918	1.34
44	3923 B		July	26.920	11.93
45	3928 B		August	1.910	10.06
46	3931 F		August	3.008	8.26
47	3935 A		August	3.993	7.36
48	3938 F		August	7.012	3.42
49	3948 D		August	14.007	6.83
50	3950 E		August	14.935	9.80
51	3980 E		September	3.912	8.58
52	3991 B		September	7.917	15.37
53	4001 A		September	14.879	5.44
54	4004 A		October	27.801	3.52
55	4321 D	1906	July	18.953	5.71
56	4345 E		July	30.883	+10.47

Venus

Plate	Date		Obs. Vel.	Comp. Vel.
1723A	1900	May 7	−13.37 km	−13.28 km
1995D		December 24	+ 8.23	+ 8.79
1996B		December 24	+ 8.7	+ 8.79
2288A	1901	October 21	−12.52	−12.13
2289B		October 21	−11.52	−12.13
[1]3573A	1904	November 28	−10.48	−10.45
[1]3574B		November 28	−10.52	−10.45
[1]3750A	1905	March 8	−12.77	−12.75
[1]3871E		July 2	+13.67	+13.47
			+13.59	
[1]3906E		July 17	+13.15	+13.30

[1] Second Mills Spectrograph.

Mars

Plate	Date	Obs. Vel.	Comp. Vel.
867D	1898 August 8	− 7.12	− 7.17
1006A	October 11	−10.29	−10.48
1976C	1900 December 4	−13.40	−13.84

An uncompleted investigation shows that an underexposed plate of a star as measured by the writer gives a positive residual, and Plates 21, 27, 47, being underexposed, were omitted from the discussion. Plate 29 was omitted on account of an irregular temperature-change.

Using Plates 2–17, inclusive, the following preliminary elements were obtained by the method of Lehmann-Filhés:

$$U = 20^{d}5464$$
$$\mu = 17^{\circ}5213$$
$$T = \text{J. D. } 2414572.12$$
$$\omega = 348^{\circ}07$$
$$K = 6.85 \text{ km}$$
$$e = 0.099$$
$$V = +6.50 \text{ km}$$

In the equations of condition

$$x = \delta V$$
$$y = \delta K$$
$$z = [2469.]\ \delta\mu$$
$$u = [7.036]\ \delta e$$
$$v = [2.543]\ \delta T$$
$$w = [6.902]\ \delta\omega$$
$$n = [0.5]\ n.$$

The resulting normal equations are:

[aa]	[ab]	[ac]	[ad]	[ae]	[af]	[an]
$+16.000x$	$+0.093y$	$+1.728z$	$+1.109u$	$+1.343v$	$+1.131w$	$+1.330=0$
	$+8.968$	$+0.548$	-3.942	-0.337	-0.323	$+2.344$
		$+6.044$	-0.042	$+5.011$	$+5.806$	-0.519
			$+6.356$	$+1.024$	$+1.425$	-2.002
				$+5.072$	$+5.673$	-0.957
					$+6.748$	-1.366

The values of the unknowns are:

$$\delta V = -0.165 \qquad \delta e = +0.0333$$
$$\delta K = -0.368 \qquad \delta T = -0.313$$
$$\delta\mu = +0.00743 \qquad \delta\omega = -11^{\circ}86.$$

The preliminary elements found by using Plates 19–54 (new Mills) are:

$U=20^{d}541$	$K=$ 7.58 km
$\mu=17^{\circ}.5259$	$e=$ 0.13
$T=$ J. D. 2416682.97	$V=+7.42$ km
$\omega=293^{\circ}.42$	

In the equations of condition:

$x=\delta V$	$v=[2.989]\ \delta T$
$y=\delta K$	$w=[8.470]\ \delta\omega$
$z=[4006]\ \delta\mu$	$n=[0.1]\ n.$
$u=[7.901]\ \delta e$	

The resulting normal equations are:

[aa]	[ab]	[ac]	[ad]	[ae]	[af]	[an]
$+32.000x$	$+2.196y$	$-1.416z$	$-3.071u$	$-0.603v$	$+0.004w$	$+0.513=0$
	$+13.379$	-1.255	-5.039	$+0.651$	$+0.842$	$+0.580$
		$+8.175$	$+0.109$	$+8.901$	$+9.834$	-0.525
			$+14.933$	-0.351	-0.753	$+0.170$
				$+12.511$	$+13.587$	-0.557
					$+15.012$	-0.579

from which

$\delta V=-0.126$	$\delta e=-0.0438$
$\delta K=-0.511$	$\delta T=+0.486$
$\delta\mu=+0.00459$	$\delta\omega=-7.62.$

FINAL ELEMENTS

OLD SERIES (1899)		NEW SERIES (1905)	
$U=20^{d}538$		$20^{d}546$	
$\mu=17^{\circ}.529$	± 0.014	$17^{\circ}.521$	± 0.006
$T=$ J. D. 2414571.81	± 0.37	2416683.46	± 0.39
$\omega=336.2$	± 7.6	301.0	± 7.6
$K=6.48$ km	± 0.19	7.07	± 0.16
$e=0.132$	± 0.038	0.086	± 0.018
$V=+6.34$ km	± 0.13	7.43 km	± 0.10
$a \sin i=1,810,000$ km		1,990,000 km	
Probable error of single obsn.,	± 0.50 km	± 0.51 km	

The change in elements, if real, may account for the rather large probable error of a single plate. The excellent lines in this star would lead one to expect somewhat better results. The large probable errors of the elements are due in a measure to the small range in velocity of the star.

By applying a constant correction to one series and changing the period to $20^{d}.523$ the observations can be satisfied, after a fashion, but there is no apparent reason why either step should be taken. At best the probable error is materially increased and the first observation is not represented. A series of plates taken after the lapse of a few years would no doubt establish or disprove the reality of the change

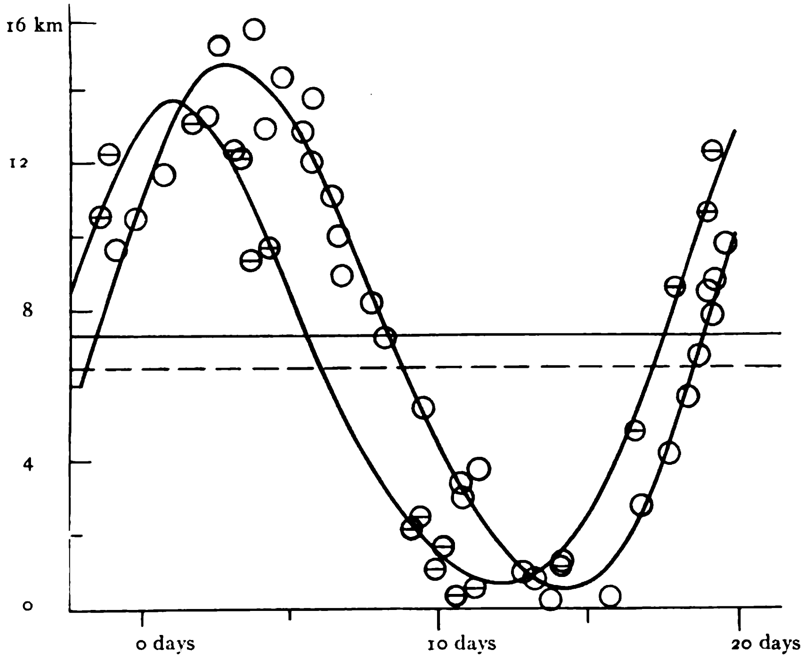

FIG. 1.—Barred circles represent observations of the first series; open circles those of the second series. The dotted line is the velocity of the center of mass for the old series; the continuous line that for the new series.

in elements. Such a series should be taken in a single cycle of $20^{d}.5$ if possible, in order to eliminate the effects of changing elements and error in period.

The discrepancy of the two sets of elements, if established, will no doubt be found to be due to a third body It is quite likely the line of apsides will be found to rotate, as the difference between ω in the two sets is some five times the probable error. The variation in V is

probably real, as the check-plates show no decided difference between plates taken with the first and second Mills spectrograph.

In the figure the *diameter* of the small circles represents the probable error of a single plate. The epochs for the velocity-curves are those given under the heading "final elements."

Lick Observatory,
Mount Hamilton, October 10, 1906.

THE TEMPERATURE OF THE MOON

By FRANK W. VERY

In a paper with the above title presented at the Ithaca meeting of the Physical Society, June 29 to July 3, 1906, of which an abstract appears in the *Physical Review*,[1] Mr. W. W. Coblentz announces the discovery that various common minerals which reflect only diffusively in the region of the spectrum of shorter wave-length than 8 μ have "bands of metallic reflection from 8.5 to 10 μ," and he draws the conclusion that "the Earth and the Moon may be considered selectively reflecting surfaces with a band of metallic reflection from 8.5 to 10 μ."

Admitting the value of this part of his conclusion, the next step of the argument does not necessarily follow. "The radiation from the Sun," he asserts, "is so much more intense than that of the Moon that, with a reflecting power of 50 to 90 per cent., at 8 to 10 μ one would expect the amount of reflected energy to be, at all times, greater than the direct radiation from the Moon." On the assumption that the extreme infra-red maximum in the energy-curve of the lunar spectrum is due to metallic reflection, Dr. Coblentz arrives at the result that "Langley's first conclusion of a lunar temperature of—225° C. again obtains."

This extension of E. F. Nichols' discovery[2] that quartz has bands of metallic reflection beyond 8 μ by which Dr. Coblentz proves that the property is shared by various common silicates, is of interest as a physical fact, but it cannot be accepted as proving "the radiation from the Moon at 8.5 μ to 10 μ to be reflected from the Sun," nor as indicating that the temperature of the sunlit Moon is about 225° C. below zero, for the following reasons:

1. If the Moon reflected specularly, there should be a maximum of heat in the lunar image at first quarter at a point seven-tenths of the distance from the center of the lunar disk to the west limb. Instead of this, the maximum heat is at the west limb, where it should

[1] 23, 247, September 1906.

[2] *Physical Review*, 4, 297, 1897.

be if the Moon radiates as a hot body, heated by the Sun's rays. Indeed, a comparison of the isothermals of the lunar surface in Figs. 11 and 12 of my *Prize Essay on the Distribution of the Moon's Heat and its Variation with the Phase*,[1] where the Moon's age is respectively 7.7 and 10.6 days, suggests that possibly the maximum point may be a little on the farther side of the Moon at first quarter.

2. The spectral energy-curve of the Moon's radiation is not a band from 8.5 to 10 μ but a continuous spectrum. After elimination of a portion of shorter wave-length than 3 μ which agrees in form with the corresponding solar curve, and is obviously reflected, the lunar curve exhibits a similarity in its limits, and when corrected for the absorption by the Earth's atmosphere, it also shows a similarity in its form with the spectral energy-curve of a heated body.

It is true, the observed lunar curve has its maximum in the region of the bands discovered by Nichols and Coblentz, but this curve is obtained after the radiation has suffered an enormous absorption by atmospheric water-vapor in the region of the great water bands between 5 and 8 μ. These bands transform an original maximum in the lunar spectrum, if it could be observed outside our atmosphere, into a minimum. When corrections for atmospheric absorption are applied, the restored curve is found to be that appertaining to bodies not much below the temperature of boiling water; and as the measurements which have been made in the spectrum are integrations including the radiations from extensive areas on which the Sun is shining at a low angle, and which are much below the maximum temperature, the latter may easily exceed 100° C., as I have shown it probably does in my paper on "The Probable Range of Temperature on the Moon."[2]

These statements may be proved in another way, by measuring the radiation at selected points in the lunar spectrum beyond 10 μ, where the atmospheric absorption does not interfere very much. At such points in the lunar spectrum, although the energy is small and not easy to measure, there is substantial agreement with the radiation of screens containing hot water.

3. The Moon reflects diffusely about one-eighth of the solar rays

[1] Published by the Utrecht Society of Arts and Sciences, The Hague, 1891.

[2] *Astrophysical Journal*, 8, 199 and 265, 1898.

between 0.3 and 3.0 μ, giving a spectral energy-curve of reflected radiation about half as high as the observed extreme infra-red maximum. But at 8.5 to 10 μ, the energy in the solar spectrum is hardly one-hundredth part as great as at 1 μ. Theory as developed by Wien, Paschen, and others, gives a ratio much smaller than even this, a point on which I have commented in my paper on "The Solar Constant."[1] Taking the measured ratio of intensities, however, a lunar maximum at about 9 μ, if due to solar rays of that wave-length specularly reflected, should bear a ratio to the diffusely reflected maximum of shorter wave-length, determined by this ratio of original intensities in the given regions of the solar spectrum, by the ratio of reflecting powers which may approach 1:8,[2] and by the ratio of the solid angle filled by the Sun to the hemisphere, or approximately

$$(\tfrac{1}{100}\times 8\times 100{,}000):1=8{,}000:1.$$

Owing to the roughness of the Moon's surface, it cannot reflect like a polished spherical mirror, and an unknown correction for the spreading of the specularly reflected rays remains to be applied; still, there is, no doubt, abundant radiant energy to affect an instrument of small aperture if sufficiently sensitive. A search for this small solar image in rays of 8.5 to 10 μ wave-lengths should be made at the proper point in the lunar image at a distance from the center of the disk,

$$V=\cos \tfrac{1}{2} \text{ Moon's elongation},$$

with a delicate heat-measuring instrument, covered by a screen with a pin-hole aperture. There is a strong probability that it will be found, whenever the experiment is made under the right conditions. But this solar image at a circumscribed point in the lunar image formed by a concave mirror, is not the source of the spectrum observed at all parts of the lunar disk. This spectrum must be due to rays emitted from a heated body.

4. It is not true that the drop in lunar radiation during a lunar eclipse "from a maximum to a minimum in about 1.5 hours . . . . would seem to indicate that the observed radiation is reflected from

[1] *Monthly Weather Review*, August 1901; reprinted as Weather Bureau Publication, No. 254 (see p. 23).

[2] I note that Mr. Coblentz speaks of the reflection of short waves as "practically zero," but this is an exaggeration.

the Sun, since the lapse of time is so short." For, let a mass of rock of considerable size, having a boring in which a thermometer is inserted, be heated in an oven until a nearly uniform temperature of between 100 and 200° C. is attained throughout its mass, and let it then be transferred to a receiver (preferably vacuous, to more nearly resemble lunar conditions) surrounded by a freezing mixture. I have never made the experiment in exactly that form, but have approached near enough to it to know that in an hour and a half a surface pellicle of rock will have cooled until it radiates only feebly, while, nevertheless, the interior of the mass is still hot. Under these circumstances, if the body is put back into the oven, its surface temperature will be restored quickly, because only a thin layer has to be reheated.

Now, this is just what happens on the Moon during an eclipse. It has taken two weeks of sunshine to produce the high temperature of the full Moon; but after the eclipse is over, the radiation is not much smaller than before. A few hours of sunshine replenish the thin layer cooled during the eclipse.

I have made several measurements of the heat derived from the image of the eclipsed Moon. In one of these "the umbra of the eclipsed Moon was found sufficiently hot to appreciably affect a bolometer covering as little as one thirty-fourth of the Moon's disk, by an amount equal to about 1 per cent. of the heat which was to be expected from the full Moon, the radiations being of a quality to which glass was impermeable."[1] Other observations showed a progressive diminution of this small remnant during totality.

In my "Photometry of a Lunar Eclipse"[2] it is shown that the light of the eclipsed Moon may be as small as

$$0.0000000042$$

of that from the full Moon. These facts prove that there is no resemblance between the behavior during an eclipse of the infra-red rays from the Moon of longer wave-length than 4μ, and of those rays which are undoubtedly reflected from the Sun.

WESTWOOD, MASS.,
October 1906

[1] *Prize Essay on the Distribution of the Moon's Heat*, p. 40.

[2] *Astrophysical Journal*, **2**, 293, 1895.

ON THE CONNECTION BETWEEN DISTURBED AREAS OF THE SOLAR SURFACE AND THE SOLAR CORONA

By A. L. CORTIE

That there exists a general connection between the state of disturbance of the Sun's surface and immediate surroundings, and the solar corona, is well established. There is a great similarity in type in the photographs of the solar corona of the years 1870, 1882, 1893, and 1905, all of them years of maximum solar activity, marked by many and great outbursts of sun-spots and prominences. A quite different type of corona corresponds to the years of minimum solar activity, as witness the photographs of 1878, 1889, and 1901; while yet again a type of corona intermediate between those of years of maximum and minimum solar activity is associated with the years of waning sun-spots and prominences, 1886 and 1896. But it is open to question whether the sun-spots or the prominences are the more intimately connected with the streamers which constitute the characteristic forms of these variations in coronal type. In the Stonyhurst photographs of the solar corona of 1905 August 30, taken at Vinaroz, Spain, the streamers seem in general to mark the regions of the prominences rather than of the sun-spots. The longest streamers photographed, consisting of two long wings with an intermediate shorter streamer, extend from latitude $-40°$ to $-90°$ S., outside the limits of the spot zones, while another streamer is placed near the Sun's north pole. Moreover, the general tendency of all the more marked streamers is poleward, while the fine group of prominences in the northeast quadrant is manifestly associated with three fine streamers. In the lower corona the complicated structures of arches and vortex rings are attached to the prominences. In this connection it will be interesting to quote from Mr. Evershed's account of the prominences of 1905:[1] "The polar regions of the Sun have again been nearly quiescent, although the high latitude zones of activity have made a distinct advance toward the poles. These prominences were found to be most frequent between the parallels 65° and 70° in both hemispheres, and the limits of promi-

nence-formation at the close of the year may be put at $+75°$ and $-75°$."[1] It would seem that the poleward tendency noticed in the streamers of the corona, in the main, was also characteristic of the prominences observed during the year, which again were most frequent in latitudes greater than those of the spot-zones.

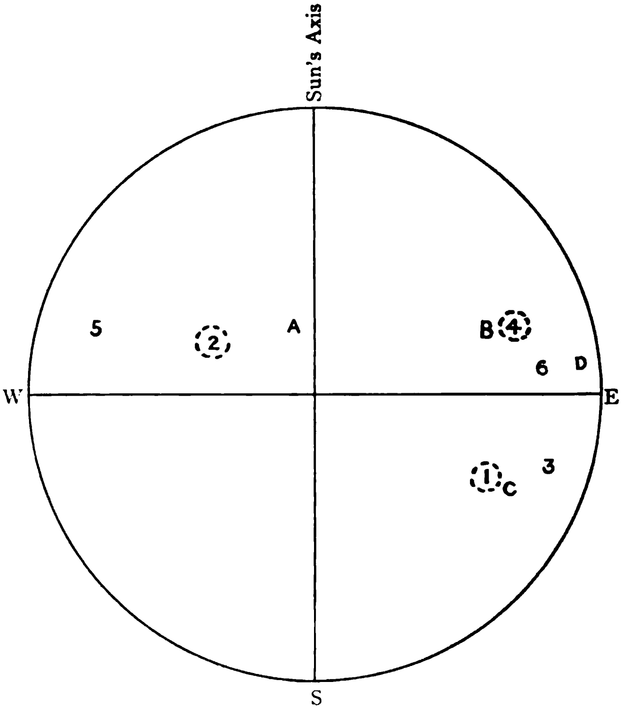

FIG. 1.—Positions of centers of sun-spot activity 1905, August 30. A, B, C, D, are positions of spots visible on the day of eclipse. The 1 within a dotted circle is the sun-spot center, January 5–July 14.

In the photographs of the corona of 1905 August 30, the sun spot zones are marked by some straight radial bright rays in the southwest quadrant, and by some beautiful plumes in the south-east quadrant.[2] These rays and plumes are well shown in the third

[1] *Monthly Notices*, 66, 227, February 1906.

[2] See Capt. Carpenter's drawing from the photographs taken by the party from the U. S. Naval Observatory. *Astrophysical Journal*, 23, Plate IX, opposite p. 13c. 1906.

plate of the Stonyhurst series, exposed for 15 seconds near the beginning of totality on a Schleussner's "Observatory" plate. The instrument employed was the full aperture of the 4-inch lens of the 20-foot coronagraph belonging to the Royal Irish Academy. In the discussion of the results of the eclipse, presented to the Royal Irish Academy, it was shown that the area on the solar surface to which the projected images of these plumes converged was situated in the southeast quadrant, the mean latitude of the position being approximately $-21°$ S. Four sun-spot groups were visible on the Sun's surface on the day of the eclipse, and the largest of the four, marked C. on the diagram, with an area of one-thousandth of the Sun's visible disk, had a mean latitude $-20°$ S., and was removed about 41° from the Sun's east limb. Moreover, of the two limbs, the east limb on the day of eclipse had the greater number of sun-spot centers of greater activity during the year in its vicinity, including among them, though on the invisible hemisphere of the Sun and removed some 51° from the limb in latitude $-16°$, that center which had been the seat of the outburst of the great sun-spot group of February 1905, the greatest observed for thirty years. If the plumes converged to an area behind the visible disk, then it was possible that they were connected with the great February spot; if to an area on the visible hemisphere, the possibility was that they were attached to the spot seen on the day of the eclipse, especially as it was near the place of a large group, marked 3 on the diagram, visible to the naked eye during the preceding July. Since writing the memoir for the *Transactions* of the Royal Irish Academy, the life-histories of the spots and faculae that occupied these two regions of disturbance have been carefully studied from the Stonyhurst sun-spot drawings. The results of this study are set forth in the following tables, and the discussion which accompanies them. The measures, unless otherwise stated, have been made on the Stonyhurst drawings.

After this date the region becomes quiescent and remains so during the period of the eclipse. In the next table the region of a great spot visible during July is similarly treated, the spot marked 3 on the diagram.

TABLE I

1905 Region of Disturbance on the Sun Connected with the Great February Spot

Appearance	Date, 1905	Mean Longitude	Mean Latitude	Remarks
I	January 5.42	319°52	−18°60	A small spot, built of great spot.
	January 11.44	322.63	−18.39	A group of small spots.
II	January 31.40	329.30	−17.50	A very large spot.
III	March 2.46	319.34	−18.75	A fairly large spot.
IV	March 27.66	324.35	−16.30	The spot very much smaller.
	April 6.33	320.71	−20.50	April 5 a single dot; April 6 positions of two dots in bright faculae near W. limb.
		318.71	−26.50	
V	May 3.48	325.75	−22.96	Faculae near W. limb.
VI	May 29.66	320.88	−26.40	Very faint faculous patch near W. limb.
VII	June 15.29	331.19	−22.20	Small spot in faculae near the position of the great spot, died on June 21.
	June 25.34	327.66	−25.84	Bright faculae in region of small spot.
VIII	July 14.32	320.51	−29.74	Bright faculae E, about same region as that of June 25.

TABLE II

1905. Region of Disturbance on the Sun Connected With a Great July Spot

Appearance	Date, 1905	Mean Longitude	Mean Latitude	Remarks
I	May 17.34	68°31	−15°23	Leading spot of an extended large group.
		53.81	−16.23	Following spot of the same group.
II	June 15.20	70.25	−16.80	Leading spot of the group.
		53.25	−16.21	Following spot of the group.
III	July 14.32	52.52	−15.65	Center of fine group of scattered spots visible to the naked eye. Position of following spot of former group.
IV	August 6.55	61.24	−16.78	Center of a large spot.
V	August 30.33	60.20	−18.70	Biggest spot on day of eclipse. Greenwich measure.

In studying this group, a set of measures, kindly supplied to me by Mr. Maunder, of Greenwich, has been of great assistance for purposes of identification. This region was disturbed, according to the Stonyhurst drawings, from May 11 and was active on August 30, the day of the eclipse, and afterward. Therefore it follows that the connection, if any, between the coronal plumes and a disturbed area on the Sun must be postulated of this second area. Is the connection merely one of coincidence of position, and not a real con-

vergence of the plumes to the spot area? The probability of the reality of the connection seems greatly enhanced by the study of a similar region in 1893, which was the area of convergence of a like system of rays or plumes in the lower corona of April 16. The material used in this last table has been taken from the Greenwich volume of ***Photographic Results*** of that year. The comparison is made with Professor Schaeberle's discussion of the eclipse photographs in the "Report of the Total Solar Eclipse of April 16, 1893" (*Contributions from the Lick Observatory*, No. 4).

TABLE III

1893. INTERMITTENTLY DISTURBED REGION OF THE SUN, MARCH 15–NOVEMBER 16

Appearance	Date, 1893	Greenwich Number	Mean Longitude	Mean Latitude
I	March 15–21	2881	42°.10	−8°.16
II	April 5–18	2914	44.56	−8.46
III	May 3–10	2959	45.86	−7.54
IV	Sept. 17–27	3187	49.30	−8.08
V	Nov. 12–16	3275	47.02	−8.26

This area, which was continuously active from a month before the eclipse to nearly a month afterward, and again broke out into disturbance in September and November, was placed on the eclipse day about 34° from the Sun's west limb with position angle 237°. In Professor Schaeberle's Skeleton Diagram of Corona, Plate IX, facing p. 89 (*loc. cit.*), is an object marked π, of which we read (p. 100, *loc. cit.*): "The heaviest of a number of much inclined and conspicuous streamers issuing between P. A. 230° and P. A. 250° of the Sun's outline . . . . on account of the numerous crossing of streamers from greater position angles the individual directions cannot be ascertained with certainty. At about P. A. 210° the bunching up from intensities is so great that from height $=10'$ to height $=15'$ an apparent increase of density in the corona takes place." In his discussion of results Professor Schaeberle distinguishes coronal arches and coronal streamers; but, so far as can be made out from a comparative study of the eclipse photographs of Professor Schaeberle and those of the English observers, Mr. Kearney and Mr. Taylor, the structure described in the above quotation is very like that named as plumes on the Stonyhurst plates of 1905.

Without subscribing entirely to the mechanical theory of the solar corona as advocated by Professor Schaeberle, and above all to the statement (p. 126, *loc. cit.*) that "there is no evidence to indicate that either electricity or magnetism has anything to do with the arrangement of coronal matter in the Sun's neighborhood;" and, moreover. admitting the sorting effect on the lighter materials ejected from the Sun due to the pressure of light, yet, in any discussion of the constitution of the corona, the great part played by the ejective forces in the Sun, actively displayed in sun-spots and prominences, must be fully admitted, and any theory which rejects or overlooks the effect of such forces must be regarded as incomplete. The present paper gives highly probable evidence that not only in general is a characteristic type of solar corona associated with sun-spot and prominence activity, but that definite structures in the corona are associated with definite areas of activity of sun-spots and faculae.

STONYHURST OBSERVATORY,
July 27, 1906.

MINOR CONTRIBUTIONS AND NOTES

VARIABLE RADIAL VELOCITY OF δ *CAPRICORNI*

The following observations of the velocity in the line of sight of δ *Capricorni* show it to be a spectroscopic binary of rather short period.

1906, September	25,	Radial	Velocity,	—30 km
October	3,	"	"	—79

The spectrum of this star belongs to Miss Maury's class *Xa,b*. The lines, although numerous, are rather broad and not suited to accurate measurement.

V. M. SLIPHER.

LOWELL OBSERVATORY,
October 5, 1906.

Reviews

STARK'S THEORY OF RADIATION.

During the present year considerable work on the radiation of light by canal rays has been done by J. Stark and his co-workers[1] at Göttingen. Their experimental results are interesting, and some of the conclusions which Stark draws from them, if justified, are of very great importance.

It has been known for some time that canal rays consist of positively charged particles, whose mass is of the order of magnitude of that of the atom of the gas in which they are produced; that they move with a high velocity (10^7 cm sec., or more); and that light is emitted by that part of the tube through which they move. If this light, wholly or in part, comes from the moving particles, it should exhibit the "Doppler effect" when viewed in a direction parallel to the motion. Stark has observed the spectrum of the canal rays in hydrogen, nitrogen, potassium, and mercury, and finds the effect only in case of line-spectra, the band-spectra showing none whatever. This indicates that line-spectra owe their origin to the positive ion. He finds also that all the lines belonging to the same series exhibit the same Doppler effect, while different series may show the effect in different amounts. In mercury, for example, the series of triplets give roughly twice the displacement shown by certain other lines which he arranges in a series of doublets; while the line λ 4078.1 gives a displacement greater than that of the triplets. Assuming the mass of the moving particle as that of a mercury atom, knowing the cathode fall of potential in the tube, and measuring the Doppler effect, he finds that the series of triplets is emitted by atoms which have lost two electrons; his series of doublets, by atoms which have lost only one; while the line λ 4078.1 is emitted by atoms which have lost more than two electrons. The hydrogen series and the principal series of potassium, both of which consist of doublets, are similarly shown to be due to atoms which have lost one electron.

Stark assumes that band-spectra owe their origin to the system composed of the positive ion and one or more electrons when in the act of recombining to form a neutral atom, and he supports this idea by some very ingenious experiments with a mercury tube.

Another question taken up by Stark, one of fundamental importance, is the cause of those vibrations of the electrons which give rise to light-

[1] *Physikalische Zeitschrift*, 6, 892, 1905; 7, 249, 251, 353, 355, 564, 567, 1906.

waves. The most natural assumption to make is that the collision or its equivalent which expels an electron from the neutral atom, thus forming the positive ion, sets up violent oscillations of the remaining electrons, and that these gradually lose this energy by radiation until another collision gives them a fresh supply. He points out that if this be the only cause of radiation immediately behind the cathode the slower canal particles ought to radiate more energetically than the faster ones; for the latter are evidently formed farther in front of the cathode than the former, and hence just behind the cathode the fast moving ones have been radiating for a longer time, and the amplitude of the vibration of their electrons would accordingly have decreased more than in the case of the slower ones. Besides, positive ions are evidently also formed in the negative glow, and hence pass through the Crookes dark space; this dark space ought then to show the line-spectra, which it does not at all in case of hydrogen. Stark's photograph also shows that between the normal line and the displaced line, there is a decided minimum of intensity, indicating that the slow particles are very feeble radiators.

To explain these facts, Stark suggests that the cause of the radiation is to be sought in the velocity of the ion with reference to the ether. Some idea of the mechanism of this may be formed by supposing that the "elastic forces" acting on the electron inside the ion depend upon the relative motion of the ion and the ether. His observations indicate that this force is diminished with increasing relative velocity; and, furthermore, that the force in the direction of motion is diminished more than at right angles to the motion. If this is so, then it is quite conceivable that when an ion is in rapid motion relative to the ether, its electrons may be caused to oscillate in such a manner as to send out light-waves. The pressure of the light emitted by such an ion will decrease its velocity, and by falling on other atoms of the gas will increase their velocity if this be less than that of the radiating ion; here is, then, a cause which tends to produce an equalization of the internal energy of the ions and their energy of translation.

Stark's spectrograms show that the minimum of intensity between the normal line and the displaced line is wider the shorter the wave-length, which indicates that the velocity necessary for the radiation of the shorter wave-lengths is greater than for longer wave-lengths, other things being equal. In case of mercury, the line λ 2536.72 (one of his series of doublets) shows a very narrow minimum, the triplets show a wider minimum, and the line λ 4078.1 a still wider one. The line λ 2536.72 should accordingly appear with relatively low velocity of the ions (low mean tempera-

ture), the triplets should require a somewhat greater velocity (higher temperature), and the line λ 4078.1 a still greater velocity. This has actually been verified, since the line λ 2536.72 may be seen in the Bunsen flame, while a considerable pressure in the mercury arc is necessary to give the line λ 4078.1 with any intensity.

That emission of light as a result of collisions is possible is evident from the fact that all the spectrograms show the lines in their normal positions as well as the displaced lines. Stark seems to regard this as "irregular radiation" or "luminescence," while the radiation due to translation is regarded as "regular" or "pure temperature radiation."

J. A. ANDERSON.

JOHNS HOPKINS UNIVERSITY.

Resolution of Spectral Lines by Means of Interference Points Produced by Plane-parallel Plates.[1] By E. GEHRCKE and O. VON BAEYER. (Communication from the *Physikalisch-Technische Reichsanstalt.*)

Since the advent of spectroscopic apparatus of high resolving power, the structure of spectral lines has been made the subject of numerous investigations. A comparison of the results obtained with different types of apparatus showed such poor agreement among themselves that it was quite impossible to decide upon the number of components accompanying well-known spectral lines. The discrepancies and contradictions were especially marked when the results obtained with the Lummer and Gehrcke[2] interference plates were compared with those obtained by means of other forms of interference apparatus. As, for example, such lines as the cadmium red (λ 643.9 $\mu\mu$) which the echelon[3] and the Fabry and Pérot[4] interferometers gave as single, appeared to be accompanied by a large number of components when analyzed by means of the Lummer and Gehrcke plate. It was suggested by Fabry and Pérot[5] that many of these supposed components might, in reality, be nothing but ghosts occasioned by imperfections in the interference plates. From all this it is obvious that some criterion by means of which it would be possible to decide upon the position and number of components was highly desirable. In their efforts

[1] *Annalen der Physik*, (4) **20**, 200–202, 1906.

[2] *Ibid.*, (4) **10**, 457–477, 1903.

[3] L. Janicki, *ibid.*, (4) **19**, 36–70, 1906.

[4] *Annales de Chim. et de Phys.*, (7) **22**, 564, 1901.

[5] *Journal de Physique*, (4) **3**, 28–32, 1904.

to find such a criterion the authors were led to the following considerations:

Let it be supposed that a parallel beam of monochromatic light of wave-length λ be first reflected from a Lummer and Gehrcke plate lying in a horizontal position and then from a similar plate standing vertically. The resulting phenomenon as viewed in a telescope would be seen to consist of a number of "interference points" arranged in horizontal and vertical rows. Let it first be assumed that the horizontal plate (I) be perfect, and that the other plate (II) be defective in such a manner as to produce a ghost. The resulting interference pattern would take the form

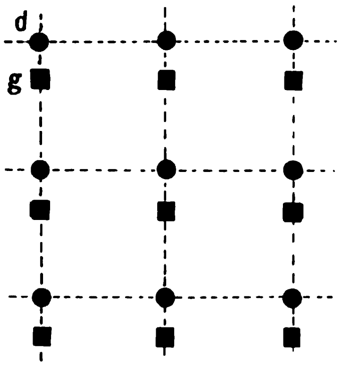

FIG. 1

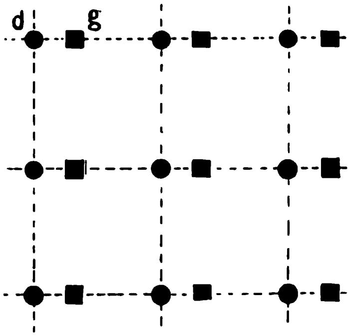

FIG. 2

shown in Fig. 1, where the circular disks *d* represent true interference maxima, while the squares *g* represent ghosts. If on the other hand Plate II be perfect and Plate I defective, it is obvious that the condition of affairs shown in Fig. 2 would exist. This means, then, that ghosts will appear only in the horizontal and vertical rows and not along diagonals. The only condition under which a ghost could appear on a diagonal would be in case the imperfections in both plates were absolutely identical. This state of affairs could, however, be detected at once by the concomitant existence of points in the vertical and horizontal rows completing the rectangle.

If, now, the source of light be changed to one giving rise to the wave-lengths λ and $\lambda+d\lambda$ (both plates being considered perfect), the resulting arrangement of interference points would be that shown in Fig. 3. Here the shaded disks *c* represent the interference maxima of the components $\lambda+d\lambda$. Thus it is evident that true components are not accompanied by corresponding points in the vertical and horizontal rows. To show how

this arrangement would work under the conditions actually met with in practice, let it be supposed that, as before, the two wave-lengths λ and $\lambda+d\lambda$ are present in the incident light and that both plates be defective—each giving rise to one ghost. In such a case the interference points would be arranged as in Fig. 4. Here d represents an interference point due to λ; g and g_2 are the ghosts of this point; c is the interference point of the component $\lambda+d\lambda$, and g'_1 and g'_2 are its ghosts. From the fact that there are no points on AB and FG completing the rectangle of which d and c are diagonally opposite corners, it is clear that c can be put down as a *true* component to the exclusion of g_1, g_2, g'_1, and g'_2.

In order to test these inferences the authors set up an interference

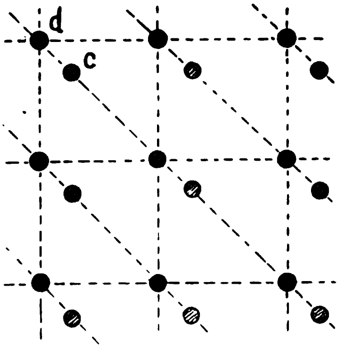

FIG. 3

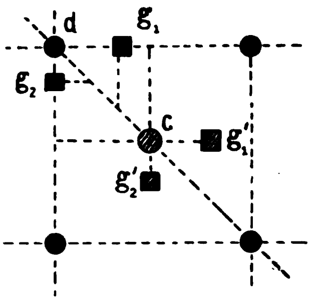

FIG. 4

apparatus of the kind described, using plates which were known to be defective. The magnitude of these defects is shown in the following table;

Plate	μ	Length in cm	Thickness in cm	Variations in Thickness ($\lambda=436\ \mu\mu$)
A.	1.53	20.0	0.908	$1\frac{1}{2}\lambda$
B	1.53	12.5	0.205	$\frac{1}{2}\lambda$
C	1.51	20.0	0.503	$\frac{1}{4}\lambda$
D.	1.53	15.0	0.317	$<\frac{1}{8}\lambda$

In order to obtain photographic records in a reasonable length of time, the authors were forced to use Heraeus quartz lamps containing various metals. The remarkable consistency of the results obtained with different plate combinations is shown in the following tables, chosen at random from a long list of results presented by the authors. In addition, the best results obtained with the echelon by Janicki and those obtained by Fabry and Pérot are given for the sake of comparison.

GREEN MERCURY LINE
(λ 5461)

Gehrcke and Baeyer			Janicki	Pérot and Fabry
Plate A and B	Plate B & C	Plate C & D	Echelon	Silvered Plates
−2.40*	−2.42	−2.41	−2.32	−2.24
−1.20	−1.11	−1.03	−0.99	−0.76
−0.72	−0.71	−0.55	−0.66	−0.52
				+0.08
+0.81	+0.88	+0.93	+0.88	+0.82
+1.25	+1.37	+1.40	+1.33	+1.36

* The results are expressed in terms of 10^{-3} $\mu\mu$.

In accord with all previous observations, the authors found no components accompanying the red cadmium line (λ 6439).

To show the effectiveness of this new method of telling true components from ghosts, the authors state that in their examination of the green mercury line (λ 5461) by means of plates B and C they were able to count at least nine ghosts—and still the position and number of true components as obtained with these plates is almost identical with that obtained by means of entirely different forms of interferometers. As the object of this review is merely to present the principles involved, it will be necessary to refer to the original article for a detailed account of the results obtained.

The authors further point out that their method which is simply a modification of the well-known "crossed prism method," might equally well be applied to the grating and echelon. By their very ingenious application of this method the authors have most assuredly proved their point; and it must be highly gratifying to all spectroscopists to feel that it is at last possible to determine with definiteness the structure of spectral lines.

A. H. PFUND.

JOHNS HOPKINS UNIVERSITY.

A Popular Guide to the Heavens: A Series of Eighty-three Plates with Explanatory Text and Index. By SIR ROBERT S. BALL. 8vo, pp. 95, with 83 plates and 12 key-maps. London: George Philip & Son, 1905. 15s.

This volume is the result of a revision of the author's *Atlas of Astronomy*, published in 1892. Besides replacing many of the old illustrations with examples of more recent work, many new plates have been added. The new subjects are well chosen and are excellently reproduced. It is to be regretted that all of the drawings of the Moon have been retained.

The exaggerated contrast and unlikeness to what is actually seen in a telescope makes them appear grotesque, and it is doubtful if they have any advantage for purposes of identification over the three photographic reproductions added to the new edition. There are nineteen lunar plates altogether, accompanied by fifteen full-page key-maps. Half this number would have been ample.

Among many features valuable to the amateur and the general reader are to be especially noted the index to planetary phenomena for the next fifty years (fifteen years would have been sufficient), and the star maps. The latter are admirably adapted for ease in identification of the constellations and the brighter stars. While the book is of exceptional value to the amateur and the general public, it will also be found a useful addition to the professional library. S. B. B.

Fünfstellige mathematische und astronomische Tafeln. Von F. BIDSCHOF und A. VITAL. Wien und Leipzig: Franz Deuticke, 1905. Pp. 219. M. 7.50.

These tables are particularly intended for the use of astronomers, geographers, and mariners. The logarithms of the numbers from 1 to 10,000 are so arranged that the arguments are also expressed in arc or time, as in many of the older tables. Addition and subtraction logarithms follow. The trigonometric tables depart from the usual arrangement in adding the values for the cosecant and secant. A total of fifty-eight tables is given, of which about fifty would be of frequent service in astronomical work. Among these may be named Bessel's refractions, table for reducing circum-meridian altitudes, and for latitude from *Polaris;* a compressed table for computing precession, with values for 1900; tables for approximate solution of Kepler's equation, for computing the true anomaly in the parabola. The last fourteen pages of this convenient work contain a summary of the principal formulae of spherical, practical, and theoretical astronomy.

The Royal Society, or Science in the State and in the Schools. By SIR WILLIAM HUGGINS. London: Methuen & Co., 1906. Pp. 131, with 25 illustrations. 4*s.* 6*d.*

This handsome volume appropriately records in a form adapted to a general circulation, four of the official addresses of Sir William Huggins during his five years of service as president of the Royal Society. A preliminary chapter gives an interesting account of the early history of the

society. The topics of the four addresses are: (1902) Supreme Importance of Science to the Industries of the Country, which Can Be Secured only Through making Science an Essential Part of All Education; (1903) The Relation of the Royal Society to the Specialized Scientific Societies; (1904) The Advisory Relation of the Royal Society to the State, and the Responsible Public Duties which Rest Permanently upon the Society; (1905) The Profound Influence which Science, Represented by the Royal Society, Has Had upon the Life and Thought of the World; and the Place of Science in General Education; Science in Education.

The book is embellished with several illustrations due to the accomplished hand of Lady Huggins, with halftone reproductions which include the death mask-of Sir Isaac Newton, and his original reflecting telescope, and with excellent portraits "of ten Fellows of immortal fame in science."

ERRATA

Astrophysical Journal, Vol. **23**, June 1906, in Professor Vogel's article on "Reflecting Telescopes of Relatively Short Focus":

Page 372, tenth line from top, *for* where these wave-lengths unite, *read* where the rays of these wave-lengths unite.

Page 373, ninth line from foot, *add* **19**, 21, 1898: H. C. Plummer, "On the Star-Image Formed by a Parabolic Mirror?"

Page 379, ninth line from foot, *for* center, *read* diameter.

Page 382, fifteenth and sixteenth lines from top, *transpose* from the vertex, *so that it shall follow the word* figure.

Page 388, eighteenth line from top, *for* a, *read* the.

Page 388, seventh line from foot, *for* should, *read* could.

Books Received

HERMANN KOBOLD: *Der Bau des Fixsternsystems.* Pp. 256, Figs. 19 and 3 charts. Braunschweig: F. Vieweg & Sohn, 1906. M. 6.60.

L. BOLTZMANN: *Leçons sur la théorie des gaz*, traduites par A. GALLOTTI et H. BENARD, avec une introduction et des notes de M. BRILLOUIN. Seconde Partie. Pp. 280. Paris: Gauthier-Villars, 1905.

L. PFAUNDLER: *Müller-Pouillets Lehrbuch der Physik und Meteorologie.* Zehnte umgearbeitete und vermehrte Auflage. Band I. Pp. 801, Figs. 838. Braunschweig: F. Vieweg & Sohn, 1906. M. 10.50.

O. D. CHWOLSON: *Lehrbuch der Physik.* Dritte Band: *Die Lehre von der Wärme*, übersetzt von E. BERG. Pp. 988, Figs. 259. Braunschweig; F. Vieweg & Sohn, 1905. M. 16.

O. D. CHWOLSON: *Traité de physique*, ouvrage traduit sur les éditions russe et allemande par E. DAVAUX. Édition revue et considérablement augmentée par l'auteur, suivie de notes sur la physique théorique par E. COSSERAT et F. COSSERAT. Paris: A. Hermann, 1906.

Tome premier: Introduction, Mécanique, Méthodes et Instruments de mesure; L'état gazeux des corps. Pp. 559, Figs. 279. Fr. 22.

Tome deuxième: Émission et absorption de l'énergie rayonnante, Vitesse de propagation, Réflexion et Réfraction. L'indice de réfraction, Dispersion et transformation de l'énergie rayonnante. Avec une note de M. de GRAMONT sur lés methodes modernes en analyse spectrale. Pp. 431, Figs. 262. Fr. 16.

R. A. MILLIKAN and H. G. GALE: *A First Course in Physics.* Pp. 493, and portraits of scientists. Boston: Ginn & Co., 1906. $1.40.

R. A. MILLIKAN and H. G. GALE: *A Laboratory Course in Physics for secondary schools.* Pp. 134, Figs. 80. Boston: Ginn & Co., 1906. $0.40.

R. W. WILLSON: *Laboratory Astronomy.* Pp. 189, Figs. 89. Boston: Ginn & Co., 1906. $1.35.

F. R. MOULTON: *An Introduction to Astronomy.* Pp. 557, Figs. 201 and 4 star maps. New York: The Macmillan Co., 1906. $1.25.

W. H. WHEELER: *A Practical Manual of Tides and Waves.* Pp. 201, Figs. 19. London: Longmans, Green & Co., 1906.

T. E. HEATH: *Our Stellar Universe, A Road Book to the Stars.* Pp. 74, Figs. 8. London: King, Sell & Olding, 1905. 5 s. net.

T. E. HEATH: *Our Stellar Universe, Stereoscopic Star Chart and Spectroscopic Key Maps.* Pp. 26, with 25 maps and 26 stereoscopic star charts. London: King, Sell & Olding, 1906. 10s. net.

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AUTHORS